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Self Magnetic Field and Current-loop of Electron with Five Different Radii and Intrinsic Properties

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Abstract

Here we study the behavior of the self-magnetic field and current due to the rotations of charge in the semi-classical modified relativistic spinning sphere model of electron. Though the original model is Compton-sized, we have tested below and above Compton-radius and α -quantized results come out as consequence.

Keywords: Electron-radii, Electron-model, α-quantization.

Introduction

Studies of the properties of the charged particles can be explored by probing about its behavior in uniform [1] and non-uniform [2] magnetic fields. Charged particles are studied along with its dynamics for more than a century. The dynamics of charged particles was focused by Maxwell who gave birth to electrodynamics [3]. Maxwell's macroscopic theory [3] was replaced by Lorentz's microscopic theory [3] after the discovery of electron.

Electron is the lightest charged particle and this is called as a point particle in the Standard Model of Particle Physics. But different electromagnetic phenomenon revealed eight different electron radii (Table-I) [4] [5] which give the signature of some extended electron model. Lorentz and Abraham made the first attempt to arrive at the structure of electron with the help of the dynamics of its charge [3]. Afterwards quite a large number of models of electron were proposed. [3] [6]. Relativistic Spinning Sphere model by M.H. MacGregor [4] [6] is proposed in recent-days. This is a semi-classical model which correlates the spectroscopic properties of the electron accurately to first order of α . This behaves as a relativistically spinning mechanical sphere of matter with an equatorial point charge e [4]. In this article we have considered the rotation of the equatorial charge with the speed c and examined the nature of current and magnetic field produced due to that rotation. Charge e is in fact residing in a very small space compared to the volume of the electron as charge radius $R_E < 10^{-19}\,m$ which is predicted from recent LEP experiment [6] [7].

Table I: Eight different electron radii.

Symbols	Name	Values
R_0	Classical electron radius	$\frac{e^2}{mc^2}$
R_C	Compton radius	$\frac{\hbar}{mc}$
R_{QMC}	Quantum mechanical Compton radius	$\sqrt{3}\,\frac{\hbar}{mc}$
R^{α}_{QMC}	QED-corrected quantum mechanical Compton radius	$\sqrt{3}(1+\frac{\alpha}{2\pi})\frac{\hbar}{mc}$
R_{em}	Classical electromagnetic radius	$\frac{\hbar^2}{mc^2}$
R_H	Magnetic field radius	$\geq 0.106 R_C$
R_{QED}	Observed QED charge distribution for a bound electron	$\approx R_C$
R_E	Charge-radius of electron	Yet to be calculated

In this process we don't consider any external field. Formulating the current-loop calculation of the charge within the radius of electron we proceed here. Out of the eight different electron radii, five are formulated with $\alpha, \hbar, e, c, m$, where α is fine structure constant, $\hbar$ is reduced Planck's constant, e is charge of electron, m is mass of the electron and c is the velocity of light in free space. Hence we use classical electron radius, Compton radius of electron, Quantum mechanical Compton radius, QED-corrected Quantum mechanical Compton radius and electromagnetic radius of electron to study the magnetic field originated from the rotation of the charge on the equator of the relativistic spinning sphere.

Rotation of charge and Ampere's law

Charge passing per unit time per unit area is known as current, $I = \frac{Q}{T}$, where Q is the charge and T is the time by which Q amount of charge passes unit area. To deal with electron we say the charge as e. When a small charge e rotates in a circular path of radius R with linear velocity v around the axis of rotation, the current comes out as

$$I = \frac{ev}{2\pi R} \tag{1}$$

In electrodynamics, current *I* can also be written with the help of current density *J* as

$$I = \int J.da\,, \tag{2}$$

where *da* is the area of the element. For magnetic field *B*, we have

$$\nabla \times B = \mu_0 J\,. \tag{3}$$

where μ_0 is the free space permittivity. According to Stoke's theorem, for a surface *S*, closed by the curve *C*

$$\int_S (\nabla \times B).da = \oint_C B.dl \tag{4}$$

where *dl* is the small line element on the curve *C*. Using equation (2) and (3) together in equation (4) we get Ampere's law

$$\oint_C B.dl = \mu_0 I \tag{5}$$

As we are studying RSS model and the charge is assumed to be rotating in the equator of the sphere with a velocity of *c*, we get current-loop corresponding to each radius for relativistically spinning spherical electron model.

Compton radius of electron is known as $R_C = \frac{\hbar}{mc}$, where $\hbar$ is reduced Planck's constant, *m* is the mass of electron, *c* is the velocity of light in free space.

Using Compton radius in equation (1), we have

$$I_C = \frac{ec}{2\pi R_C} = \frac{c^2}{4\pi}\left(\frac{em}{\frac{\hbar}{2}}\right) \tag{6}$$

where *e* is the charge of electron, *m* is the mass of electron and $\frac{\hbar}{2}$ is the spin of electron. Therefore in other words this current-loop can be written in terms of three intrinsic properties (charge, mass and spin) of electron as

$$I_C = \frac{c^2}{4\pi}\left(\frac{Charge.Mass}{Spin}\right).$$

Classical electron radius is mathematically expressed as $R_0 = \frac{e^2}{mc^2}$. This is also known as Thomson scattering cross-section or Lorentz-radius. The current-loop for classical electron radius is

$$I_0 = \frac{ec}{2\pi R_0} = \alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})] . \tag{7}$$

Classical electromagnetic radius $\frac{\hbar^2}{me^2}$ is also known as Bohr radius of the hydrogen atom. This is a larger one than the classical electron radius and the Compton radius of electron. The current-loop expression for this radius comes out as

$$I_{em} = \frac{ec}{2\pi R_{em}} = \alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{8}$$

Quantum mechanical Compton radius (R_{QMC}) and QED-corrected quantum mechanical Compton radius (R^{α}_{QMC}) are defined by M. H. MacGregor [4]. The formalism of quantum mechanical spin and magnetic moment projection factors lead to an electron radius $R_{QMC} = \sqrt{3}\frac{\hbar}{mc}$, quantum mechanical Compton radius of electron. Hence the current-loop of quantum mechanical Compton radius is calculated as

$$I_{QMC} = \frac{1}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})] \tag{9}$$

Applying magnetic self-energy corrections R_{QMC} becomes $R^{\alpha}_{QMC} = \sqrt{3}(1+\frac{\alpha}{2\pi})\frac{\hbar}{mc}$, QED-corrected quantum mechanical Compton radius. The corresponding current-loop comes out as

$$I^{\alpha}_{QMC} \approx \frac{1}{\sqrt{3}}(1-\frac{\alpha}{2\pi})[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})] . \tag{10}$$

For R_C, Ampere's law can be calculated by putting equation (6) into equation (5) as

$$\oint_C B_C.dl = \mu_0[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \quad (11)$$

Similar calculations using equations (7), (8), (9), and (10) for R_0, R_{em}, R_{QMC} and R_{QMC}^{α} respectively produce

$$\oint_C B_0.dl = \mu_0\alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})], \quad (12)$$

$$\oint_C B_{em}.dl = \mu_0\alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})], \quad (13)$$

$$\oint_C B_{QMC}.dl = \frac{\mu_0}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})], \quad (14)$$

and

$$\oint_C B_{QMC}^{\alpha}.dl = \frac{\mu_0(1-\frac{\alpha}{2\pi})}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \quad (15)$$

Magnetic moment of electron was calculated by Uhlenbeck and Goudsmit as $\mu = \frac{e\hbar}{2mc}$. But the experimental results differed from the theoretical by 0.01%. Solution to this problem was given by Schwinger in 1949 [4] [8]. From the virtual emission and absorption of light quanta the logarithmically divergent self-energy of a free electron arises. The electromagnetic self-energy of a free electron can be described as electromagnetic mass of the electron and this must be added to the mechanical mass of the electron to give the experimental mass. This electromagnetic mass is the above-mentioned correction to the mechanical mass of the electron. Hence the corrected magnetic moment written as $\mu = \frac{e\hbar}{2mc}(1+\frac{\alpha}{2\pi})$, where $\alpha \approx \frac{1}{137}$ is the fine-structure constant and $\frac{\alpha}{2\pi}$ is known as Schwinger correction term [4][8].

Using the total mass of electron (= mechanical mass + electromagnetic mass), the expressions (6) – (10) can be re-written with the introduction of $m(1+\frac{\alpha}{2\pi})$

$$I_C = [\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \quad (16)$$

$$I_0 = \alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{17}$$

$$I_{em} = \alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{18}$$

$$I_{QMC} = \frac{1}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{19}$$

$$I_0 = \frac{1}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{20}$$

In the same way the corrections can be made in expressions (11) - (15) for Ampere's law as

$$\oint_C B_C .dl = \mu_0[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{21}$$

$$\oint_C B_0 .dl = \mu_0\alpha^{-1}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{22}$$

$$\oint_C B_{em} .dl = \mu_0\alpha[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{23}$$

$$\oint_C B_{QMC} .dl = \frac{\mu_0}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}) \tag{24}$$

and

$$\oint_C B^{\alpha}_{QMC} .dl = \frac{\mu_0}{\sqrt{3}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{25}$$

Approximation for long straight wire and B in terms of charge, mass and spin

The expressions (21) – (25) produce Ampere's law in terms of charge, mass and spin of the electron. But mathematically B is inside the integral and having product with line element dl. To get the value of B separately long straight current carrying wire's approximation [9] is used here which gives

$$B = \frac{\mu_0 I}{2\pi R}. \tag{26}$$

As equation (26) is a modified version of equation (5), the expressions (21) – (25) can be modified respectively as

$$B_C = \frac{\mu_0}{2\pi R_C}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{27}$$

$$B_0 = \frac{\mu_0 \alpha^{-1}}{2\pi R_0}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{28}$$

$$B_{em} = \frac{\mu_0 \alpha}{2\pi R_{em}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{29}$$

$$B_{QMC} = \frac{1}{\sqrt{3}}\frac{\mu_0}{2\pi R_{QMC}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{30}$$

$$B_{QMC}^{\alpha} = \frac{1}{\sqrt{3}}\frac{\mu_0}{2\pi R_{QMC}^{\alpha}}[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]. \tag{31}$$

Generalized current-loop and magnetic field

For a rotational motion of charge on RSS model, we have the current-loop expressions (16) – (20). The remarkable thing is that all of these five expressions carry a common factor $[\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]$. We say this common factor as generalized current $I_G = [\frac{c^2}{4\pi}(\frac{em}{\frac{\hbar}{2}})]$. In fact all of the above current-loops (equations (16) – (20)) can be re-written respectively in terms of the generalized current-loop as

$$I_C = I_G(1+\frac{\alpha}{2\pi}), \tag{32}$$

$$I_0 = \alpha^{-1} I_G(1+\frac{\alpha}{2\pi}), \tag{33}$$

$$I_{em} = \alpha I_G(1+\frac{\alpha}{2\pi}), \tag{34}$$

$$I_{QMC} = \frac{1}{\sqrt{3}} I_G (1 + \frac{\alpha}{2\pi}), \tag{35}$$

$$I^{\alpha}_{QMC} = \frac{I_G}{\sqrt{3}}. \tag{36}$$

In the expressions (27) - (31) of magnetic field also the term generalized current-loop is present. Hence the equations (27) – (31) can be re-written as

$$B_C = \frac{\mu_0 I_G}{2\pi R_C}(1 + \frac{\alpha}{2\pi}), \tag{37}$$

$$B_0 = \frac{\alpha^{-1}\mu_0 I_G}{2\pi R_0}(1 + \frac{\alpha}{2\pi}), \tag{38}$$

$$B_{em} = \frac{\alpha\mu_0 I_G}{2\pi R_{em}}(1 + \frac{\alpha}{2\pi}), \tag{39}$$

$$B_{QMC} = \frac{1}{\sqrt{3}} \frac{\mu_0 I_G}{2\pi R_{QMC}}(1 + \frac{\alpha}{2\pi}), \tag{40}$$

$$B^{\alpha}_{QMC} = \frac{1}{\sqrt{3}} \frac{\mu_0 I_G}{2\pi R_{QMC}}. \tag{41}$$

The current-loop expressions ((32) – (36)) for different radii can now be related with each other as

$$I_C = \alpha I_0 = \alpha^{-1} I_{em} = \sqrt{3} I_{QMC} = \sqrt{3}(1 + \frac{\alpha}{2\pi}) I^{\alpha}_{QMC}. \tag{42}$$

Using the equations (37) – (41) we have similar relation for the self-magnetic field produced for those above-mentioned radii as

$$B_C = \alpha^2 B_0 = \alpha^{-2} B_{em} = 3 B_{QMC} = 3(1 + \frac{\alpha}{2\pi}) B^{\alpha}_{QMC}. \tag{43}$$

α - quantized mass-leap and approximation for radius of muon and tau

In QED, the fine structure constant α is a coupling constant too. Comparison of the electron to the other particle mass data set has been yielded two different α-quantized masses, and they appear in two different forms know as fermionic (with half integral spin) and bosonic (with integral spin). These two α-masses are (1) $m_f = \frac{3}{2}\frac{m_e}{\alpha} = 105 MeV$ mass quanta that is created in the "α-leap" from the electron to the muon; (2) $m_b = \frac{m_e}{\alpha} = 70 MeV$ mass quantum that is created as part of a hadronically bound particle-antiparticle pair in the "α-leap" from an electron-positron pair to the pion (where m_e is the electron mass) [10].

The factor $(\frac{m_e}{\alpha})$ is found in the expression of current-loop for classical electron radius (equation (17)) and by re-writing equation (17), we have

$$I_0 = [\frac{c^2}{4\pi}(\frac{e\frac{m_e}{\alpha}}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{44}$$

where $m_e = m =$ mass of electron. Using $m_f = \frac{3}{2}\frac{m_e}{\alpha}$ in equation (44) we get

$$I_0 = [\frac{c^2}{6\pi}(\frac{em_\mu}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}), \tag{45}$$

where m_μ is the mass of muon. Comparing the current-loop expression for muon in a similar way to electron we have the radius of muon as

$$R_\mu = \frac{3}{2}\frac{\hbar}{m_\mu c} \tag{46}$$

Compton radius of electron is known as $R_C = \frac{\hbar}{m_e c}$ with m_e as the mass of the electron. Equation (46) looks like the Compton radius of electron. Also right hand side carries a dimension of length which is essential for radius. Hence R_μ can be called as radius of muon.

Mass of tau is almost 17 times of the mass of the muon. Therefore in the same way with the help of equation (17) and the α-leap of the fermionic mass, we have

$$I_0 = \frac{2}{51}[\frac{c^2}{4\pi}(\frac{em_\tau}{\frac{\hbar}{2}})](1+\frac{\alpha}{2\pi}). \tag{47}$$

Therefore radius of tau comes out as

$$R_\tau = \frac{51}{2}\frac{\hbar}{m_\tau c}. \tag{48}$$

This equation (48) gives a form of radius just like equation (46) and this also looks like Compton radius.

Conclusion

Current (equations (16) – (20)) and magnetic field (equations (27) – (31)) are expressed in terms of three intrinsic properties of electron; i.e. charge, mass and spin. This is an interesting feature that mass and spin are also involved in describing current and magnetic field.

With the help of equations (32) – (36) the relation amongst the current-loops is developed in equation (42). Similar relation amongst the equations (37) – (41) is derived in equation (43).

It is also remarkable that though Compton radius, classical electron radius and electromagnetic radius are originated from different electromagnetic phenomenon, their current and the magnetic field are expressed in a generalized way with α-quantization. α-leap of the mass of elementary particles is discussed by MacGregor [10] and that helped to formulate equations (44) – (48) about muon and tau. Equations (46) and (48) prompt to suggest lower radius and higher mass-density for rising in mass of leptons.

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