An EOQ Model for Weibull Distribution Deterioration with Exponential Demand under Linearly Time Dependent Shortages

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Abstract

Uncertainty is the natural phenomena for any type of business transaction. Deterioration is not always constant during market period. This paper considers an inventory model for exponential demand rate and weibull distribution deterioration. Shortages are allowed and linearly time dependent. Truncated Taylor series approximation is used for finding closed form solution. Mathematical model is derived to find optimal cycle time and total cost. We show that the total cost function is convex with respect to cycle time. Numerical example is give to validate the model. Sensitivity analysis is provided with the validations of several parameters.

Keywords: EOQ, Weibull distribution, Exponential Demand, Shortage.

INTRODUCTION

Manufacturing processes generally experience failures and recoveries in their operation. Re cooking process play an important role in reducing wastages and cost of production in the manufacturing processes. At present deteriorating inventory problems were crucially attract by developing mathematical model. Wu[1] established the property of the convexity of the total variable cost function of the inventory model. Covert and Philip [2] considered an inventory model for items with


At present inflation is a major problem for large scale business. Chung [22] established the discounted cash-flows approach for the analysis of the optimal inventory in the presence of the trade credit. Jaggi and Aggarwal [23] generalized model Chung [22] to established an EOQ model for getting the optimal order quantity of deteriorating items in the presence of trade credit using the DCE approach. Some related research papers in this direction done by Wu et al. [24], Sarker et al. [25], Buzacott [26], Aggarwal [27], Sarkar and Moon [28] and Chen [29] etc.

The shortage cost and lost sale have been considered by Dye et al. [30] in their inventory model. Ghiami et al. [31] presented a two echelon EOQ model for deteriorating items under partial backlogging. Taleizadeh & Nemtollahi [32] proposed an EOQ model for deteriorating items under financial considerations and shortages. An inventory model for deteriorating items with shortages and time varying demand were discussed by Sicilia et al. [33]. Bhunia et al. [34] considered an EOQ for two warehouses with partial backlogging under trade credits. Jaggi et al. [35] discussed credit financing in economics ordering policies for defective items with allowable shortages.

In this paper inventory model for Weibull deteriorating items and exponential time dependent demand is developed. Shortages are allowed and fully backlogged.

In the next section assumptions and notations are given. Mathematical formulation is discussed for finding optimal solution. Numerical example is provided to illustrate the proposed model. Total cost function is proved by graphical representation. Sensitivity
analysis of the optimal solution with respect to the several parameters is carried out. The paper ends with concluding remarks and future research.

ASSUMPTION & NOTATIONS
While developing mathematical model we used following mathematical notation & assumptions as given bellow

Notations
- I(t) : Positive inventory level at time t.
- Q: Maximum level of positive inventory
- s : Cost of shortage per unit and per unit time
- T: Time when inventory level become zero
- T: Shortage occurrence time.
- p : Constant ordering cost per order
- c3 : Unit purchases cost of an item

Assumptions
- Lead time is taken as zero.
- h represent holding cost/unit.
- Where demand rate D (t) = γ e^{gt} where γ > 100 and 0 < δ < 1.
- θ (t) = α β t^{β-1} : the two parameter Weibull distribution deterioration rate.
  Where 0 ≤ α < 1 is the scale parameter; β > 0 is the shape parameter.

MATHEMATICAL MODEL AND OPTIMAL SOLUTION
As we know that we consider inventory model with weibull distribution deterioration and decline demand rate as exponential. As we see by fig.(1) the inventory level reduce due to deterioration rate and demand rate, between the interval [0, T_1], which can represented by following deferential equation:-

\[ \frac{dI_1}{dt} + \theta(t) I_1 = -\gamma e^{\delta t} \quad 0 < t < T_1 \]  \hspace{1cm} (I)

We use initial and boundary condition as I_1 (T_1) = I_2(T_1) = 0 and I_1 (0) = Q.
And shortages
\[ \frac{dI}{dt} = -(a + bt) \quad T_1 < t < T \] (2)

Where \( a > 0 \) and \( 0 < b < 1 \).

Now the solution of equ.(1) & (2):

\[ I_1(t) = \gamma e^{-at^2} \left[ T_1 + \delta T_1^2 + \frac{\delta^2 T_1^3}{3} + \frac{\alpha T_1^{(\beta+1)}}{(\beta+1)} + \frac{\alpha T_1^{(\beta+2)}}{(\beta+2)} \right. \]
\[ \quad + \frac{\alpha \delta T_1^{(\beta+3)}}{3} - \left( t + \frac{\delta t^2}{2} + \frac{\delta^2 t^3}{3} + \frac{\alpha t^{(\beta+1)}}{\beta+1} \right) \]
\[ \left. + \frac{\alpha \delta t^{(\beta+2)}}{(\beta+2)} + \frac{\alpha \delta^2 t^{(\beta+3)}}{(\beta+3)} \right] \]

And

\[ I_2(t) = a(T_1 - t) + \frac{b}{2} (T_1^2 - t^2) \] (4)

Now the positive inventory \( I_1(0) = Q_1 \), by

Equ. (3)

\[ I_1(0) = Q_1(T_1) = \gamma \left[ T_1 + \frac{\delta T_1^2}{2} + \frac{\delta^2 T_1^3}{3} + \frac{\alpha T_1^{(\beta+1)}}{(\beta+1)} + \right. \]
\[ \frac{\alpha \delta T_1^{(\beta+2)}}{(\beta+2)} + \frac{\alpha \delta^2 T_1^{(\beta+3)}}{(\beta+3)} \right] \]

Figure 1: Inventory level \( I_1(t) \) and shortage \( I_2(t) \)
An EOQ Model for Weibull Distribution Deterioration with Exponential Demand.

As to find various component of total cost first we find holding cost of inventory system:

\[
HC = \int_0^T h \cdot I_1(t) \, dt
\]

\[
= h \gamma \left[ T_1^3 + \frac{\delta T_1^3}{2} + \frac{\delta^2 T_1^4}{3} + \frac{\alpha T_1^{(\beta+2)}}{(\beta+1)} + \frac{\alpha \delta T_1^{(\beta+3)}}{(\beta+2)} + \frac{\alpha \delta^2 T_1^{(\beta+4)}}{(\beta+3)} \right]
\]

Now we have shortage cost

\[
SC = -s \int_{T_1}^T I_2(t) \, dt
\]

\[
= s \left\{ \frac{a}{2} (T - T_1)^2 + \frac{b}{6} (T_3 + 2T_1^2 - 3TT_1^2) \right\}
\]

Now ordering Cost

\[
OC = p : \text{Where } p \text{ be constant.}
\]

Purchase cost

\[
PC = C_s Q(T)
\]

\[
= C_s [Q_1(T) + Q_2(T)]
\]

Where

\[
Q_2(T) = -I_2(T) = -[a(T_1 - T) + \frac{b}{2} (T_1^2 - T^2)]
\]
\[ PC = c_1 \gamma \left[ T_1 + \frac{\delta T_1^2}{2} + \frac{\delta^2 T_1^3}{3} + \frac{\alpha T_1^{(\beta+1)}}{(\beta+1)} + \right. \]
\[ + \frac{\alpha \delta T_1^{(\beta+2)}}{(\beta+2)} + \frac{\alpha \delta^2 T_1^{(\beta+3)}}{(\beta+3)} \left. \right] - \left[ a(T_i - T) + \frac{b}{2}(T_i^2 - T^2) \right] \]  

(11)

Hence the total cost of the system

\[ U = \frac{1}{T} (OC + HC + SC + PC) \]  

(12)

\[ = \frac{1}{T} \left[ p + h \gamma \left( T_1^2 + \frac{\delta T_1^2}{2} + \frac{\delta^2 T_1^3}{3} + \frac{\alpha T_1^{(\beta+2)}}{(\beta+1)} + \frac{\alpha \delta T_1^{(\beta+3)}}{(\beta+2)} \right) ight. \]
\[ + \frac{\alpha^2 T_1^{(2\beta+2)}}{(\beta+1)^2} - \frac{\alpha^2 \delta T_1^{(2\beta+3)}}{(\beta+1)(\beta+2)} + \left. \frac{\alpha^2 \delta^2 T_1^{(2\beta+4)}}{(\beta+1)(\beta+3)} \right) \left( \frac{T_1^2}{2} + \right. \]
\[ \left. \frac{\delta T_1^3}{6} + \frac{\delta^2 T_1^4}{12} + \frac{\alpha T_1^{(\beta+2)}}{(\beta+1)(\beta+2)} + \frac{\alpha \delta T_1^{(\beta+3)}}{(\beta+2)(\beta+3)} + \frac{\alpha \delta^2 T_1^{(\beta+4)}}{(\beta+3)(\beta+4)} \right) + \frac{b}{6} \left( \frac{T_1^3 + 2T_1^2 - 3TT_1^2}{2} \right) + c_i \left[ - \gamma \left[ T_1 + \frac{\delta T_1^2}{2} + \frac{\delta^2 T_1^3}{3} + \frac{\alpha T_1^{(\beta+1)}}{(\beta+1)} \right. \right. \]
\[ + \left. \frac{\alpha \delta T_1^{(\beta+2)}}{(\beta+2)} + \frac{\alpha \delta^2 T_1^{(\beta+3)}}{(\beta+3)} \right] - \left[ a(T_i - T) + \frac{b}{2}(T_i^2 - T^2) \right] \]  

(13)

Now to minimize the total cost \( U \) per unit time, we obtain the optimum value of \( T_1 \) and \( T \) with help of following equations

\[ \frac{\partial U}{\partial T_1} = 0 \text{ and } \frac{\partial U}{\partial T} = 0. \]  

(14)
An EOQ Model for Weibull Distribution Deterioration with Exponential Demand.

\[
\frac{\partial U}{\partial T_i} = \frac{1}{T_i} \left[ C_i \left[ \gamma \left[ \left( 1 + \delta T_i + \delta^2 T_i^2 + \alpha T_i^{(\beta_2)} \right) - \left[ a + b T_i \right] \right] + \right.ight.
\]

\[
+s \left\{ -a (T - T_i) + \frac{b}{3} (2T_i - 3T_i^2) \right\} + \]

\[
h \gamma \left[ 2T_i \left( \frac{3}{2} \delta T_i^2 + \frac{4}{3} \delta^2 T_i^3 + \frac{(\beta + 2)}{(\beta + 1)} \alpha T_i^{(\beta_2)} \right) \right.
\]

\[
+ \frac{(\beta + 3) \alpha \delta T_i^{(\beta_2)}}{(\beta + 2)} + \frac{(\beta + 4) \alpha \delta^2 T_i^{(\beta_4)}}{(\beta + 3)} - \]

\[
\left. \left( \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)} - \frac{(\beta + 3) \alpha \delta T_i^{(\beta_3)}}{2(\beta + 1)} \right) \right]- \]

\[
\left. \frac{(\beta + 4) \alpha \delta^2 T_i^{(\beta_4)}}{3(\beta + 1)} - \frac{(2\beta + 2) \alpha T_i^{(2\beta_2)}}{(\beta + 1)^2} \right) \right]- \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\left. \frac{(\beta + 3) \alpha \delta^2 T_i^{(\beta_3)}}{3} + \frac{(\beta + 2) \alpha \delta^3 T_i^{(\beta_2)}}{(\beta + 1)} + \frac{(\beta + 2) \alpha T_i^{(\beta_2)}}{(\beta + 1)^2} \right) \]

\[
\frac{\partial U}{\partial T} = \frac{1}{T} \left[s \left\{ a (T - T_i) + \frac{b}{2} (T_i^2 - T_i^2) \right\} + \right.
\]

\[
\left. C_i \left\{ a + b T_i \right\} - \frac{1}{T} \left[ OC + HC + SC + PC \right] \right] \]
We observed during all calculation that it difficult to find second order derivative for sufficient condition of model optimality condition.

Above figure is convex, which proves that the optimal total cost with respect to time $t_1$ and $T$ is minimum. That is, second derivative of $\frac{d^2U}{dT^2}$ and $\frac{d^2U}{dT_1^2}$ are positive.

**NUMERICAL EXAMPLE**

In developed model which is proposed by us is explained below by given following example:

For numerical and graphical analysis of solution of the developed model, value of parameters is taken as proper units, When $\alpha = 0.7$, $\beta = 4$, $\gamma = 101$, $\delta = 0.2$, $h = 10$, $s = 50$, $a = 100$, $b = 0.3$, $p = 100$, $C_3 = 10$. After putting these values in model we get optimal solution as $T_1 = T_1^* = 1.7627$ and $T = T^* = 2.0578$, $Q = Q^* = 548.775$ and $U = U^* = 3965.67$.

**SENSITIVITY ANALYSIS**

<table>
<thead>
<tr>
<th>p</th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>547.033</td>
<td>4086.7</td>
</tr>
</tbody>
</table>
**An EOQ Model for Weibull Distribution Deterioration with Exponential Demand.**

**Figure 3:** Graphical representation shows change in $p$, $T_1$, $T$, $Q$ and $U$.

**Table 2:** shows effect of change in $\delta$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
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<td>2.17049</td>
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<td>5016.14</td>
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<tr>
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</tr>
<tr>
<td>0.6</td>
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<td>2.30309</td>
<td>1021.49</td>
<td>6277.16</td>
</tr>
<tr>
<td>0.7</td>
<td>1.78059</td>
<td>2.3756</td>
<td>1181.81</td>
<td>6975.75</td>
</tr>
</tbody>
</table>

**Figure 4:** Graphical representation shows change in $\delta$, $T_1$, $T$, $Q$ and $U$. 
Table 3: shows effect of change in $\gamma$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1666.21</td>
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<td>2236.17</td>
<td>12693.6</td>
</tr>
<tr>
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<tr>
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<td>3.61354</td>
<td>3965.49</td>
<td>19401.3</td>
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</tbody>
</table>

Figure 5: Graphical representation shows change in $\gamma$, $T_1$, $T$, $Q$ and $U$.

Table 4: shows effect of change in $\beta$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>1.85994</td>
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<td>1.41386</td>
<td>1.70838</td>
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<td>3957.22</td>
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</tbody>
</table>
Figure 6: Graphical representation shows change in $\beta$, $T_1$, $T$, $Q$ and $U$.

Table 5: shows effect of change in $s$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th>$s$</th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
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<tbody>
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</tr>
</tbody>
</table>

Figure 7: Graphical representation shows change in $s$, $T_1$, $T$, $Q$ and $U$. 
Table 6: shows effect of change in a, T₁, T, Q and U

<table>
<thead>
<tr>
<th>a</th>
<th>T₁</th>
<th>T</th>
<th>Q</th>
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<td>4209.39</td>
</tr>
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</table>

Figure 8: Graphical representation shows change in a, T₁, T, Q and U.

Table 7: shows effect of change in α, T₁, T, Q and U

<table>
<thead>
<tr>
<th>α</th>
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<th>T</th>
<th>Q</th>
<th>U</th>
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<td>4043.71</td>
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</table>
Figure 9: Graphical representation shows change in $\alpha$, $T_1$, $T$, $Q$ and $U$.

Table 8: shows effect of change in $C_3$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th>$C_3$</th>
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<th>$Q$</th>
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<td>1.94599</td>
<td>2.55921</td>
<td>809.986</td>
<td>8672.08</td>
</tr>
</tbody>
</table>

Figure 10: Graphical representation shows changes in $C_3$, $T_1$, $T$, $Q$ and $U$. 
Table 9: shows effect of change in $C_3$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.44007</td>
<td>2.4556</td>
<td>1219.79</td>
<td>4173.51</td>
</tr>
<tr>
<td>3</td>
<td>2.0169</td>
<td>2.29738</td>
<td>941.251</td>
<td>3820.7</td>
</tr>
<tr>
<td>4</td>
<td>1.9413</td>
<td>2.20951</td>
<td>801.97</td>
<td>3697.04</td>
</tr>
<tr>
<td>5</td>
<td>1.88298</td>
<td>2.15469</td>
<td>708.652</td>
<td>3668.09</td>
</tr>
<tr>
<td>6</td>
<td>1.85096</td>
<td>2.11841</td>
<td>662.143</td>
<td>3688.84</td>
</tr>
</tbody>
</table>

Figure 11: Graphical representation shows changes in $h$, $T_1$, $T$, $Q$ and $U$.

Table 10: shows effect of change in $C_3$, $T_1$, $T$, $Q$ and $U$

<table>
<thead>
<tr>
<th></th>
<th>$T_1$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
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<td>3964.21</td>
</tr>
<tr>
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<tr>
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<td>2.05399</td>
<td>548.694</td>
<td>3959.36</td>
</tr>
</tbody>
</table>

Figure 12: Graphical representation shows changes in $b$, $T_1$, $T$, $Q$ and $U$. 
The observations mentioned above can be summed up as follows:

(i) If $\gamma$, $\delta$, $Q$, and $U$ moves in the same direction.
(ii) The increase of $\beta$, $a$, $p$, and $s$ shows an opposite change in $Q$ and $U$.
(iii) The increase in $\alpha$, $C_3$, $h$, $a$, $b$, and $p$ will cause a decrease in $Q$ and $U$.

CONCLUSION

In several cases supply is not sufficient to fulfill the demand of customers. In this situation, customers will have to wait until supply and demand are balanced. In this situation, shortages occur.

In this paper, we have developed a model with exponential demand under linearly time-dependent shortages with Weibull distribution deterioration. Mathematical formulation and optimal solutions are obtained with respect to positive inventory and cycle time. Numerical examples and sensitivity analysis are provided to validate the proposed model.

For a managerial point of view, the following inference can be followed:

- Increment in $\gamma$, $p$, $\delta$, $s$, $a$, and $C_3$ results in an increment of total cost.
- Increment in $\beta$, $h$, $b$, and $\alpha$ comes with an opposite outcome in the total cost $U$.

The model established in this paper is generalized for including advertisement costs and freight charges. The model can be expanded for quadratic time-dependent demand and shortages.

REFERENCES


