M/M/1/N Fuzzy Queueing Models with Discouraged Arrivals under Wingspans Fuzzy Ranking Method

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Abstract: This paper presents an analysis of performance measures for M/M/1/N Fuzzy Queueing models with Discouraged arrivals by Wingspans fuzzy ranking method. Ranking techniques are very noteworthy in the fuzzy numbers system for defuzzification. Many authors have already proposed various types of techniques to find out the performance of fuzzy queues. It is possible to convert from fuzzy environment to crisp environment by our proposed ranking method in order to analyze the performance measures of fuzzy queues. Finally, the effectiveness and the accurate values of our proposed method have been successfully solved by an example and also compared by graphically.

Key words: Fuzzy sets, Fuzzy numbers, M/M/1/N Fuzzy Queueing models with Discouraged arrivals ,Wingspans fuzzy ranking.

INTRODUCTION

Queueing theory plays an important role in real life problems. Now-a-days, we face a lot of congesting problems in the queueing environment such as ATM points, Medical shops, Reservation centers, Ration shops, Hospitals, Making calls in Telecommunications, etc…, . Stress the importance of time management is the ultimate aim of the researcher. At this juncture, queueing models take a very prominent role.

The basic preliminaries [6],[8],[13], and models of queueing are very essential for our research purpose. In our day to day life situation, most of the time we apply the Fuzzy logic and applications[14],[25]. Queues with discouraged arrivals[2],[3],[4] have applications in computers with job processing where job submissions are discouraged when the system is used
frequently and arrivals are modeled as a Poisson process with state dependent arrival rate. The discouragement affects the arrival rate of the queueing system. In computer and communication systems, the congestion control mechanisms is employed to prevent the formation of long queues by controlling the transmission rates of packets based on the queue length of packets at source or destination. Kumar and Sharma [15] apply M/M/1/N queuing system with retention of reneged customers. Natvig [16] studies the single server birth-death queuing process with state dependent parameters $\lambda_n = \frac{1}{n+1} \lambda$, $n \geq 0$, $\mu_n = \mu$, $n \geq 1$. He reviews state dependent queuing models of different kind and compares his results with M/M/1, M/D/1 and D/M/1 and the single server birth-and-death queuing model with parameters $\lambda_n = \lambda$, $n \geq 0$ and $\mu_n = n\mu$, $n \geq 1$ numerically.

Many authors have so far applied various ranking techniques to measure the performances of the fuzzy queues. Area based ranking techniques [5],[7],[10] are considered as very well known ranking techniques. Particularly some of the authors applied centroid based ranking techniques[19],[22],[23]. Westman and Wang applied Ranking Fuzzy Numbers by Their Left and Right Wingspans [24]. Our proposed ranking technique to measure the performance of M/M/1/N Fuzzy Queueing models with Discouraged arrivals. This is a very easy method to compute the actual crisp values of the queueing models.

PRELIMINARIES

**Fuzzy Set:** A Fuzzy Set $\tilde{A} = \{(x, \phi_{\tilde{A}}(x)); x \in U\}$ is concluded by a membership function $\phi_{\tilde{A}}$ mapping from elements of a universe of discourse $U$ to the unit interval $[0,1]$.

(i,e) $\phi_{\tilde{A}}: U \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\tilde{A}$ and $\phi_{\tilde{A}}(x)$ is called the membership value of $x \in U$ in the fuzzy set $\tilde{A}$.

**Triangular Fuzzy Number:** A Triangular Fuzzy Number $\tilde{A}(x)$ is represented by

$$\tilde{A}(a_1, a_2, a_3)$$ with the membership function $\phi_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & x = a_2 \\ \frac{x-a_3}{a_2-a_3}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$
Trapezoidal Fuzzy Number: A Trapezoidal Fuzzy Number \( \tilde{A}(x) \) is represented by

\[
\tilde{A}(a_1, a_2, a_3, a_4) \text{ with the membership function } \\
\phi_{\tilde{A}}(x) = \\
\begin{cases} 
\frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\
1, & a_2 \leq x \leq a_3 \\
\frac{x-a_4}{a_3-a_4}, & a_3 \leq x \leq a_4 \\
0, & \text{otherwise}
\end{cases}
\]

M/M/1/N QUEUEING MODELS WITH DISCOURAGED ARRIVALS

Let us consider the customers arrive at single channel queue in respect of a Poisson process with the fuzzy rate \( \tilde{\lambda} \) and that they are waiting for service in FIFO discipline with the service time as an exponential distribution with fuzzy rate \( \tilde{\mu} \). There is one server and a customer finding the server busy arrive with arrival rate that depends on the number of customers present in the system at that time i.e. if there are \( n \) (\( n > 1 \)) customers in the system, the new customer enters the system with rate \( \tilde{\lambda}_n = \frac{\tilde{\lambda}}{n+1} \). The capacity of the system is taken as finite, say \( N \). Each customer upon joining the queue will wait a certain length of time, say \( T \) for his service to begin, if it does not begin by then, he may get reneged and may leave the queue without getting service with probability \( p \) and may remain in the queue for his service with probability \( q (= 1 - p) \) if certain customer retention strategy is applied. The times \( T \) follow exponential distribution with parameter \( \tilde{\xi} \). Let \( P_n \) be the probability that there are \( n \) customers in the system. Using the Markov chain theory, the steady-state equations of the model are

\[ \tilde{\lambda} P_0 = \tilde{\mu} P_1 \] .............................................................(1)

\[
\begin{pmatrix} \frac{\tilde{\lambda}}{n+1} + \tilde{\mu} + (n-1)\tilde{\xi}p \end{pmatrix} P_n = [\tilde{\mu} + n\tilde{\xi}p] P_{n+1} + \left( \frac{\tilde{\lambda}}{n} \right) P_{n-1} \] .................................(2)

\[ , 1 \leq n \leq N-1 \]

\[
\left( \frac{\tilde{\lambda}}{N} \right) P_{N-1} = [\tilde{\mu} + (N-1)\tilde{\xi}p] P_N \] .............................................................(3)
By the queuing theory concepts under above steady-state conditions we get

(i) The Expected Number of Customers in the System \((\text{N}_S) = \sum_{n=1}^{N} nP_n\)

\[\Rightarrow (\text{N}_S) = \sum_{n=1}^{N} n \left[ \frac{1}{n!} \prod_{k=1}^{n} \frac{\tilde{\lambda}}{\tilde{\mu} + (k-1)\tilde{\xi}p} \right] P_0\]

(ii) The Expected Number of Customers Served \((\text{E(C S)}) = \sum_{n=1}^{N} \tilde{\mu}P_n\)

\[\Rightarrow (\text{E(C S)}) = \tilde{\mu} \sum_{n=1}^{N} \left[ \frac{1}{n!} \prod_{k=1}^{n} \frac{\tilde{\lambda}}{\tilde{\mu} + (k-1)\tilde{\xi}p} \right] P_0\]

(iii) Average Reneging

Rate \((R_r) = \sum_{n=1}^{N} (n-1)\tilde{\xi}pP_n\)

\[\Rightarrow (R_r) = \sum_{n=1}^{N} (n-1)\tilde{\xi}p \frac{1}{n!} \left[ \prod_{k=1}^{n} \frac{\tilde{\lambda}}{\tilde{\mu} + (k-1)\tilde{\xi}p} \right] P_0\]

(iv) Average Retention

Rate \((R_R) = \sum_{n=1}^{N} (n-1)\tilde{\xi}qP_n\)

\[\Rightarrow (R_R) = \sum_{n=1}^{N} (n-1)\tilde{\xi}q \frac{1}{n!} \left[ \prod_{k=1}^{n} \frac{\tilde{\lambda}}{\tilde{\mu} + (k-1)\tilde{\xi}p} \right] P_0\]

Where \(P_0 = \frac{1}{1 + \sum_{n=1}^{N} \frac{1}{n!} \prod_{k=1}^{n} \frac{\tilde{\lambda}}{\tilde{\mu} + (k-1)\tilde{\xi}p}}\)
WINGSPANS FUZZY RANKING METHOD – ALGORITHM

Let \( \tilde{A} \) be a fuzzy number, \( \phi_{\tilde{A}} \) be its membership function and let \( a_0 \) be a core point of \( \tilde{A} \). Then \( W_\tilde{A} = a_0 - \frac{1}{2} \int_{-\infty}^{a_0} \phi_{\tilde{A}}(x)dx + \frac{1}{2} \int_{a_0}^{\infty} \phi_{\tilde{A}}(x)dx \) is called the Wingspans center of \( \tilde{A} \). It is evident that for any fuzzy number, the wingspans center is its symmetric center. This is based on the area between the curve of the membership function and the horizontal real axis.

By our calculations,

For triangular fuzzy number

\( \tilde{A} = [a_l \ a_0 \ a_r] \), its wingspans center and also our proposed ranking function is

\[ R(\tilde{A}) = \frac{1}{2} a_0 + \frac{1}{4} (a_l + a_r). \]

For trapezoidal fuzzy number

\( \tilde{A} = [a_l \ a_b \ a_c \ a_r] \), its wingspans center and also our proposed ranking function is

\[ R(\tilde{A}) = \frac{1}{4} (a_l + a_b + a_c + a_r). \]

NUMERICAL EXAMPLE

In India the Goddess Mariamman temple is situated at Tamilnadu state, Trichy district, Samayapuram village. Every year there is celebrated a great chariot festival at the first Tuesday of Tamil month Chithirai. In that festival a large number of devotees come and worship the Goddess Mariamman. At that time a very long queue is formed by the devotees. If arriving devotees see a large number in the system they may not join the queue. So the mean arrival rate progressively decreases as the state of the system goes up. This is as if we are discouraging new arrivals (Discouraged arrivals) to this queue, when there are more devotees waiting in the queue for service. In this situation we consider only a finite number of devotees (N) and single server modeled queue (cost free queue(M/M/1/N)). The devotees are arrived by a fuzzy arrival rate \( \tilde{\lambda} \) and served by a fuzzy service rate \( \tilde{\mu} \) with time distribution parameter \( \xi \).

Here we take the probabilities of the reneged (p), retained devotees (q) as \( p = 0,0.1,0.2,\ldots,0.9,1, \quad q=1,0.9,0.8,\ldots,0.1,0 \) and \( N = 10 \). At this juncture, we are calculating the Expected Number of Customers in the System, the Expected Number of Customers Served, the Average Reneging Rate and Average Retention Rate.
A: For Triangular fuzzy number

Consider the arrival rate $\tilde{\lambda} = [1,2,3]$, the service rate $\tilde{\mu} = [4,4.5,5]$ and time distribution parameter $\tilde{\xi} = [0.1,0.2,0.3]$ per hour respectively.

Now the membership function of the triangular fuzzy number $[1,2,3]$ is

$$\phi_\lambda(x) = \begin{cases} 
\frac{(x-1)}{(2-1)}, & 1 \leq x \leq 2 \\
1, & x = 2 \\
\frac{(x-3)}{(2-3)}, & 2 \leq x \leq 3 \\
0, & \text{otherwise}
\end{cases}$$

Similarly we can proceed for all remaining triangular fuzzy rates in this same way.

Now we apply the Wingspans Fuzzy Ranking Method to the triangular fuzzy numbers.

$$R(\tilde{\lambda}) = R(1,2,3) = \frac{1}{2} (2) + \frac{1}{4} (1+3) = 2.$$ Similarly $R(\tilde{\mu}) = 4.5$, $R(\tilde{\xi}) = 0.2$

By our calculations we tabulate the following numerical results for different measures of performance. Here we calculate the variation in performance measures with respect to $q$.

<table>
<thead>
<tr>
<th>S.No</th>
<th>$q$</th>
<th>$N_s$</th>
<th>$E(CS)$</th>
<th>$R_r$</th>
<th>$R_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.4549</td>
<td>1.6867</td>
<td>0.0379</td>
<td>0</td>
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<tr>
<td>2</td>
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<td>1.6890</td>
<td>0.0341</td>
<td>0.0037</td>
</tr>
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<td>1.6915</td>
<td>0.0304</td>
<td>0.0076</td>
</tr>
<tr>
<td>4</td>
<td>0.3</td>
<td>0.4577</td>
<td>1.6934</td>
<td>0.0266</td>
<td>0.0114</td>
</tr>
<tr>
<td>5</td>
<td>0.4</td>
<td>0.4588</td>
<td>1.6957</td>
<td>0.0228</td>
<td>0.0152</td>
</tr>
<tr>
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<td>0.4598</td>
<td>1.6974</td>
<td>0.0190</td>
<td>0.0190</td>
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<tr>
<td>7</td>
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<td>0.4607</td>
<td>1.6995</td>
<td>0.0151</td>
<td>0.0227</td>
</tr>
<tr>
<td>8</td>
<td>0.7</td>
<td>0.4618</td>
<td>1.7012</td>
<td>0.0114</td>
<td>0.0267</td>
</tr>
<tr>
<td>9</td>
<td>0.8</td>
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<td>1.7035</td>
<td>0.0076</td>
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<tr>
<td>10</td>
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<td>0.4637</td>
<td>1.7048</td>
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<td>0.0381</td>
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</table>

Table 1: Performance measures Vs $q$
The above results states that as we increase the probability of retaining a reneging customer there is a steady increase in the average number of customers in the system, average number of customers served and average retention rate. Here, the average reneging rates decrease subsequently on increasing q. Thus, one can study the effect of different probabilities of retaining reneging customers on the different performance measures. When q = 0, there is no customer retention and therefore \( R = 0 \). Also, when q = 1, \( R = 0 \), that is, all reneging customers are retained.

**B: For Trapezoidal fuzzy number**

Consider the arrival rate \( \tilde{\lambda} = [1,2,3,4] \),

the service rate \( \tilde{\mu} = [3.5,4,4.5,5] \) and time distribution parameter \( \tilde{\xi} = [0.1,0.2,0.3,0.4] \) per hour respectively.

Now the membership function of the trapezoidal fuzzy number \([1,2,3,4]\) is

\[
\phi_{\tilde{\lambda}}(x) = \begin{cases} 
\frac{(x-1)}{(2-1)}, & 1 \leq x \leq 2 \\
1, & 2 \leq x \leq 3 \\
\frac{(x-4)}{(3-4)}, & 3 \leq x \leq 4 \\
0, & \text{otherwise}
\end{cases}
\]

Similarly we can proceed for all remaining trapezoidal fuzzy rates in this same way.

Now we apply the Wingspans Fuzzy Ranking Method to the trapezoidal fuzzy numbers.

\[
R(\tilde{\lambda}) = R([1,2,3,4]) = \frac{1+2+3+4}{4} = 2.5
\]

Similarly \( R(\tilde{\mu}) = 4.5 \), \( R(\tilde{\xi}) = 0.25 \)
By our calculations we tabulate the following numerical results for different measures of performance. Here we calculate the variation in performance measures with respect to q.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Q</th>
<th>N_s</th>
<th>E(CS)</th>
<th>R_f</th>
<th>R_R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.5738</td>
<td>2.0510</td>
<td>0.0302</td>
<td>0</td>
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<tr>
<td>2</td>
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<td>0.5757</td>
<td>2.0541</td>
<td>0.0272</td>
<td>0.0030</td>
</tr>
<tr>
<td>3</td>
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<td>0.5777</td>
<td>2.0575</td>
<td>0.0243</td>
<td>0.0060</td>
</tr>
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<td>0.0157</td>
</tr>
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<td>0.5913</td>
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</tr>
<tr>
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<td>1.0</td>
<td>0.5933</td>
<td>2.0837</td>
<td>0</td>
<td>0.0318</td>
</tr>
</tbody>
</table>

Table 2 : Performance measures Vs q

The above results states that as we increase the probability of retaining a reneging customer there is a steady increase in the average number of customers in the system, average number of customers served and average retention rate. Here, the average reneging rates decrease subsequently on increasing q. Thus, one can study the effect of different probabilities of retaining reneging customers on the different performance measures. When q = 0, there is no customer retention and therefore R_R = 0. Also, when q = 1, R_f = 0, that is, all reneging customers are retained.

RESULT & DISCUSSION

In this section, we have made an attempt to analysis the results obtained from the section V. The graphical representations of the results are given as follows:
Figure 1: Analysis for Ns

Figure 2: Analysis for E(CS)

Figure 3: Analysis for Rr

Figure 4: Analysis for RR
From above figures, we get minimum scores for $N_s$ and $E(CS)$ and maximum scores for $R_r$ and $R_R$ for triangular fuzzy numbers comparing with the trapezoidal fuzzy numbers corresponding to the various probability values. Moreover, we are getting the minimum score difference for $N_s$ and $E(CS)$ and maximum score difference for $R_r$ and $R_R$ for triangular fuzzy numbers comparing with the trapezoidal fuzzy numbers corresponding to the various probabilities. So triangular fuzzy would give better accuracy for $R_r$ and $R_R$ whereas trapezoidal fuzzy number would give better accuracy for $N_s$ and $E(CS)$ corresponding to the various probability values based on the proposed method under Wingspans Fuzzy Ranking function.

CONCLUSION

In this paper, we have analyzed the new methodology for finding the performance measures of $M/M/1/N$ Fuzzy Queueing models with discouraged arrivals under Wingspans Fuzzy Ranking Method. We may use this methodology for various Fuzzy Queues instead of using the existing methods. This methodology not only gives the crisp values but also gives more accuracy than the other values. This methodology will be useful and helpful to all the researchers and inventors in the days to come.

REFERENCES


