Laminar flow heat transfer analysis inside coiled tubes of Newtonian and Non-Newtonian Fluids

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**ABSTRACT**

It is seen that the average heat transfer coefficient depends on the geometrical and physical factors. The correlation between them is not so simple because of the complex relationship between the large numbers of variables involved. The velocity profile itself, which affects the temperature profile, depends on geometrical factors, such as the impeller diameter and its nature, position and speed; the diameter of the tank, the diameter and pitch of the coils, the diameter of the coil cube, the fluid height in the tank and also on the position and the dimensions of the baffles. Physical parameters are normally grouped in termed of Prandtl number, Reynolds number and viscosity correlation factors whereas geometrical parameters are considered in the Reynolds number and various geometric dimensionless groups. All these coefficients will be determined under the steady state condition and the data correlation will be based on the following theoretical background.

**Keywords:** helical coiled tube, Prandtl number, Reynolds number, Friction factor

1. **INTRODUCTION**

Laminar forced convection in curved pipes has received considerable attention in recent years. From the work of Kubair and Kuloor (2) and Akiyama,M et al. (3) Dravid (4), Kaya O, Teke I (13), S.S. Pawar et al. (14), it is seen that very little work has been done for non-Newtonian fluids in curved pipes. The existing heat transfer results in the literature for fully developed laminar forced convection in curved pipe with uniform wall temperature are rather limited and incomplete. For Newtonian fluids, perturbation method was applied by Mujawar,B.A., & Raja Rao,M (8) for extremely low Dean Number. The boundary layer approximation near the wall was presented by Mori and Nakayama (1) for high Dean Number of order one. Dravid et al. (4) presented the numerical results in the thermal entrance region for Dean Number less than 225 and \( \text{N}_\text{pr} = 5 \). The improved results of Akiyama (3) shows that the ratio of heat transfer coefficients in coil and in straight pipe is a function of Dean Number as well as Prandtl number.

Thus there is possibility of obtaining a suitable correlation of the following form

\[
\frac{\text{Nu}_\text{ic}}{\text{Nu}_\text{is}} = 1 = \varnothing_1 (N_D, N_{\text{pr}}) \quad (1.1)
\]

The advantage of the above form of correlation is that for small \( N_D \), the equation satisfies the condition that \( \text{Nu}_\text{ic} \approx \text{Nu}_\text{is} \) for very very small \( N_D \). \( \text{Nu}_\text{is} \) may be calculated for isothermal
laminar heat transfer in straight tube for the case of uniform wall temperature and parabolic velocity distribution by the following correlation:

\[ N_{Nuis} = 1.75 \left( \frac{N_GZ}{\sigma} \right)^{\frac{1}{3}} \]  \hfill (1.2)

For non-Newtonian fluids equations (1.3) and (1.4) take the following form:

\[ N_{Nuis} = 1.75 \left( \frac{3n+1}{4n} \right) \left( \frac{N_GZ}{\sigma} \right)^{\frac{1}{3}} \]  \hfill (1.3)

Here again it is important to note that Dean Number contains Reynolds number and the viscosity term appears in both the Reynolds and Prandtl numbers. It is suggested that the effective viscosity \( \mu_2 \) at the shear stress prevailing at the wall should be used for evaluating the Reynolds and Prandtl numbers. From the above discussions it may be concluded that the correlation may be written as below:

\[
\frac{N_{Nuis}}{N_{Nuis}} - 1 = C_3 N_{Dz}^{b_5} N_{Prz}^{b_6} \]  \hfill (1.4)

2. EXPERIMENT

In order to obtain exponents \( b_5 \) and \( b_6 \) and constant \( C_3 \) in equation (1.4), the proposed form of correlation suggested for laminar forced convection to non-Newtonian fluids in helical coils 0.5, 1 and 2% CMC-A solutions were investigated in a helical coil of 34 cm diameter made from a tube of diameter 1.898 cm. Before conducting the heat transfer runs, pressure drop measurements were made to verify the fitness of the coil. Coil flow is compared with capillary shear flow in figure 2.A, friction factor, \( f_c \), is shown in figure 2.B against the Reynolds number \( N_{Rez} \) defined by equation

\[ N_{Rez} = \frac{Dz \rho}{\mu_2} \]  \hfill (4)

Where

\[ \mu_2 = \frac{w}{8U'\tau_w} = K' \left( \frac{8U'}{D_c} \right)^{n'_1} = K' \left( \frac{\tau_w}{\pi} \right)^{n'_1} \]  \hfill (1.6)

The use of Reynolds number \( N_{Rez} \) seems to be more appropriate since it is defined with a viscosity term evaluated at prevailing wall shear stress. Rheological parameters \( K' \) and \( n' \) too, are evaluated at \( \tau_w \).

For laminar flow of non-Newtonian fluids flowing through straight pipe, all the three definitions of Reynolds number \( N_{Re} \), \( N_{Re_1} \) and \( N_{Rez} \) are found to be the same.

According to present approach the Dean number is defined as

\[ N_{Dz} = N_{Rez} \left( \frac{D_t}{D_c} \right)^{1/2} \]  \hfill (1.7)

And for non-Newtonian fluids the correlation of the friction factor will take the following form

\[ \frac{f_c}{f_s} = \varphi \left( N_{Dz} \right) \]  \hfill (1.8)

Where

\[ f_{SL} = \frac{16}{N_{Rez}} \]  \hfill (1.9)
The ratio of friction factor, $f_c$, for coil to that of straight tube, $f_{SL}$, evaluated at Reynolds number, $N_{Re2}$, from equation $f_{SL} = 16/N_{Re2}$ is plotted against $N_{D2}$ in figure 2.B.

Three non-Newtonian fluids: 0.5 (n= 0.937), 1(n= 0.851) and 2% (n= 0.793) CMC-A in water, representatives of pseudo plastic fluids obeying power law, and water, a Newtonian fluid were investigated. Most of the data obtained were in laminar region and very few data of non-Newtonian fluids could fall in turbulent region. Due to lack of turbulent flow data for non-Newtonian fluids in coil, only laminar region data are processed and correlated.

3. RESULT AND DISCUSSION

The ratio of friction factor, $f_c$, for coil to that of straight tube, $f_{SL}$, evaluated at Reynolds number, $N_{Re2}$, from equation $f_{SL} = 16/N_{Re2}$ is plotted against $N_{D2}$ in figure 3.A.

Three non-Newtonian fluids: 0.5 (n= 0.937), 1(n= 0.851) and 2% (n= 0.793) CMC-A in water, representatives of pseudo plastic fluids obeying power law, and water, a Newtonian fluid were investigated. Most of the data obtained were in laminar region and very few data of non-Newtonian fluids could fall in turbulent region. Due to lack of turbulent flow data for non-Newtonian fluids in coil, only laminar region data are processed and correlated.
In order to find the effect $N_{D2}$ on the ratio of heat transfer in coil to heat transfer in straight pipe, the first attempt was to plot the ratio $(\frac{N_{Nuic}}{N_{Nuis}})$ against $N_{D2}$ on a logarithmic scale as shown in figure (3.B).

No net conclusion could be drawn even from this figure except for some qualitative ones. The heat transfer coefficient in coil is always higher than in straight pipe as it is seen that $\frac{N_{Nuic}}{N_{Nuis}}$ is always greater than unity. The ratio $h_{ic}/h_{is}$ increases with Dean number, $N_{D2}$. These data do not follow any particular trend. This may be explained by the fact that due to non Newtonian behavior Prandtl number changes by change of flow rate affecting the heat transfer rate. Data plotted for three fluids in figure 3.3. have wide range of Prandtl number. However, an exponent of Dean Number $\frac{1}{2}$ correlated the data much better. This trial has been made on the basis of theoretical results presented by previous investigators.

4. CONCLUSION

As a result of the present investigation on the heat transfer to agitated fluids flowing through immersed helical coils in jacketed vessels, the following conclusions are drawn:

For small temperature driving forces the non isothermal correlation is negligible.

The heat transfer data for agitated Newtonian and non-Newtonian fluids have been successfully correlated by using the viscosity of the fluid evaluated at the impeller tip assuming a cylinder of diameter equal to that of impeller rotating in an
infinite fluid. Data of 1, 2 and 4% CMC-A, for three impeller diameters, have been correlated by the following equation:

\[ N_{Nuoc} = 0.036 N^{n_f/3} Rea^{1/3} Pr_a^{1/3} \left( \frac{D_a}{D_c} \right)^{0.1} \]

(Standard deviation 15.03%)

Using the above concepts of Reynolds and Prandtl numbers it is also possible to correlate the available published data for other non-Newtonian fluids obtained with different impeller geometries.

Under the identical flow conditions, laminar flow heat transfer rate in helical coils are higher than those in straight pipes. The ratio \( \frac{N_{Nuic}}{N_{Nuis}} \) is found to be a function of \( N_{D_2} \) and \( N_{Pr_2} \). The data obtained have been correlated by the following equation.

\[ \frac{N_{Nuic}}{N_{Nuis}} = 1 + 0.0666 N_{D_2}^{1/2} N_{Pr_2}^{0.12} \]

(Standard deviation 15.52%)

\[ 24 < N_{D_2} < 2000, 40 < N_{Pr_2} < 225 \] and \[ 0.793 < n < 1 \]

**NOMENCLATURE**

- \( b_3, b_4 \): constants in equation (1.2)
- \( b_5, b_6 \): constants in equation (1.6)
- \( D_a \): agitator diameter, cm
- \( D_c \): diameter of the coil helix, cm
- \( D_t \): inner diameter of the straight or coil tube
- \( D_o \): outer diameter of the straight or coil tube
- \( D_T \): diameter of the agitated vessel
- \( f_c \): friction factor in coil
- \( f_a \): laminar flow friction factor in straight pipe
- \( h_j \): heat transfer coefficient for jacketed vessel wall to fluid, Kcal/hr m² °C
- \( h_{oc} \): coil outside heat transfer coefficient, Kcal/hr m² °C
- \( h_{ic} \): coil inside heat transfer coefficient, Kcal/hr m² °C
- \( k \): thermal conductivity
- \( n \): flow behavior index
- \( n' \): generalized flow behavior index
- \( r \): radial distance, cm
- \( t \): time, sec
- \( T_{ci} \): inlet fluid temperature in the coil, °C
- \( T_{co} \): outlet fluid temperature in the jacket, °C
- \( T_i \): inlet temperature
- \( T_s \): surface temperature
- \( U \): average velocity, cm
- \( U_j \): jacket overall heat transfer coefficient, Kcal/hr m²
- \( U_{oc} \): coil overall heat transfer coefficient, Kcal/hr m² °C
- \( u \): local velocity in the x direction at r or y, cm/sec

**Dimensionless groups**

- \( N_{Nu} \): Nusselt number, \( h D/k \)
- \( N_{Naj} \): Nusselt number, \( h_j D_T/k \)
- \( N_{Nuoc} \): Nusselt number, \( h_{oc} D_k/k \)
- \( N_{Naic} \): Nusselt number, \( h_{ic} D_k/k \)
- \( N_{Nuis} \): Nusselt number, \( h_{is} D_k/k \)
- \( N_{Re} \): Reynolds number, \( D_T U_p/\mu \)
- \( N_{Re2} \): Reynolds number defined by equation
- \( N_{Re''} \): Reynolds number defined by equation
- \( N_{Pr} \): Prandtl number defined by equation
- \( N_{Pr_2} \): Prandtl number defined by equation
- \( N_D \): Dean Number
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