Accurate Split Domination, Accurate Non - Split Domination of a Graph

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Abstract

In this paper, we define the notions of accurate split and non-split domination in a graph. We get many bounds on accurate split and non-split domination numbers. Exact values of these new parameters are obtained for some standard graphs. Nordhaus - Gaddum type results are also obtained for these new parameters.

Keywords: Dominating set, split dominating set, non-split dominating set, accurate dominating set, accurate split dominating set, accurate non- split dominating set, accurate split and non-split domination numbers.

1. INTRODUCTION

Graphs are among the most ubiquitous models of both natural and human-made structures. Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs. During the later part of the twentieth century and the beginning of twenty first century the areas of graph theory, computer engineering, and operations research has had an explosive growth. In computer science, graphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc. For instance, the link structure of a website can be represented by a directed graph, in which the vertices represent web pages and directed edges represent links from one page to another.

A similar approach can be taken to problems in travel, biology, computer chip design, and many other fields. The development of algorithms to handle graphs is therefore of major interest in computer science. The transformation of graphs is often formalized and represented by graph rewrite systems. Complementary to graph transformation
systems focusing on rule-based in-memory manipulation of graphs are graph databases geared towards transaction-safe, persistent storing and querying of graph-structured data. A graph structure can be extended by assigning a weight to each edge of the graph. Graphs with weights, or weighted graphs, are used to represent structures in which pair wise connections have some numerical values. For example if a graph represents a road network, the weights could represent the length of each road.

All graphs considered here are simple, finite and undirected. Let \( n \) and \( m \) denote the order and size of a graph \( G \). We use the terminology [4]. Let \( \Delta(G) \) (\( \delta(G) \)) denote the maximum (minimum) degree and \( \lceil x \rceil (\lfloor x \rfloor) \) the greatest (least) integer less (greater) than or equal to \( x \). A non empty set \( D \subseteq V(G) \) is said to be a dominating set of \( G \) if every vertex not in \( D \) is adjacent to at least one vertex in \( D \). A dominating set \( D \subseteq V \) of a graph \( G \) is a split (non-split) dominating set if the induced sub graph \( <V-D> \) is disconnected (connected). The split (non-split) domination number \( \gamma_s(G) \), \( (\gamma_{ns}(G)) \) is the minimum cardinality of a split (non-split) dominating set. A dominating set \( D \) of a graph \( G \) is an accurate dominating set, if \( V-D \) has no dominating set of cardinality \( |D| \). The accurate dominating number \( \gamma_a(G) \) of \( G \) is the minimum cardinality of an accurate dominating set. (V.R. Kulli and M.B. Kattimani [9])

**DEFINITION 1.1.**
**ACCURATE SPLIT DOMINATING SET:**
A dominating set \( D \) of \( G \) is an accurate split dominating set if the induced sub graph \( <V-D> \) is disconnected and it has no split dominating set of cardinality \( |D| \). The accurate split domination number \( \gamma_{as}(G) \) of \( G \) is the minimum cardinality of an accurate split dominating set.

The upper accurate split dominating number \( \Gamma_{as}(G) \) of \( G \) is the maximum cardinality of a minimum accurate split dominating set.

**Example: 1.2**

![Figure 1: Upper Accurate Split Dominating Graph](image_url)
Let $D = \{2, 7\}, D_1 = \{1, 3, 4, 5, 8\}$ are accurate split dominating set.

\[ \therefore \gamma_{as}(G) = 2 \text{ and } \Gamma_{as}(G) = 5 \]

2. EXACT VALUES OF $\gamma_{as}(G)$ FOR SOME STANDARD GRAPHS.

2.1: Observation:
1. For any path $P_n$, for $n \geq 3$

\[ \gamma_{as}(P_n) = \left\{ \begin{array}{ll}
\left\lceil \frac{n}{3} \right\rceil + 1, & \text{if } n = 3p + 1, 3p + 2, p \geq 1 \\
\frac{n}{3}, & \text{if } n = 3p, p \geq 1
\end{array} \right. \]

2. For any cycle $C_n$, for $n \geq 3$

\[ \gamma_{as}(C_n) = \left\{ \begin{array}{ll}
\left\lceil \frac{n}{3} \right\rceil + 1, & \text{if } n = 3p + 1, 3p + 2, p \geq 1 \\
\frac{n}{3} + 2, & \text{if } n = 3p, p \geq 1
\end{array} \right. \]

3. For any wheel graph $W_n$

\[ \gamma_{as}(W_n) = \left\{ \begin{array}{ll}
3, & \text{if } n = 4, 5 \\
4, & \text{if } n = 6, 7 \\
5, & \text{if } n \geq 5
\end{array} \right. \]

4. For any complete bipartite graph $K_{m,n}$

\[ \gamma_{as}(K_{m,n}) = \left\{ \begin{array}{ll}
m, & \text{if } m < n \\
m + 1, & \text{if } m = n
\end{array} \right. \]

5. For any complete graph $K_n$

\[ \gamma_{as}(K_n) = n - 2, \text{ for } n \geq 3 \]

DEFINITION 2.2.

ACCURATE NON- SPLIT DOMINATING SET:

A dominating set $D$ of $G$ is an accurate non-split dominating set if the induced subgraph

$\langle V - D \rangle$ is connected and it has no non-split dominating set of cardinality $|D|$. The accurate non-split domination number $\gamma_{as}(G)$ of $G$ is the minimum cardinality of an accurate non-split dominating set.
The upper accurate non-split dominating number $\Gamma_{\text{ans}}(G)$ of $G$ is the maximum cardinality of a minimal accurate non-split dominating set.

**Example 2.3:**

![Accurate non-split dominating Graph](image)

**Figure 2**: Accurate non-split dominating Graph

Let $D = \{1,2,7,8\}$, $D_1 = \{1,2,3,6,7,8\}$, $D_2 = \{1,3,6,8\}$ are accurate non-split dominating set.

\[ \gamma_{\text{ans}}(G) = 4 \text{ and } \Gamma_{\text{ans}}(G) = 4 \]

### 2.4. Exact values of $\gamma_{\text{ans}}(G)$ for some standard graphs.

**Observation:**

1. For any path $P_n$ for $n \geq 5$
   \[ \gamma_{\text{ans}}(P_n) = n - 2 \]
2. For any cycle $C_n$, for $n \geq 5$
   \[ \gamma_{\text{ans}}(C_n) = n - 2 \]
3. For any complete bipartite graph $K_{m,n}$
   \[ \gamma_{\text{ans}}(K_{m,n}) = 0 \]
4. For any complete graph $K_n$
   \[ \gamma_{\text{ans}}(K_n) = \left\lfloor \frac{n}{2} \right\rfloor + 1 \]
5. For any wheel graph $W_n$
   \[ \gamma_{\text{ans}}(W_n) = 1, \text{ for } n \geq 4 \]

**Theorem 2.5**: For any graph $G$

(i) $\gamma_a(G) \leq \gamma_{\text{ans}}(G)$

(ii) $\gamma_a(G) \leq \gamma_{\text{ans}}(G)$
**Proof:** Every accurate split dominating set of G is an accurate dominating set of G,
We have $\gamma_a(G) \leq \gamma_{as}(G)$

Every accurate non-split dominating set of G is an accurate dominating set of G,
We have $\gamma_{a}(G) \leq \gamma_{ans}(G)$

**Theorem 2.6**
For any graph G, $\gamma_a(G) \leq \min(\gamma_{as}(G), \gamma_{ans}(G))$

**Proof:** Every accurate split dominating set and every accurate non-split dominating set of G are an accurate dominating set of G,
We have $\gamma_a(G) \leq \gamma_{as}(G)$ and $\gamma_{a}(G) \leq \gamma_{ans}(G)$

Hence $\gamma_a(G) \leq \min(\gamma_{as}(G), \gamma_{ans}(G))$

**Theorem 2.7:** For any graph G, (i) $\gamma_{a}(G) \leq \gamma_{as}(G)$

(ii) $\gamma_{as}(G) \leq \gamma_{ans}(G)$

**Proof:** Every accurate split dominating set of G is split dominating set of G,
We have $\gamma_s(G) \leq \gamma_{as}(G)$

Every accurate non-split dominating set of G is a non-split dominating set of G,
We have $\gamma_{as}(G) \leq \gamma_{ans}(G)$

**Theorem 2.8:** For any graph G,

(i). $\gamma(G) \leq \gamma_{as}(G)$

(ii). $\gamma_{a}(G) \leq \gamma_{ans}(G)$

**Proof:** Every accurate split dominating set and every accurate non-split dominating set of G are dominating sets of G,
We have $\gamma(G) \leq \gamma_{as}(G)$ and $\gamma(G) \leq \gamma_{ans}(G)$

Hence $\gamma(G) \leq \min(\gamma_{as}(G), \gamma_{ans}(G))$.

**Proposition 2.9:** For any graph G and $\overline{G}$ with no isolated vertices,

(i) $2 \leq \gamma_{as}(G)$ (ii) $2 \leq \gamma_{as}(\overline{G})$ (iii) $2 \leq \gamma_{ans}(G)$ (iv) $2 \leq \gamma_{ans}(\overline{G})$
Proposition 2.10: For any graph G and $\overline{G}$ with no isolated vertices,

(i) $\gamma_{as}(G) \leq n - 2$  (ii) $\gamma_{as}(\overline{G}) \leq n - 2$  (iii) $\gamma_{ans}(G) \leq n - 2$  (iv) $\gamma_{ans}(\overline{G}) \leq n - 2$

3. NORDUAUS – GADDUM TYPE RESULTS

Theorem 3.1: For any graph G and $\overline{G}$ with no isolated vertices,

(i) $4 \leq \gamma_{as}(G) + \gamma_{as}(\overline{G}) \leq 2(n - 2)$  
(ii) $4 \leq \gamma_{as}(G)\gamma_{as}(\overline{G}) \leq (n - 2)^2$

Proof: The result follows from Proposition 2.9 and Proposition 2.10.

Theorem 3.2: For any graph G and $\overline{G}$ with no isolated vertices,

(i) $4 \leq \gamma_{ans}(G) + \gamma_{ans}(\overline{G}) \leq 2(n - 2)$  
(ii) $4 \leq \gamma_{ans}(G)\gamma_{ans}(\overline{G}) \leq (n - 2)^2$

Proof: The result follows from Proposition 2.9 and Proposition 2.10.

Theorem 3.3: For any graph G which is not regular, 

$\gamma_{as}(G) \leq n - \gamma_s(G) + 1$

And this bound is sharp.

Proof: Let D be a minimum split dominating set of G with $|D| < \frac{n}{2}$. Then for any vertex $v \in D, (V - D) \cup \{v\}$ is an accurate split dominating set of G.

$\therefore \gamma_{as}(G) \leq |(V - D) \cup \{v\}| = n - \gamma_s(G) + 1$

The path $P_4$, cycle $C_4$, $C_6$ achieve this bound.

Theorem 3.4: For any graph G which is not regular, 

$\gamma_{ans}(G) \leq n - \gamma_{as}(G) + 1$

Theorem 3.5: For any graph G, (i) $\gamma_{as}(G) \leq n - \gamma(G) + 1$

(ii) $\gamma_{ans}(G) \leq n - \gamma(G) + 1$
Proof: We know that $\gamma(G) \leq \gamma_s(G)$ & $\gamma(G) \leq \gamma_{ns}(G)$.

$\therefore$ It satisfies the above inequality.

Theorem 3.6: For any star graph, $\gamma_{as}(G) = n - k$, where $n$ is the number of vertices and $k$ is the number of end vertices.

Theorem 3.7: For any Tree $T$ with $n \geq 5$, $\gamma_{as}(G) \leq n - k$, where $n$ is the number of vertices and $k$ is the number of end vertices.

Note 3.8: For any Tree $T$, $\delta(G) = 1$

$\therefore \gamma_{as}(G) \leq n - 1$

Theorem 3.9: For any tree $T$ which is not star $\gamma_{as}(T) \leq n - 2$ for $n \geq 5$ and the bound is sharp.

Proof: If $T$ is not a star, Let $D$ be an accurate split dominating set with $|D| \leq \frac{n}{2} + 1$.

In $D$ there exists two adjacent cut vertices $u$ and $v$ with degree $\geq 2$. This implies $V - \{u, v\}$ is an accurate split dominating set. The bound is sharp in $T_6$.

Definition: Let $G$ be a connected graph of diameter $d$

a) $(d - 1)$ coloring $c$ requires that $d(u, v) + |c(u) - c(v)| > d$, for every two distinct vertices $u$ and $v$ of $G$.

b) $(d - 1)$ radio coloring $c$ is also refereed as a radio antipodal coloring (or simply an antipodal coloring).

Since $c(u) = c(v)$ only if $u$ and $v$ are an antipodal vertices of $G$. The antipodal coloring coloring number $c(G)$ of an antipodal coloring $c$ of $G$ is the maximum color assigned to a vertex of $G$. The antipodal chromatic number $c(G)$ of $G$ is $\min (g)$ over all antipodal colorings $c$ of $G$. An antipodal coloring $c$ of $G$ is a minimum antipodal coloring if $c(c) = c(G)$.

In order to provide additional information about the antipodal chromatic numbers of paths, we present a sufficient condition for the antipodal chromatic number of a connected subgraph of a connected graph $G$ having diameter $d$ to be bounded above by $c(G)$.
The concepts colorings and radio k-chromatic number of graphs were inspired by the so-called channel assignment problem, where channels are assigned to FM radio stations according to the distances between the stations (and some other factors as well). Radio k-colorings provide a generalization of ordinary colorings of graphs. The radio d-chromatic number was studied in the previous chapter and was also called the radio-number. Radio d-colorings are also referred to as radio labelings since no two vertices can be colored the same in a radio d-coloring. Thus, in a radio-labeling of a connected graph of diameter d; the labels (colors) assigned to adjacent vertices must differ by at least d times.

To illustrate these concepts, consider the graph G of figure 3. Since diam(G) = 3; it follows that in any antipodal coloring of G, the colors of every two adjacent vertices must differ by at least 2, the colors of every two vertices whose distance is 2 must differ by at least 1; and the colors of two antipodal vertices can be the same. Thus the coloring of G given in figure 3 is, an antipodal coloring. In fact, it can be verified that the antipodal coloring of G given in figure 3 is a minimum antipodal coloring of G and so c(G) = 5.

**Theorem 3.10:** For any tree $T$, $\gamma_{as}(G) \leq \gamma_{ant}(G)$ for $n \geq 5$.

**Theorem 3.11:** For any graph G which is not complete,

$$\gamma_{as}(G) \leq n - \beta_0(G) + 1$$

Where $\beta_0(G)$ is maximum independent set. The bound is sharp in $P_4$ and $C_5$.

**Corollary: 3.12:** For any graph G which is not complete, where $\gamma_{as}(G) \leq \alpha_0(G) + 1$, $
\alpha_0(G)$ is a covering set of G.
**Theorem 3.13:** For any graph $G$ which is not complete,
\[ \gamma_{\text{ans}}(G) \leq \beta_0(G) + 2, \text{ for } n \geq 5 \]
Where $\beta_0(G)$ is maximum independent set. The bound is sharp in $p_7$ and $C_5$.

**Corollary: 3.14:** For any graph $G$ which is not complete, $\gamma_{\text{ans}}(G) \leq \alpha_0(G) + 2$, where $\alpha_0(G)$ is a covering set of $G$.

**Note:3.15:**
In a complete graph (i) $\gamma_{\text{as}} > \beta_0(G)$
(ii) $\gamma_{\text{ans}} > \beta_0(G)$

**Theorem3.16:** For any graph $G$ which is not complete, $(i) \gamma_{\text{as}}(G) \leq n - \Delta(G) + 2$
(ii) $\gamma_{\text{ans}}(G) \leq n - \Delta(G) + 3$, for $n \geq 5$

**Note 3.17:**
In a complete graph (i) $\gamma_{\text{as}} \leq 2n - \Delta(G)$
(ii) $\gamma_{\text{ans}} \leq 2n - \Delta(G)$

**4. CONCLUSION**
In this paper, we define the notions of accurate split and non split domination in graphs. We got many bounds on accurate split and non-split domination numbers. Exact values of these new parameters are obtained for some standard graphs. Nordhaus - Gaddum type results are also obtained for these new parameters. As a future work the readers can extend the results and study the applications of the parameters in a wider sense.
REFERENCES


