On Solving Multi-objective Generalized Intuitionistic Fuzzy Linear Programming Problem

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ABSTRACT:

In this paper, Multi-objective Generalized Intuitionistic Fuzzy Linear Programming Problem (MOGIFLPP) has been solved. Ranking technique is applied to convert the given generalized intuitionistic fuzzy number into simple form of membership and non-membership functions. Weighting factor approach is used to transform the Multi-objective Linear Programming Problem into single objective Linear Programming Problem. The constrains are defuzzified through similarity measures. Finally, the formulated crisp linear programming problem has been solved through TORA software for various combinations of weights and the solutions obtained were independent of weights. A numerical example is given to show the efficiency of the proposed approach.

Keywords: Membership/non-membership functions, Weighting Factor, Similarity Measures, Generalized Intuitionistic Fuzzy Number.

1. INTRODUCTION:


Based on these papers, Here Multi-objective generalized Intuitionistic Fuzzy Linear Programming Problem has been solved.
This paper is organized as follows:

Preliminaries which are required to solve the MOGIFLPP are given in section 2. The procedure for solving the given MOGIFLPP is discussed in section 3. A numerical which describes the solution procedure is explained in section 4. This paper is concluded at the end.

2. PRELIMINARIES

2.1. Generalized Intuitionistic Fuzzy Number and Ranking procedure:

A generalized triangular intuitionistic fuzzy number
\[
\tilde{a} = \{(\mu_a, a, \bar{\mu}_a, w_{\tilde{a}})(\nu_a, a, \bar{\nu}_a, u_{\tilde{a}})\}
\]
is an intuitionistic fuzzy set, whose membership and non-membership functions are defined as,

\[
\mu_a(x) =
\begin{cases}
(x - a^{\mu})w_{\tilde{a}} & a^{\mu} \leq x \leq a \\
\frac{a - a^{\mu}}{w_{\tilde{a}}} & x = a \\
\frac{(a^{\mu} - x)w_{\tilde{a}}}{\bar{a}^{\mu} - a} & a \leq x \leq a^{\mu} \\
0 & \text{otherwise}
\end{cases}
\]

(2.1)

\[
\nu_a(x) =
\begin{cases}
\frac{a - x + u_{\tilde{a}}(x - a^{\nu})}{a - a^{\nu}} & a^{\nu} \leq x \leq a \\
\frac{u_{\tilde{a}}}{a^{\nu} - a} & x = a \\
\frac{x - a + u_{\tilde{a}}(a^{\nu} - x)}{\bar{a}^{\nu} - a} & a \leq x \leq a^{\nu} \\
1 & \text{otherwise}
\end{cases}
\]

(2.2)

The values \(w_{\tilde{a}}\) and \(u_{\tilde{a}}\) respectively represent the maximum degree of the membership and non-membership such that \(0 \leq w_{\tilde{a}} \leq 1\), \(0 \leq u_{\tilde{a}} \leq 1\) and \(0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1\).
According to S. Salashour [6], if $\bar{a} = \{(a^\mu, a, a^\mu; w_\bar{a}) \ (a^\nu, a, a^\nu; u_\bar{a})\}$ be a generalized triangular intuitionistic fuzzy number, then the value and ambiguity of $\bar{a}$ are given as follows.

$$V_\mu(\bar{a}) = \frac{(a^\mu + 4a + a^\mu) w_\bar{a}}{6} \quad V_\nu(\bar{a}) = \frac{(a^\nu + 4a + a^\nu) (1 - u_\bar{a})}{6} \quad (2.3)$$

and

$$A_\mu(\bar{a}) = \frac{(a^\mu - a^\mu) w_\bar{a}}{3} \quad A_\nu(\bar{a}) = \frac{(a^\nu - a^\nu) (1 - u_\bar{a})}{3} \quad (2.4)$$

With reference to Sophia Porchelvi et al. [7] and by the use of score function, the value $P(\bar{a})$ is defined as,

$$P(\bar{a}) = S_\mu(\bar{a}) - S_\nu(\bar{a}) \quad (2.5)$$

Where,

$$S_\mu(\bar{a}) = \frac{V_\mu(\bar{a})}{1 + A_\mu(\bar{a})} \quad (2.6)$$

$$S_\nu(\bar{a}) = \frac{V_\nu(\bar{a})}{1 + A_\nu(\bar{a})} \quad (2.7)$$

If $\bar{a}$ and $\bar{b}$ are two generalized triangular intuitionistic fuzzy numbers, then

(i) $\bar{a} < \bar{b}$ if and only if $P(\bar{a}) < P(\bar{b})$

(ii) $\bar{a} > \bar{b}$ if and only if $P(\bar{a}) > P(\bar{b})$

(iii) $\bar{a} \approx \bar{b}$ if and only if $P(\bar{a}) = P(\bar{b})$

The symbol " $<$ " is intuitionistic fuzzy version of the order relation “$<$” in the real number set and has the interpretation as “essentially less than”. The symbols $>$ and $\approx$ are explained similarly.

2.2.Similarity Measures of Intuitionistic Fuzzy Sets:

Let $\varphi(x)$ be the set of all IFS on $X$. Let $s: \varphi(x)^2 \rightarrow [0,1]$ then the degree of similarity between $A \in \varphi(x)$ and $B \in \varphi(x)$ has defined on the matching function as:

$$s(A,B) = \frac{\sum_{j=1}^{n}(\mu_A(x_j) \mu_B(x_j) + \nu_A(x_j) \nu_B(x_j) + \pi_A(x_j) \pi_B(x_j))}{\max(\sum_{j=1}^{n}\mu_A^2(x_j) + \nu_A^2(x_j) + \pi_A^2(x_j) \sum_{j=1}^{n}\mu_B^2(x_j) + \nu_B^2(x_j) + \pi_B^2(x_j))} \quad (2.8)$$

which satisfies the following properties:

$0 \leq s(A,B) \leq 1$;

$s(A,B) = 1$ iff $A=B$

$s(A,B) = s(B,A)$

$s(A,C) \leq s(A,B)$ and $s(A,C) \leq s(B,C)$ if $A \subseteq B \subseteq C, C \in \varphi(x)$
2.3. Weighting Factor:

An ordered weighted averaging (OWA) operators of dimension $n$ is a mapping $f : R^n \rightarrow R$ that has an associated $n$ vector $W$

$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$$

such that (1) $w_i \in [0,1]$  
(2) $\sum_i w_i = 1$  

The Multi-objective generalized Intuitionistic Fuzzy Linear Programming Problem with Intuitionistic fuzzy coefficients can be formulated as

$$\max_{x \in X} \{ z_1(x), z_2(x), \ldots, z_k(x) \}$$

Subject to

$$\sum x_{ij} \geq 0 \left\{ (a_{ij}, b_{ij}, c_{ij}) (a_{ij}', b_{ij}', c_{ij}') \right\} \leq \left\{ (t_i, u_i, v_i) (t_i', u_i', v_i') \right\}$$

$$0 \leq i \leq m; 0 \leq j \leq n$$

Where $z_i : R^n \rightarrow R^i$

Where $R$ be the set of all real numbers and $R^n$ be an $n$-dimensional Euclidean space.

By considering the weighting factor, the Multi-objective generalized Intuitionistic Fuzzy Linear Programming Problem is defined as

$$\max_{x \in X} \{ W_1 z_1(x), W_2 z_2(x), \ldots, W_k z_k(x) \}$$

i.e. $\max_{x \in X} \sum_{m=1}^{k} W_m z_k(x)$

Subject to

$$\sum x_{ij} \geq 0 \left\{ (a_{ij}, b_{ij}, c_{ij}) (a_{ij}', b_{ij}', c_{ij}') \right\} \leq \left\{ (t_i, u_i, v_i) (t_i', u_i', v_i') \right\}$$

$$0 \leq i \leq m; 0 \leq j \leq n$$

Where $W_m \in [0,1]$ and $\sum_{m=1}^{k} W_m = 1$
3. SOLUTION PROCEDURE:

**Step 1:** Consider the MOGIFLPP in which the coefficients of the objective functions and the constraints are in generalized intuitionistic fuzzy numbers.

**Step 2:** Calculate $S_μ(\bar{a})$ and $S_ν(\bar{a})$ as in (2.6) and (2.7) for each generalized intuitionistic fuzzy numbers considered in step 1.

**Step 3:** Defuzzify the objective functions through Ranking technique $P(\bar{a})$ as in (2.5).

**Step 4:** Transform Multi-objective functions into single objective function using weighting factor as in (2.10)

**Step 5:** Defuzzify the constraints through similarity measures as in (2.8).

**Step 6:** Formulate the crisp linear programming problem obtained in step 4 subject to the constraints in step 5.

**Step 7:** Solve the formulated crisp linear programming problem obtained in step 6 for various combinations of weights using TORA software.

4. NUMERICAL EXAMPLE:

Consider the Multi-objective Generalized Intuitionistic fuzzy linear programming problem

Max $Z_1 = \{(1.6,2,2.6,0.5),\}
\{(1.5,2,2.5,0.5)\}x_1 + \{(2.4,3,3.3,0.5),\}
\{(2.3,3,3.2,0.2)\}x_2 + \{(3.3,4,4.3,0.3),\}
\{(3.2,4,4.2,0.4)\}x_3$

Max $Z_2 = \{(0.6,1,1.4,0.5),\}
\{(0.8,1,1.3,0.4)\}x_1 + \{(1.8,2,2.4,0.3),\}
\{(1.4,2,2.3,0.7)\}x_2 + \{(2.4,3,3.6,0.3),\}
\{(2.5,3,3.2,0.4)\}x_3$

Max $Z_3 = \{(0.6,1,1.3,0.6),\}
\{(0.4,1,1.2,0.2)\}x_1 + \{(1.9,2,2.2,0.6),\}
\{(1.8,2,2.3,0.4)\}x_2 + \{(2.6,3,3.4,0.6),\}
\{(2.2,3,3.2,0.1)\}x_3$

Subject to the constraints,

$\{(0.6,1,1.2,0.6),\}
\{(0.7,1,1.4,0.2)\}x_1 + \{(1.2,2,2.2,0.4),\}
\{(1.6,2,2.4,0.3)\}x_2 + \{(2.8,3,3.2,0.5),\}
\{(2.3,3,3,0.2)\}x_3 \leq \{(2.4,3,3.4,0.3),\}
\{(2.1,3,3,2,0.2)\}$

$\{(0.8,1,1.4,0.3),\}
\{(0.7,1,1.6,0.4)\}x_1 + \{(1.6,2,2.5,0.6),\}
\{(1.5,2,2.8,0.3)\}x_2 + \{(2.3,3,3,3,0.2),\}
\{(2.5,3,3,50.5)\}x_3 \leq \{(2.8,3,3,2,0.6),\}
\{(2.5,3,3,6,0.3)\}$

$\{(0.9,1,1.2,0.4),\}
\{(0.6,1,1.4,0.6)\}x_1 + \{(1.4,2,2.6,0.2),\}
\{(1.3,2,2.4,0.6)\}x_2 + \{(2.2,3,3,6,0.4),\}
\{(2.4,3,3,8,0.2)\}x_3 \leq \{(2.2,3,3,2,0.2),\}
\{(2.6,3,3,3,0.4)\}$
Using $S_μ(\bar{a})$ & $S_ν(\bar{a})$ as in equation (2.6) & (2.7), the given problem becomes

Max $Z_1 = (0.87,0.85)x_1 + (1.29,0.55)x_2 + (2.24,1.38)x_3$

Max $Z_2 = (0.44,0.38)x_1 + (0.58,0.54)x_2 + (1.64,1.08)x_3$

Max $Z_1 = (0.36,0.18)x_1 + (1.14,1.10)x_2 + (1.08,0.28)x_3$

Subject to the constraints,

\[
\begin{align*}
(0.52,0.68)x_1 + (0.67,1.18)x_2 + (1.40,1.85)x_3 & \leq (1.69,0.54) \\
(0.29,0.39)x_1 + (1.71,1.11)x_2 + (0.55,1.28)x_3 & \leq (1.14,0.82) \\
(0.39,0.36)x_1 + (0.37,0.60)x_2 + (1.11,2.08)x_3 & \leq (1.57,0.54)
\end{align*}
\]  \quad (4.1)

Using score function $P(\bar{a})$ as in (2.5), the coefficients of the objectives are converted into the simple form as,

Max $Z_1 = (0.02)x_1 + (0.74)x_2 + (0.86)x_3$

Max $Z_2 = (0.06)x_1 + (0.04)x_2 + (0.56)x_3$

Max $Z_3 = (0.18)x_1 + (0.04)x_2 + (0.80)x_3$

Using similarity measures as in (2.8) the constraints are converted into the following form,

\[
\begin{align*}
(0.25)x_1 + (0.22)x_2 + (0.22)x_3 & \leq (1.15) \\
(0.14)x_1 + (0.43)x_2 + (0.20)x_3 & \leq (0.32) \\
(0.44)x_1 + (0.12)x_2 + (0.13)x_3 & \leq (1.03)
\end{align*}
\]  \quad (4.2)

Using weighting factor as in (2.9), multiobjective function becomes,

Max $Z = w_1Z_1 + w_2Z_2 + w_3Z_3$

Subject to the constraints (4.2)

Using TORA software we obtain the solution for different weights.

For example, $w_1 = 0, w_2 = 0, w_3 = 1$

Max $Z = (0.18)x_1 + (0.04)x_2 + (0.80)x_3$
Subject to the constraints (4.2)

The Optimal solution is \( Z = 1.280, x_1 = 0, x_2 = 0, x_3 = 1.6 \)

Following table lists the solution for the above problem for various combinations of weights.

<table>
<thead>
<tr>
<th>S.NO</th>
<th>( W_1 )</th>
<th>( W_2 )</th>
<th>( W_3 )</th>
<th>((x_1, x_2, x_3))</th>
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<td>0</td>
<td>1</td>
<td>(0,0,1.6)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
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</tr>
<tr>
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<td>1</td>
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<td>(0,0,1.6)</td>
</tr>
<tr>
<td>4</td>
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<td>0.3</td>
<td>(0,0,1.6)</td>
</tr>
<tr>
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</tr>
<tr>
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<td>0.6</td>
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<td>0.2</td>
<td>(0,0,1.6)</td>
</tr>
<tr>
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<td>0.3</td>
<td>(0,0,1.6)</td>
</tr>
<tr>
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<td>0.3</td>
<td>0.1</td>
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<tr>
<td>9</td>
<td>0.2</td>
<td>0.7</td>
<td>0.1</td>
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</tr>
<tr>
<td>10</td>
<td>0.6</td>
<td>0.4</td>
<td>0</td>
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</tr>
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</table>

**CONCLUSION:**

In this paper, Generalized Intuitionistic Fuzzy Linear Programming Problem has been solved. Ranking Technique and Similarity Measures are used for defuzzification. TORA software is used to obtain the solution and the obtained solution are identical for various combinations of weights.
References:


