Design of 2D Digital FIR Low Pass Filter using Hybrid Particle Swarm Optimization with Simulated Annealing

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Abstract
In this paper, a hybrid technique of particle swarm optimization algorithm and simulated annealing algorithm (HPSOSA) is investigated for the design of two-dimensional (2D) low pass filter (LPF) using $L_1$-error fitness function. The $L_1$-error fitness function produces flatter response with comparable transition width with normal transition width. Here, the filter’s coefficients are calculated using HPSOSA algorithm. Finally, design example of 2D LPF is demonstrated in terms of various optimal characteristics. The comparison of obtained simulation results is done with the conventional Particle Swarm Optimization (PSO). It is observed that the HPSOSA algorithm provides promising results for 2D FIR LPF by overcoming the problem of early convergence of PSO algorithm.

Keywords: Two-dimensional digital filters, $L_1$-error, particle swarm optimization, simulated annealing, low-pass filter.

Introduction
In the digital signal processing (DSP) domain, the study of digital filtering process have drawn a lot of attention from research scholars. The digital filters are the most broadly used systems and they have a crucial role in both one-dimensional (1D) and two-dimensional (2D) digital signal processing. 2D digital FIR filters are extensively applied in the area of image processing. Over the past three decades, a significant improvement has been observed in this area. Depending on evolving hardware technologies and ease of computation, new design methods have been developed. These recently developed methods have been successfully applied in several areas like imaging, electronics, medicine and robotics. 2D digital filtering process plays a fundamental role in problems related to the field of satellite image processing [1], pattern recognition [2] and seismic data processing [3]. There are two main methodologies to design 2D digital FIR filters: the transformation-based method and the optimization-based method. The transformation-based method comprises of McClellan transformation and least square contour mapping [4], the transformation of variable [5] and a lot more [6], [7]. From studies, it is discovered that the design of 2D digital FIR filter is a multi-dimensional as well as multi-parameter problem with several local optima [8]. Thus, the use of metaheuristic optimization approach is required to reduce computation complexity. The design and simulation of 2D digital FIR filters using various metaheuristic optimization from 1992 till 2012 are studied in [9]. Similarly, review and analysis of various evolutionary optimization techniques for 2D filters are given in [10]. The various optimization algorithms employed for the designing 2D FIR filters are genetic algorithm (GA) and its variations, particle swarm optimization (PSO), bacterial foraging optimization (BFO), differential evolution (DE), cat swarm optimization (CSO), simulated annealing (SA), cuckoo search (CS) algorithm, bat algorithm (BA), seeker optimization algorithm (SOA), artificial bee colony (ABC), gravitational search algorithm (GSA) and HPSO-GSA.

The approach followed in this paper is based on the modified form of PSO. The PSO algorithm is a swarm (population) based evolutionary algorithm [11], [12]. One of the key advantages PSO algorithm is simple to implement with few parameters required to control its convergence. The problems encountered in conventional PSO are premature convergence and local trapping[13]. In order to remove these problems, a hybrid PSO algorithm is evaluated by [14] called hybrid particle swarm optimization with simulated annealing (HPSOSA). Here, design and simulation of 2D digital FIR LPF are performed using HPSOSA algorithm; giving the optimized filter coefficients. The objective function utilised here is $L_1$-error fitness function. The algorithm utilised in this paper has the benefits of both conventional PSO and SA algorithms.

Remaining paper is arranged as follows: Introduction is presented in Section 1. 2D digital FIR LPF design problem is discussed in Section 2. A concise discussion of the employed algorithms is given in Section 3. Section 4 presents the discussion and simulation results obtained using PSO and HPSOSA algorithm. In Section 5, paper is concluded.

Two-Dimensional Digital FIR Low Pass Filter Design
The ideal response 2D digital filter is represented as:

$$I_p(\omega_1, \omega_2) = \begin{cases} 1, & \text{Passband: } \omega \in R_p \\ 0, & \text{Stopband: } \omega \in R_s \end{cases}$$

(1)

where $\omega_1$, $\omega_2$ are the two frequency components in the range $[-\pi, \pi]$ with $\omega = \sqrt{\omega_1^2 + \omega_2^2}$ and $R_p$, $R_s$ are passband and stopband regions.
stopband regions. The $z$-transfer function of 2D digital FIR filter is written as:

$$H(z_1, z_2) = z_1^{-M_1}z_2^{-M_2} \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} h(m_1, m_2) z_1^{-m_1} z_2^{-m_2}$$

(2)

where $h(m_1, m_2)$ is impulse response matrix and $M_1$, $M_2$ are the length of the filter.

The quadrantal symmetric impulse response fulfills the condition given below:

$$h(m_1, m_2) = h(-m_1, -m_2) = h(m_1, -m_2) = h(-m_1, m_2)$$

The amplitude response of 2D digital FIR filter is given as:

$$\vec{H}(\omega_1, \omega_2) = \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} a_e(m_1, m_2) \cos(m_1 \omega_1) \cos(m_2 \omega_2)$$

(3)

Further, the frequency response is written as:

$$H_e(e^{j\omega_1}, e^{j\omega_2}) = e^{-j\omega_1 M_1} e^{-j\omega_2 M_2} \vec{H}(\omega_1, \omega_2)$$

(4)

Now, in order to map 2D sequence to 1D sequence, lexicographic ordering [15] is employed, the updated amplitude response is written as:

$$\vec{H}_e(\omega_1, \omega_2) = \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} a_e(m_1, m_2) \cos(m_1 \omega_1) \cos(m_2 \omega_2)$$

(5)

where the dimension $L_i = (M_1 + 1)(M_2 + 1)$, the coefficient vector, $\mathbf{p}_k = a_e(m_1, m_2)$ and $\mathbf{u}_k(\omega_1, \omega_2) = \cos(m_1 \omega_1) \cos(m_2 \omega_2)$ with $k = m_2(M_1 + 1) + m_1 + 1, m_1 = 0, 1, ..., M_1$ and $m_2 = 0, 1, ..., M_2$.

The amplitude response is rewritten as:

$$\vec{H}_e(\omega_1, \omega_2) = \mathbf{b}_e^T \mathbf{g}_e(\omega_1, \omega_2)$$

(6)

where

$$\mathbf{b}_e = [p_1, p_2, ..., p_{L_i}]^T$$

(7)

Also

$$\mathbf{g}_e(\omega_1, \omega_2) = [\mathbf{u}_1(\omega_1, \omega_2), \mathbf{u}_2(\omega_1, \omega_2), ..., \mathbf{u}_{L_i}(\omega_1, \omega_2)]^T$$

(8)

The digital filter design problem is expressed as an optimization problem. The primary objective is to find the best coefficients $\mathbf{b}_e$ so as to minimize the difference between the obtained amplitude response, $\vec{H}_e(\omega_1, \omega_2)$, and the ideal response, $I_0(\omega_1, \omega_2)$. The objective function is written as:

$$E = \iint |\vec{H}_e(\omega_1, \omega_2) - I_0(\omega_1, \omega_2)|^2 d\omega_1 d\omega_2$$

(9)

The error objective function ($E$) utilized here is taken in $L_1$ sense, which is defined as the absolute error between desired filter response and designed filter response. The $L_1$-error approximation yields less overshoot and provides more flatness in the passband region [8], [16].

**Employed Optimization Algorithms**

This section briefly presents the optimization algorithms used in this paper. The implementation of PSO and HPSOSA algorithms is illustrated in detail.

**Particle Swarm Optimization**

The PSO algorithm is a swarm-based method that was first presented by Kennedy and Eberhart in 1995 [17]. Its mechanism is based on social behaviour displayed by various living creatures such as birds, fishes and ants considered as particles in a swarm. Every particle is treated as a possible solution. Every particle is associated with its velocity ($v_i$) and position ($x_i$) vector. At each iteration, each particle’s fitness function is calculated. The position and velocity of every particle are revised according to personal best position ($p_{best}$) and global best position ($g_{best}$) as [18]:

$$v_{i}^{n+1} = w \times v_{i}^{n} + r_1 \times c_1 \times (p_{best} - x_i^n) + r_2 \times c_2 \times (g_{best} - x_i^n)$$

(10)

$$x_{i}^{n+1} = x_i^n + v_{i}^{n+1}$$

(11)

where $w$ is the inertia weight parameter that regulates the balance between $p_{best}$ and $g_{best}$ at $i$th iteration. $c_1$ and $c_2$ are defined as the social and cognitive weight parameters. The range of random parameters $r_1$ and $r_2$ is between [0,1].

**Simulated Annealing**

The Simulated Annealing (SA) algorithm is a metaheuristic algorithm for global optima approximation. The concept of simulated annealing was first proposed by Physicist Metropolis in 1953 [19]. The groundwork of SA algorithm is based on the famous Monte Carlo procedure. The SA algorithm mainly describes the molecular movement of atoms in a material during the annealing process. In this process, the atoms in the material move towards higher stability and lower energy state if the cooling rate allows enough time for the atoms to move. So, the annealing process can be simulated in a mathematical way by using the probability criteria for the metropolis acceptance rule in order to calculate the highest stability state which corresponds to the optimal solutions. The probability criteria is given as [14]:

$$P = e^{-\Delta E/K_B T}$$

(12)

where

$\Delta E = $ Objective function

$K_B = $ Boltzmann constant

$T = $ Temperature

The temperature reduction rule is given as [14]:

$$T_{i+1} = \alpha \times T_o$$

(13)

where

$T_{i+1} =$ Temperature at $i$

$\alpha =$ cooling rate

$T_o =$ Initial temperature

**Hybrid Particle Swarm Optimization Incorporating Simulated Annealing**

The main aim of hybridizing SA algorithm with conventional PSO algorithm is to improve the exploration capability of PSO algorithm [14]. First, by using conventional PSO algorithm, population is randomly generated and initial temperature is defined. Then, new position of every particle is calculated using velocity and position update equations of the conventional PSO. This new position of the particle is automatically accepted by the conventional PSO. If there is no
improvement during the iteration, SA algorithm is introduced. The SA algorithm introduces metropolis acceptance criteria at this point. This step allows the algorithm to reject any local optimum solutions and thus improving the quality of the solution. After each iteration, temperature is dropped according to cooling rate. The complete flowchart of HPSOSA algorithm is shown in Figure 1.

**Implementation of HPSOSA for the Design of 2D digital FIR LPF:**
Following are the steps involved in designing HPSOSA based 2D digital FIR LPF.

**Step 1:** Initialize the population (swarm), $X_i$, with population size $n = 55$. The upper and the lower bounds for coefficients are in the range [-1,1]. The maximum iterations is taken as $N = 1000$. Also, initialize other parameters like $c_1$, $c_2$ having values 2.05 each, $\alpha$, $T_0$ etc.

**Step 2:** Calculate the fitness value of initial randomly generated population.

**Step 3:** Calculation of minimum $L_1$-error fitness value and calculation of $p_{best}$ and $g_{best}$

**Step 4:** Updating the velocities of all particles using Eq. (10). Every particle’s position is updated using Eq. (11) and fitness value is calculated. The updated fitness values of particles based on minimum fitness value are calculated.

**Step 5:** The $p_{best}$ and $g_{best}$ vectors are updated after every iteration according to metropolis probability acceptance criteria using Eq. (12) and the temperature is reduced according to temperature reduction rule given in Eq. (13).

**Step 6:** The algorithm continues from step 4 to the total number of iterations $(N)$. After termination of algorithm, $g_{best}$ vector contains the optimal coefficients for the 2D digital FIR LPF with an aim of minimizing fitness error.

**Simulation analysis and results**
This part of the paper evaluates the design and performance of 2D digital FIR LPF employing HPSOSA algorithm. Evaluation is done in terms of maximum passband and stopband ripple, minimum stopband attenuation, computation time, and is compared with the 2D digital FIR LPF designed using PSO algorithm. The controlling parameters selected for both the algorithms for designing 2D digital FIR LPF are given in Table 1. The order of the 2D FIR digital filter, $M_1$, $M_2$ is selected to be 10. The design example taken in this paper is low pass FIR filter. The ideal response of 2D FIR digital LPF with respect to $\omega_1$, $\omega_2$ is shown in Figure 2 having passband cut-off frequency, $\omega_p = 0.54\pi$ and stopband cut-off frequency, $\omega_s = 0.56\pi$. The frequency components $\omega_1, \omega_2$ are in the range of [-$\pi, \pi$].

**Table 1:** PSO and HPSOSA parameters for 2D FIR LPF design

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PSO</th>
<th>HPSOSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (swarm) size</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>Maximum Iterations</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Lower bound</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>Upper bound</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Social parameter($c_1$)</td>
<td>2.05</td>
<td>2.05</td>
</tr>
<tr>
<td>Cognitive parameter($c_2$)</td>
<td>2.05</td>
<td>2.05</td>
</tr>
<tr>
<td>$v_{i\min}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$v_{i\max}$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$w_{\min}$</td>
<td>0.2</td>
<td>-</td>
</tr>
<tr>
<td>$w_{\max}$</td>
<td>0.9</td>
<td>-</td>
</tr>
<tr>
<td>Initial temperature($T_0$)</td>
<td>-</td>
<td>50</td>
</tr>
<tr>
<td>Cooling rate($\alpha$)</td>
<td>-</td>
<td>0.99</td>
</tr>
</tbody>
</table>

**Figure 2:** The ideal response of 2D digital FIR Low Pass Filter
The magnitude responses of PSO and HPSOSA based 2D digital FIR LPF are shown in Figure 3. The magnitude response shown is plotted against the two frequency components $\omega_1, \omega_2$. The optimized coefficients forming even symmetric response of 2D digital FIR LPF are given in Table 2.

![Figure 3: Magnitude response of 10 × 10 order 2D digital FIR Low Pass Filter: (a) using PSO (b) using HPSOSA](image)

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>HPSOSA</th>
</tr>
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<tbody>
<tr>
<td>$h(0,0) = h(0,10) = h(10,0) = h(10,10)$</td>
<td>0.000110495116000</td>
</tr>
<tr>
<td>$h(0,1) = h(0,9) = h(10,1) = h(10,9)$</td>
<td>0.000853911888757</td>
</tr>
<tr>
<td>$h(0,2) = h(0,8) = h(10,2) = h(10,8)$</td>
<td>0.000622691578243</td>
</tr>
<tr>
<td>$h(0,3) = h(0,7) = h(10,3) = h(10,7)$</td>
<td>0.000820556776713</td>
</tr>
<tr>
<td>$h(0,4) = h(0,6) = h(10,4) = h(10,6)$</td>
<td>0.001263568893185</td>
</tr>
<tr>
<td>$h(0,5) = h(0,5)$</td>
<td>0.001351887500818</td>
</tr>
<tr>
<td>$h(1,0) = h(1,10) = h(9,0) = h(9,10)$</td>
<td>0.0001054327918852</td>
</tr>
<tr>
<td>$h(1,1) = h(1,9) = h(9,1) = h(9,9)$</td>
<td>0.002199999015488</td>
</tr>
<tr>
<td>$h(1,2) = h(1,8) = h(9,2) = h(9,8)$</td>
<td>0.003884981245398</td>
</tr>
<tr>
<td>$h(1,3) = h(1,7) = h(9,3) = h(9,7)$</td>
<td>0.002518520058888</td>
</tr>
<tr>
<td>$h(1,4) = h(1,6) = h(9,4) = h(9,6)$</td>
<td>0.000740205622188</td>
</tr>
<tr>
<td>$h(1,5) = h(9,5)$</td>
<td>-0.000470379886109</td>
</tr>
</tbody>
</table>

Table 3 shows the results of various performance parameters computed using PSO and HPSOSA. The employed HPSOSA algorithm for 10th order 2D digital FIR LPF filter results in maximum passband ripple = 1.042, maximum stopband ripple = 0.0193, maximum passband attenuation = 0.35(dB) and minimum stopband attenuation = -34.79(dB) as compared to PSO algorithm which results in maximum passband ripple = 1.061, maximum stopband ripple = 0.0202, maximum passband attenuation = 0.51(dB) and minimum stopband attenuation = -32.61(dB). Also, the execution time per 1000 iterations for HPSOSA is 32.52s and that of PSO is 34.14s. This proves that the employed algorithm, HPSOSA, for the design and simulation of 2D digital FIR LPF converges fast and shows significant improvement in terms of various performance parameters of the respective filter. On comparing responses, it is shown that the transition width of HPSOSA algorithm based 2D FIR LPF is comparable to same filter designed using PSO algorithm.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>PSO</th>
<th>HPSOSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. passband ripple</td>
<td>1.061</td>
<td>1.042</td>
</tr>
<tr>
<td>Max. stopband ripple</td>
<td>0.0234</td>
<td>0.0182</td>
</tr>
<tr>
<td>Max. passband attenuation(dB)</td>
<td>0.51</td>
<td>0.35</td>
</tr>
<tr>
<td>Min. stopband attenuation(dB)</td>
<td>-32.61</td>
<td>-34.79</td>
</tr>
<tr>
<td>Execution time</td>
<td>34.14</td>
<td>32.52</td>
</tr>
</tbody>
</table>
The convergence curve of $L_1$-error obtained from PSO and HPSOSA is shown in Figure 4. From convergence curve, it is revealed that in starting iterations PSO converges fastly because it converges to the nearest optimal solution (exploitation) i.e., gets trapped rather than exploring other possible solutions. On the other hand, HPSOSA takes little more iterations than PSO initially so as to explore all possible solutions and then exploiting the best one. So, in this way, HPSOSA resolves the problem of local trapping as seen in PSO.

![Figure 4: Convergence curve of PSO and HPSOSA for 2D digital FIR LPF design](image)

**Conclusion**

In this paper, HPSOSA algorithm is utilized for the design of 2D digital FIR low pass filter having quadrantly even symmetric properties. The performance analysis of the applied HPSOSA algorithm is done comparatively with the well-known conventional PSO algorithm. It is concluded that the HPSOSA algorithm produces lower passband, stopband ripples and higher stopband attenuation with less execution time as compared to conventional PSO algorithm. It is also concluded that HPSOSA overcomes the drawback of early convergence and local optima trapping as seen in PSO algorithm. Further, the work presented here can be applied in the designing of two-dimensional digital differentiators.

**References**


