LABELING OF SOFT GRAPH AND ITS APPLICATIONS

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Abstract:

A new notion, labeling of soft graph is introduced. We initiate various types of labeling of soft graphs. Also, we investigate some theorems and discuss an application in error correcting code problems related to this concept.

Keywords: Labeling of soft graph, hamming distance labeling of soft graph, Dominated set labeling of soft graph.

Introduction:

In 1966, A. Rosa\(^1\) introduced a new graph labeling called $\beta$ – labeling. A few years later, S.W. Golomb\(^2\) renamed $\beta$ – labeling as graceful labeling as it is known today. In the intervening years many graph labeling techniques have been studied and still getting embellished due to increasing number of application driven concepts. A novel concept called soft set theory which deals with uncertainty was introduced by Molodtsov\(^3\) in 1999. Type 2 soft set, type 2 soft graphs and their applications were discussed by Khizar Hayat and et al.\(^4\) fuzzy parameterized fuzzy soft set and some of its applications were introduced by Naim Cagman and others\(^5\). Error correcting codes concerned with improving reliability of communication over noisy channels. Cryptography is concerned with security, privacy or confidentiality of communication over an insecure channel. Some applications of coding theory in Cryptography were introduced by J.L. Massey\(^6\) in 1995. A new notion on soft graph using soft sets was introduced by S.Venkatraman et.al\(^7\). In this paper, we generate labeling of soft graph using soft sets and discussed some of its applications.

The construction of this paper is as follows. Section 1 deals with preliminaries. We introduced labeled soft graph and graceful dominated set labeling of soft graph in Section 2. A useful application of Hamming distance labeling of soft graph on coding theory was discussed in Section 3.

1. PRELIMINARIES:

Definition 1.1: Soft set

Let $U$ be an universal set and $A$ be a set of parameters. A Pair $(F, A)$ is called a soft set over $U$ if and only if $F$ is a mapping of $A$ into the set of all subsets of the set $U$.

i.e. $F : A \rightarrow \mathcal{P}(U)$ where $\mathcal{P}(U)$ is the power set of $U$. 
Definition 1.2: Absolute Soft set
The soft set \((F, A)\) over \(U\) is said to be an absolute soft set denoted by \(\tilde{A}\), if \(\forall e \in A, F(e) = U\).

Definition 1.3: Soft Graph
A soft graph \(G_s\) is defined by \(G_s = (V, E, F, e)\). \(V\) is a universal set. \(A_{v_1}\) is the set of primary Attributes (first order parameters). \(F\) is a relation such that \(F : V \rightarrow P(V)\). Let \(V = \bigcup F(e_i)\) such that \(e_i \in A_{v_1}\).

\[\xi = \{ (v_i \times v_j) / v_i, v_j \in V \}.\]
The elements of \(V\) are considered as vertices of the soft graph \(G_s\).

\(\xi\) contains all possible edges. \(F_2\) is a relation such that \(F_2 : A_{v_1} \rightarrow P(\xi)\). Let \(E = \bigcup F_2(e_i)\) such that \(e_i \in A_{v_1}\). The elements of \(E\) are the edges of \(G_s\).

Definition 1.4: Level - 1 flow
Level – 1 – flow is a relation from \(v_i\) to \(v_j\) for \(i, j \in \mathbb{N}\) with respect to \(A_{v_1}\).

Definition 1.5: Level - 1 - soft number
Level – 1 – soft number corresponding to Level – 1 – flow is defined as

\[
\begin{pmatrix}
\frac{a_i}{a_j} \\
\frac{a_i}{a_j}
\end{pmatrix}^{11 \to 1m}
= \begin{pmatrix}
\frac{a_i}{a_j} \\
\frac{a_i}{a_j}
\end{pmatrix}^{11} \begin{pmatrix}
\frac{a_i}{a_j} \\
\frac{a_i}{a_j}
\end{pmatrix}^{12} \ldots \begin{pmatrix}
\frac{a_i}{a_j} \\
\frac{a_i}{a_j}
\end{pmatrix}^{lm}, \quad 1 \leq m \leq n(A_{v_1}), \quad i \neq j \text{ and } i < j
\]

Let \(L_{1m} S \left( \begin{pmatrix}
\frac{a_i}{a_j}
\end{pmatrix}^{11} \right) \) or \(\begin{pmatrix}
\frac{a_i}{a_j}
\end{pmatrix}^{11 \to 1m}\) where \(\begin{pmatrix}
\frac{a_i}{a_j}
\end{pmatrix}^{11}\) is corresponding to first parameter of \(A_{v_1}\). Here \(a_i\) and \(a_j\) denote inward weight of \(v_i\) from \(v_j\) and outward weight of \(v_i\) to \(v_j\) respectively. Inward and outward weights may be given in percentage or it may be belong to \(\mathbb{R}\).

Definition 1.6: Graceful graph
Let \(G\) be a \((p, q)\) graph and let \(f : V(G) \rightarrow \{0, 1, ..., q\}\) be an injective function such that when each edge \(uv\) is assigned the value \(g(uv) = |f(u) - f(v)|\), the resulting edge labels are all distinct. Then \(f\) is a graceful labeling of \(G\). A graph that admits a graceful labeling is called a graceful graph.

Definition 1.7: Binary Sum
Let \(B = \{0, 1\}\), then \(B^n = \{(x_1, x_2, ..., x_n) \mid x_i \in B, i = 1, 2, ..., n\}\) is a group under the binary operation \(\oplus\) defined by \(x \oplus y = (x_1, x_2, ..., x_n) \oplus (y_1, y_2, ..., y_n) = (x_1 + y_1, x_2 + y_2, ..., x_n + y_n)\).
where \( x_i + y_i \) denotes the binary sum of \( x_i \) and \( y_i \).

**Definition 1.8: Weight of \( x \)**
Let \( x \in B^n \). Then the weight \( |x| \) of \( x \) is the number of 1’s in \( x \).

**Definition 1.9: Hamming distance**
Let \( x, y \in B^n \). Then the Hamming distance \( \delta(x, y) \) between \( x \) and \( y \) is the weight of \( x \oplus y \) which is same as the number of places \( i \) such that \( x_i \neq y_i \).

### MAIN RESULTS

#### 2. LABELED SOFT GRAPH:

**Definition 2.1: Labeled of soft graph**
Let \( V \) be the universal set. Let the parameter set \( A \) be contained in or equal to the group of integer \( \mod(n+1) \).

Let \((F, A)\) be a soft set with the relation \( F : A \rightarrow P(V) \) defined by
\[
F(l) = \left\{ v_i \in V \setminus v_i = l \text{ for } l \in A \right\}.
\]
Let \((K, A)\) be a soft set over \( \xi = V \times V \) with the relation
\[
K : A \rightarrow P(\xi) \text{ defined by } K(p) = \left\{ v_i v_j \in \xi \setminus F(v_i) \ast F(v_j) = p \right\} \text{ for all } p \in A \text{ and } p \leq |\xi| \text{ where } \ast \text{ is a binary operation.}
\]
Then the labeled soft graph (LSG) is \( \langle F, K, A \rangle \).

**Definition 2.2: \( \beta \) – valuation of soft graph**
Labeling of soft graph admits graceful or \( \beta \) – valuation if \( |\xi| = n \) and \( \ast \) as absolute difference with \( K(p) \neq K(q) \) for all \( p, q \in A \).

**Example 2.1:**
Let \( V = \{ v_1, v_2, v_3, v_4 \} \), \( \xi = \{ v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_1 \} \), \( A = \{0,1,2,3,4\} \). \( F(0) = \{ v_1 \} \), \( F(1) = \{ \} \), \( F(2) = \{ v_3 \} \), \( F(3) = \{ v_4 \} \), \( F(4) = \{ v_2 \} \), \( K(0) = \{ \} \), \( K(1) = \{ v_3, v_4 \} \), \( K(2) = \{ v_2 v_3 \} \), \( K(3) = \{ v_1 v_4 \} \), \( K(4) = \{ v_1 v_2 \} \).

![Figure 2.1: LSG = C_4](image)

**Figure 2.1: LSG = C_4**
Definition 2.3: α – labeling of soft graph
A graceful labeling of soft graph admits α – labeling if there exists an integer γ with $0 \leq \gamma \leq \lceil \xi \rceil$ such that $\min \{F(l), F(m)\} \leq \gamma < \max \{F(l), F(m)\}$ for every edge corresponding to $F(l)F(m)$ of LSG. Then it is said to be α – labeling of LSG.

Example 2.2:
In Figure 2.1, $\gamma = 2$.

Proposition 2.1:
All graceful graphs are labeled soft graphs.
Proof: Since the graph G is itself graceful and by our definition of LSG, we have LSG=G.

Definition 2.4: Dominating set labeling of soft graph
A LSG which has a minimum independent dominated set is called dominated set labeled soft graph (DLSG).

Theorem 2.1:
All $K_{m,n}$ are graceful DLSG.
Proof: In $K_{m,n}$,
(i) if $m = n$, then obviously $K_{m,n}$ is a graceful DLSG.
(ii) If $m \neq n$ and $m < n$, then the minimum independent dominated set has $m$ vertices and the maximum matching has $m$ edges in which those vertices are saturated. So, all $K_{m,n}$ are graceful DLSG.

Proportion 2.2:
Not all graceful graphs are graceful DLSG.

Remark 2.1:
$K_3$ is graceful DLSG but, $K_4$ is not a graceful DLSG.

Open problem:
Does it Sufficient for a DLSG to be graceful is the cardinality of minimum independent dominating set and the cardinality of maximum matching are equal such that the dominated vertices are saturated other than bipartite graphs ?
Definition 2.5: Hamming distance labeled soft graph
Let \( e : B^m \rightarrow B^n \) be the encoding function, where \( n > m \). Let \( V = \{ z \in B^n \mid e(x) = z \text{ for } x \in B^m \} \) be the universal set. Let the parameter set \( A \) be contained in or equal to the group of integer \( \mathbb{Z} \mod (n+1) \).

Let \( (F, A) \) be a soft set with the relation \( F : A \rightarrow p(V) \) defined by \( F(l) = \{ z \in V \mid |z| = l \} \) for all \( l \in A \). Let \((K, A)\) be a soft set over \( X \) with the relation \( K : A \rightarrow p(\xi) \) defined by \( K(p) = \{ z^1, z^2 \in \xi \mid \delta(z^1, z^2) = p \} \) for all \( p \in A \) and \( p \leq |\xi| \) where * is a binary operation.

Then the humming distance labeled soft graph (HLSG) is denoted by \( \langle F, K, A \rangle \).

Definition 2.6: Decoding function
\( K^*(q) = \{ z^* \mid z \in \xi : \delta(z^*, z) = q \} \) for all \( q \in A \), where \( z^* \) is the receiving word with \( q \) errors and \( z \) is the corrected code word, then \( K^* \) behaves like a decoding function.

Theorem 2.2:
An \((m, n)\) encoding function \( e : B^m \rightarrow B^n \) can detect \( l \) or fewer errors iff \( \min(P) \geq l + 1 \) where \( K(P) = \{ z^1, z^2 : \delta(z^1, z^2) = P \} \).

Proof: If part
Suppose \( \min(P) \geq l + 1 \). Let \( x \in B^m \) and \( e(x) = z \) be transmitted as \( z^* \).

If \( z^* \) were a code word different from \( z \), then \( K(P) = \{ z^1, z^2 : \delta(z^*, z) = P \geq l + 1 \} \).

Thus if \( z \) is transmitted with \( l \) or fewer errors \((1 \leq \delta(z^*, z) \leq l)\) then \( z^* \) cannot be a code word. So \( e \) can detect \( l \) or fewer errors.

Only if part
Suppose \( \min(P) \leq l \). Let \( z^1 \) and \( z^2 \) be two code words such that \( K(P) = \{ z^1, z^2 : \delta(z^1, z^2) = P \} \).

If \( z^2 = z^* \), then \( z^1 \) is transmitted as a code word \( z^2 \). Since \( z^2 \) is also a code word, the \( P \) errors committed in transmitting \( z^1 \) as \( z^* = z^2 \) have not been detected. This is a contradiction since \( e \) detects \( l \) or fewer errors. Hence \( \min(P) > l \) that is \( \min(P) \geq l + 1 \).
3. Application of HLSG in Error Correcting Group Code:

Consider H be the subgroup of code words of the group $B^n$.

$$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The corresponding encoding function $e_h : B^3 \to B^8$ is given by $e_h(b_1, b_2, b_3) = b_1 b_2 b_3 c_1 c_2 c_3$

where $c_1 = b_1 + b_2, c = b_1 + b_2 + b_3$

The code words are 000000, 001011, 010101, 011110, 100111, 101100, 110010, 111001.

Denote these as $z_0, z_1, z_2, z_3, z_4, z_5, z_6, z_7$ respectively.

Now without constructing decoding table we can do the detection of single error as well as double errors in a group code using HLSG.

Construct HLSG like a star (here $K_{1,7}$) as in the figure 3.1.

Let $V$ be the set of all code words received. $A = \{0, 3, 4\}$.

Take $F(l)$ and $K(p)$ as in the definition of HLSG.

$F(0) = \{z_0\}$

$F(3) = \{z_1, z_2, z_5, z_6\}$

$F(4) = \{z_3, z_4, z_7\}$

$K(0) = \{\}$

$K(3) = \{z_0^0 z_1^1, z_0^0 z_2^2, z_0^0 z_5^5, z_0^0 z_6^6\}$

$K(4) = \{z_0^3 z_3^3, z_0^4 z_4^4, z_0^7 z_7^7\}$

Now suppose the word received with single error is 101111. Since its weight is 5, it should be a pendent vertex adjacent to the vertices of F(4).

As received word has single error, the hamming distance between this received word and the vertices of F(4) should be 1. Here $z* = 101111$ and $z = z_4$, then $K*(1) = \{z^* z^4\}$. So the corrected code word is 100111 and the error has occurred in the third place. The original message is 100.
If the received word 111111 has a double error then the hamming distance between this word and the vertices of F(4) should be 2. Take \( z^* = 111111 \) then \( K^*(2) = \{ z^* z^3, z^* z^4, z^* z^7 \} \). So the corrected code words may be 011110, 100111, or 111001.

**Figure 3.1:** HLSG = \( K_{1,7} \)

**Conclusion:**
In this paper, we initiated labeling in soft graphs. Also we introduced hamming distance labeling of a soft graph, using this we dealt an application in detection of error correcting group code without constructing decoding table.

**References:**