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Optimal Design of Digital Fractional Order Differentiator using Updated Cuckoo Search Algorithm

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Abstract
A recently developed metaheuristic techniques cuckoo search algorithm and its updated version are used to design finite impulse response digital fractional order differentiator. In standard cuckoo search, the switching parameter which maintains the balance between local and global random walk is kept constant. Here, the impact of linearly increasing switching parameter is studied on convergence rate and performance parameters of digital fractional order differentiator to get better exploration in search space. The results obtained from updated algorithm are compared for magnitude and phase errors, execution time and fitness curve with standard cuckoo search algorithm and particle swarm optimization algorithm. It is discovered from extensive simulations that fractional order differentiator using updated cuckoo search algorithm outperforms other algorithms..

Keywords: Fractional calculus, fractional order differentiator, metaheuristics algorithms, cuckoo search algorithm, particle swarm optimization.

Introduction
Fractional calculus has gained huge popularity because of its wide applications in the various fields of science and engineering [1], [2] such as radar applications, sonar applications, control engineering, biomedical signal processing, image processing, electromagnetic theory, etc. Digital Fractional order differentiator (FOD) has emerged as an important area of focus in the fields of science and engineering from last two decades. Its main function is to compute time derivative of the given signal. The systems dynamic characteristics are described by using fractional order calculus. The frequency response with high degree of freedom in shaping can be achieve using fractional order system. A high order model with the lower order helps in decreasing the complexity of the system. Fractional order system is generally divided into two categories such as fractional order differentiator and integrator.

An excellent survey on fractional order differentiators and integrators is given by Krishna, B. T [3]. The approximation of obtained results of a designed differentiator or integrator to the ideal differentiator or integrator is considered as an optimization problem, and several methods have been used to attain optimal solutions in search space. To obtain linear phase finite impulse response (FIR) stable system is used. The fractional operator is used in both continuous and digital domain as well [4], [5]. Several methods are used to design continuous time FOD such as Roys method, Carlson Method, Chareffs method, Matsudas method, Oustaloups method, etc. The research trend is diverted towards discretization of fractional order system because of several advantages present in digital domain. There are several methods in literature for discretizing continuous fractional order system which are categorized as direct and indirect method [3], [6]–[9]. For complex and with multimodal fitness function problems, a robust optimization techniqueis required. Nowadays metaheuristic optimization algorithms are in research trends to solve complex optimization problems and these algorithms are applied in every application of science and engineering [10], [11]. A survey of metaheuristics optimization algorithms and is given by Yang [12]. Nature inspired metaheuristics optimization algorithms are extensively used because these algorithms take less computation time and perform in robust manner as compared to conventional optimization techniques [13]. The purpose of this paper is to use recently developed nature inspired metaheuristics cuckoo search algorithm [14] with an updated strategy of dynamically changing parameter $'Pa'$ in order to obtain fast convergence rate and better exploration. In Figure 1 the design process of FIR-FOD is shown.

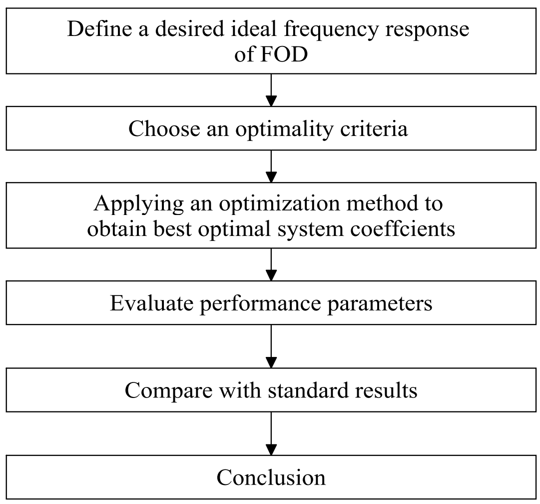

Figure 1: The design methodologies of FIR-FOD

In the following sections rest of the paper is ordered as : In section 2, the problem formulation is demonstrated for designing FOD and explanation of fractional derivative is given. In section 3, basics of the original Cuckoo search algorithm is explained. Section 4 demonstrates implementation of the updated algorithm and interpreted in detail. In section 5, simulation results for employed algorithms are shown. In section 6, finally the paper is concluded.

Problem Formulation

The integer order derivative $D^n f(x) = \frac{d^n f(x)}{dx^n}$ is generalized by fractional operator $D^v f(x) = \frac{d^v f(x)}{dx^n}$, where '$n$' represents a integer number and $'v'$ represents a real number. Where $'v'$ is a fractional number and its value can be positive or negative with different range. There are several definitions specified in literature to define fractional derivatives [4], [15]. In paper, the Grünwald Letnikov (GL) definition is used for computation of fractional derivatives which is given by

$$ {}_aD_t^v = \begin{cases} \frac{d^v}{dt^v}, & v > 0 \\ 1, & v = 0 \\ \int_a^t (d\tau)^v, & v < 0 \end{cases} \tag{1} $$

Where ${}_aD_t^v$ is a general fractional order calculus operator which is used to compute $'vth'$ fractional order and expressed as,

$$ {}_aD_t^v s(t) = \lim_{\Delta \to 0} \sum_{k=0}^{\infty} \frac{(-1)^k C_k^v}{\Delta^\alpha} s(t - k\Delta) \tag{2} $$

Where the binomial coefficient C_k^v is given by

$$ C_k^v = \binom{v}{k} = \frac{\Gamma(v+1)}{\Gamma(k+1)\Gamma(v-k+1)} $$

$$ = \begin{cases} 1 & k = 1 \\ \frac{v(v-1)(v-2).....(v-k+1)}{1.2.3...k} & k \geq 1 \end{cases} \tag{3} $$

The notation $\Gamma(.)$ represents a gamma function.
The frequency response of ideal FOD is given by,

$$ H_{id}(\omega) = (j\omega)^v \tag{4} $$

Where $'v'$ is a fractional number, $H_{id}(\omega)$ is frequency response of ideal FOD, $'w'$ is a normalized frequency between [0 1], and $j = \sqrt{-1}$.
The finite impulse response FOD transfer function is expressed as

$$ H(z) = \sum_{n=0}^{N-1} \mathrm{h(n)} z^{-n} \tag{5} $$

And correspondingly frequency response is given by

$$ H(w) = \sum_{n=0}^{N-1} \mathrm{h(n)} e^{-j\omega n} \tag{6} $$

In order to obtain linear parameters and to handle regression situation, a novel fitness function i.e., weighted least square (WLS) fitness function is used. The *fitness* is the *weighted-sum of squared* errors between the ideal FOD response $(H_{id}(\omega))$ and optimized FOD response $(H(\omega))$ with respect to phase and magnitude. Different sets of weights have identical effects on optimization results. The WLS fitness function is represented by

$$ J = J_m + J_p \tag{7} $$

$$ J = \int_0^1 W_1(\omega) \left|abs(H_{id}(\omega)) - abs(H(\omega))\right|^2 + W_2(\omega)\left|phase(H_{id}(\omega)) - phase(H(\omega))\right|^2 d\omega \tag{8} $$

In this paper, the non-negative weighting functions $W_1(\omega)$ is set to 0.9 and $W_2(\omega)$ is set to 0.1 respectively. The fitness function $'J'$ as given in equation (8) is minimized using standard cuckoo search algorithm (CSA) and results are compared with updated CSA. The obtained optimized coefficients are used to design FOD and performance parameters are evaluated which are expressed as:

The WLS absolute magnitude error (ϵ_m)

$$ = \int_0^1 W_1(\omega) \left|abs(H_{id}(\omega)) - abs(H(\omega))\right|^2 d\omega \tag{9} $$

The WLS absolute phase error (ϵ_p)

$$ = \int_0^1 W_2(\omega)\left|phase(H_{id}(\omega)) - phase(H(\omega))\right|^2 d\omega \tag{10} $$

Standard Cuckoo Search Algorithm

Cuckoo search is recently developed nature-inspired metaheuristic technique used to solve many complex and multimodal problems in the different field of science and engineering. An amazing literature survey is given on cuckoo search by many authors [16]–[19]. Its switching parameters are 25% fixed for many applications. The optimization goal of an algorithm is to discover either minimum or maximum value of the respective objective function. The algorithm is based on the aggressive reproduction approach of cuckoo species. For augmentation of the hatching probability of their eggs, cuckoo lays their own eggs in the host nests which is known as a brood parasitism. If a host bird is able to discover alien eggs it will either abandoned its own nest or throw away eggs and the probability of discovering alien eggs is given by $Pa \in [0,1]$. Cuckoo search algorithm along with $Lévy$ flight distribution is inspired by Yang and Deb [14]. $Lévy$ flight distribution for cuckoo search [20] is used to find next location in random walk manner and helps to explore search space [21]. To generate random new solutions $x^{(t+1)}$ in global space, $Lévy$ flight random walk is performed as

$$ x_i^{(t+1)} = x_i^{(t)} + \alpha_0 \oplus Lévy(\lambda) \tag{11} $$

Where α_0 is step size $(\alpha_0 > 0)$, $x_i^{(t)}$ is current location, $x_i^{(t+1)}$ is next location, the product $\oplus$ represents entry wise multiplications.

Random walk using $Lévy$ flight is expressed as

$$ Lévy \sim u = t^{-\lambda}, \quad 1 < \lambda \leqslant 3 \tag{12} $$

Where λ is mean free path random step length with infinite variance. CSA is basically a population based optimization technique similarly as particle swarm optimization (PSO). CSA is more efficient as compared to PSO because PSO suffers from local trapping problems, premature convergence, and low convergence speed at higher orders. Which can be noticed in fitness curves.

The idea behind using $Lévy$ flight distribution in cuckoos breeding behavior is that if cuckoos eggs are find to be similar to hosts eggs, then the probability of discovering cuckoos eggs by host will be less. Using $Lévy$ flight new nest are generated around the best nests and fitness is calculated. This helps in avoiding local optimum trapping and speeds up the local searching which was the main problem in standard PSO algorithm.

Updated Cuckoo Search Algorithm

In standard CSA, the switching parameter $'Pa'$ is kept constant at 0.25 value for finding global minima. This parameter also has a huge impact on convergence rate. In this paper, dynamically changing switching parameter $'Pa'$ is used which is a key difference between standard CSA and updated CSA [22]. The dynamically changing switching parameter for probability distribution function $'Pa'$ is taken as linear increasing function. This parameter is tested against the constant probability distribution function of standard CSA and results of the updated algorithm (CSLIN) are also compared with PSO. The equation for linear increasing probability distribution function is given by

$$Pa_{lin} = (Pa_{max}) * C_i \ / \ T_i \tag{13}$$

Where Pa_{lin} is linear increasing probability, Pa_{max} is the maximum value of the switching parameter which is kept 0.25, C_i is current iteration and T_i is total iteration.The flowchart for updated CSA shown in figure 2.

Simulation Results

In this section, the simulation results of the designed FOD using PSO, CSA and updated algorithm are presented. The FOD designed here is for 8th, 10th, 12th, 14th order having fractional order α = 0.5. The controlling parameters of the given algorithms are mentioned in Table 1. The results are evaluated on the basis of performance parameters like magnitude error, phase error, execution time and error fitness value. The normalized frequency is taken in a range of $\omega \in [0,1]$. The program is executed for over 100 to 1000 iterations. The fitness function employed here is WLS having a value of weights $W_1(\omega), W_2(\omega)$ fixed at 0.9 and 0.1. In the updated algorithm, the probability of discovering alien eggs (P_a) also called the switching parameter, increases linearly as the number of iterations increases. This increase depends upon the values of current and maximum no. of iterations. While in the case of standard CSA, this parameter is fixed at 0.25. All the other controlling parameters are kept same for CSA. The lower and upper boundary limits, for all the mentioned algorithms, are in the range [-1,1]. The obtained magnitude response, phase response, and fitness curve along with the calculated magnitude and phase errors using different algorithms are shown in Figure 3-6.

Table 1: PSO, CSA and CSLIN tuning parameters for FIR-FOD

Parameters	PSO	CSA	CSLIN
Initial population size	25	25	25
Iterations cycle	100-1000	100-1000	100-1000
Lower bound	-1	-1	-1
Upper bound	1	1	1
Discovering rate of alien eggs	-	0.25	-
Inertia weight	0.4	-	-
Learning parameters	c_1=2,c_2=2	-	-
Particle velocity	v_{min}=0.01, v_{max}=1	-	-

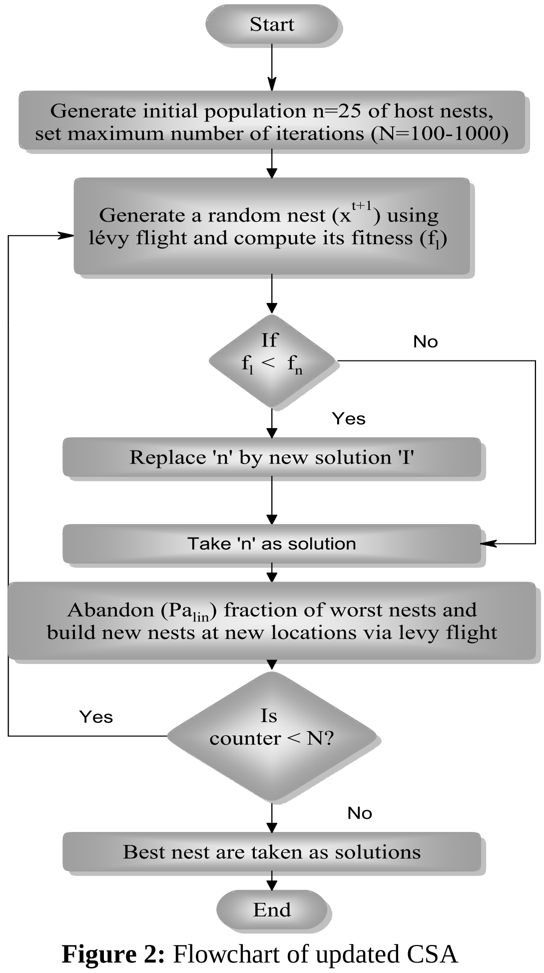

Figure 2: Flowchart of updated CSA

The (a) and (b) subfigures of Figure 3 shows the magnitude response and absolute magnitude error of 8th, 10th, 12th, 14th orders FOD while subfigures (c) and (d) shows the obtained phase response and absolute phase error. As per these obtained simulation results, a detailed comparison between the performance of mentioned algorithms is given in Table 2-5 for different orders. The performance of an updated algorithm is compared with CSA and PSO algorithm w.r.t performance parameters such as magnitude error, phase error (degree), fitness value at maximum iteration and execution time in seconds.

Table 2: Comparison of simulation results of PSO, CSA and CSLIN w.r.t. performance parameters on 8th order

Algorithm	Magnitude Error	Phase Error	Fitness Value	Execution Time(Sec)
PSO	5.2×10^{-4}	0.0986	0.00335	38.32
CSA	4.8×10^{-4}	0.0300	0.00191	25.52
CSLIN	2.8×10^{-4}	0.0156	0.00183	24.37

Table 3: Comparison of simulation results of PSO, CSA and CSLIN w.r.t. performance parameters on 10th order

Algorithm	Magnitude Error	Phase Error	Fitness Value	Execution Time(Sec)
PSO	4.6×10^{-4}	0.1006	0.00488	54.94
CSA	1.2×10^{-4}	0.0250	0.00165	45.97
CSLIN	8.5×10^{-5}	0.0059	0.00108	25.78

Table 4: Comparison of simulation results of PSO, CSA and CSLIN w.r.t. performance parameters on 12th order

Algorithm	Magnitude Error	Phase Error	Fitness Value	Execution Time(Sec)
PSO	6.5×10^{-4}	0.0861	0.00324	40.63
CSA	2.3×10^{-4}	0.0269	0.00163	30.38
CSLIN	1.9×10^{-4}	0.0105	0.00209	25.64

Table 5: Comparison of simulation results of PSO, CSA and CSLIN w.r.t. performance parameters on 14th order

Algorithm	Magnitude Error	Phase Error	Fitness Value	Execution Time(Sec)
PSO	4.4×10^{-4}	0.1482	0.00319	51.06
CSA	2.1×10^{-4}	0.0124	0.00191	35.37
CSLIN	1.5×10^{-4}	0.0098	0.00183	25.22

The magnitude error and phase error are calculated using equation (9,10). The fitness value is calculated using equation (8) and execution time is evaluated up to maximum iterations. The updated algorithm provides the lowest error for magnitude and phase as compared to other two algorithms. For 8th order, the updated CSA performs with the betterment of 57% and 41% over PSO and standard CSA, respectively for magnitude error. Whereas in case of the phase error, updated CSA shows the amendment of 84% and 48% over PSO and standard CSA respectively. Also, the fitness value achieved by an updated algorithm which is 0.00183 is also the lowest as compared to 0.00335 in case of PSO and 0.00191 in case of CSA. The overall execution time computed for the updated algorithm is 24.37 sec which shows that updated algorithm is faster in simulation and also provides minimum fitness at less number of iterations in comparison with both PSO and standard CSA. To check the robustness of updated algorithm, the results are also compared at higher orders as given in Table 2-5. In updated CSA the magnitude error 2.8×10^{-4} is reduced to 8.5×10^{-5} as the order is increased and phase error 0.0156 is reduced to 0.0059. It is being observed during simulations that cuckoo search based algorithm performs better at higher order whereas PSO algorithm is failed to converge fast and not able to approximate the ideal response of FOD as seen in Figures 3-6. The updated cuckoo search algorithm with linear increasing switching parameter provides better results as compared to standard CSA. As shown in Table 5, as the order is increasing the magnitude and phase error are becoming less and the fitness value is reached up to 0.00108 at 1000 iteration.

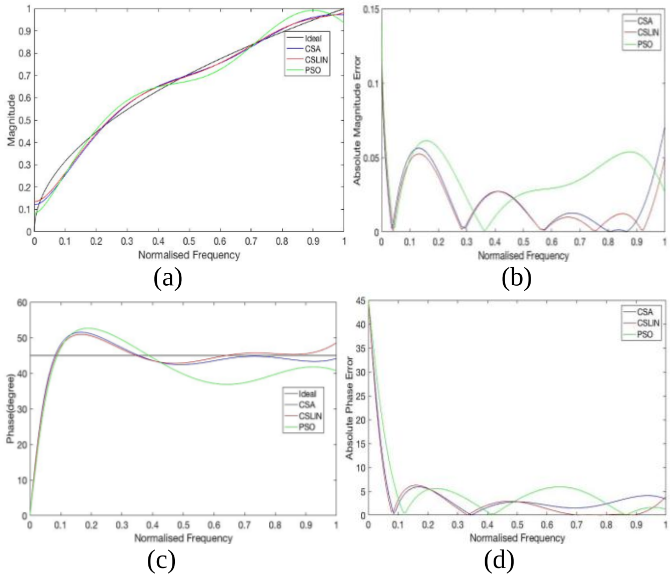

Figure 3: **(a)** Magnitude response **(b)** Magnitude error **(c)** Phase response **(d)** Phase error: comparison of 8th order FOD using PSO, CSA, CSLIN.

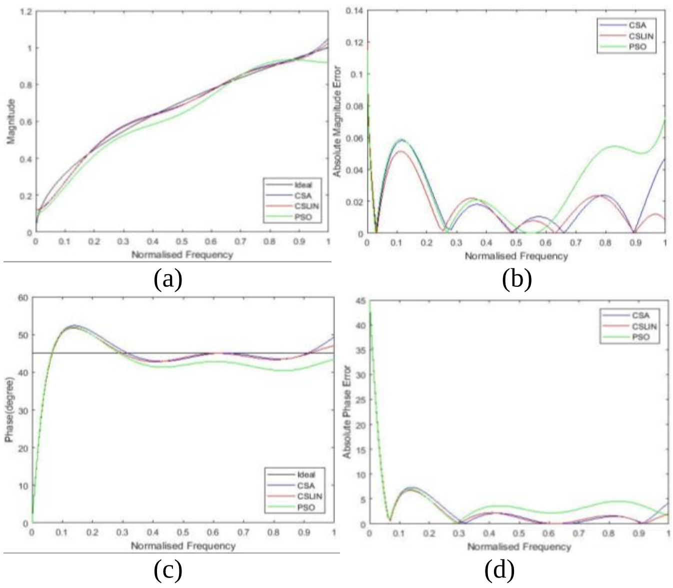

Figure 4: **(a)** Magnitude response **(b)** Magnitude error **(c)** Phase response **(d)** Phase error: comparison of 10th order FOD using PSO, CSA, CSLIN.

For higher order PSO shows poor results because of low convergence rate in iterative process also local search ability is not good as compared to standard CSA. PSO algorithm is suitable for lower order where it can approximate the ideal results however it is easy to implement.

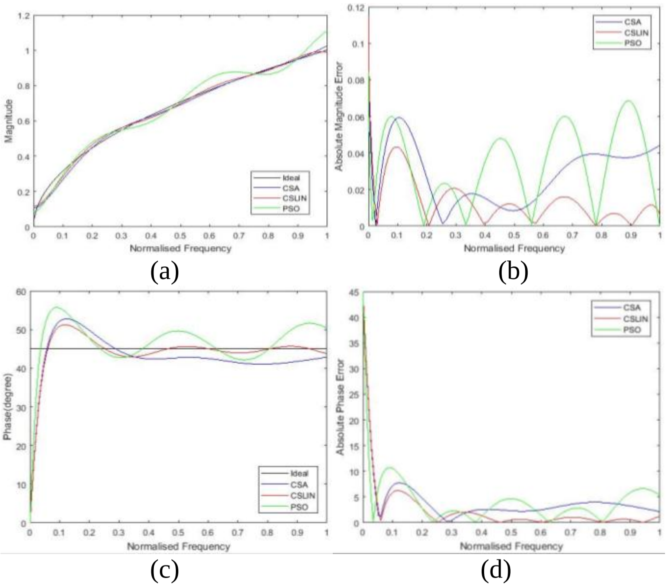

Figure 5: **(a)** Magnitude response **(b)** Magnitude error **(c)** Phase response **(d)** Phase error: comparison of 12th order FOD using PSO, CSA, CSLIN.

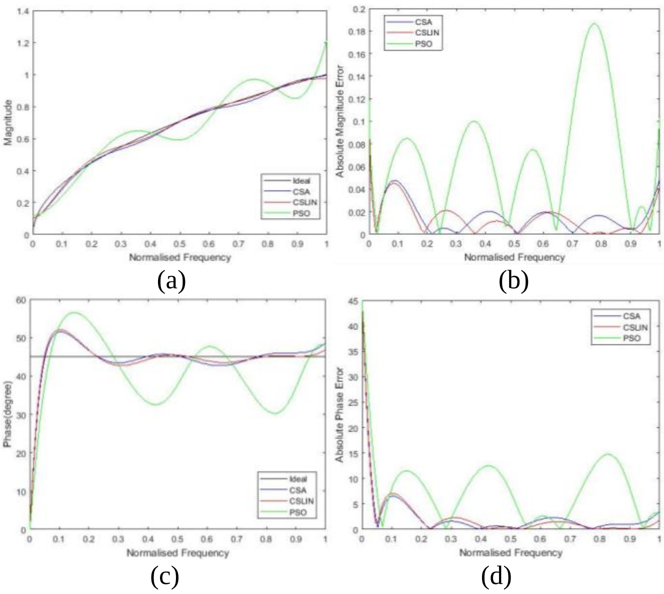

Figure 6: **(a)** Magnitude response **(b)** Magnitude error **(c)** Phase response **(d)** Phase error: comparison of 14th order FOD using PSO, CSA, CSLIN.

The fitness curves for respective orders are necessary to judge that how much the overall fitness is optimized over the number of iterations. The fitness function is a particular objection function that is used to summarize the results. The fitness curve for order 8,10,12, and 14 is shown in figure 7.

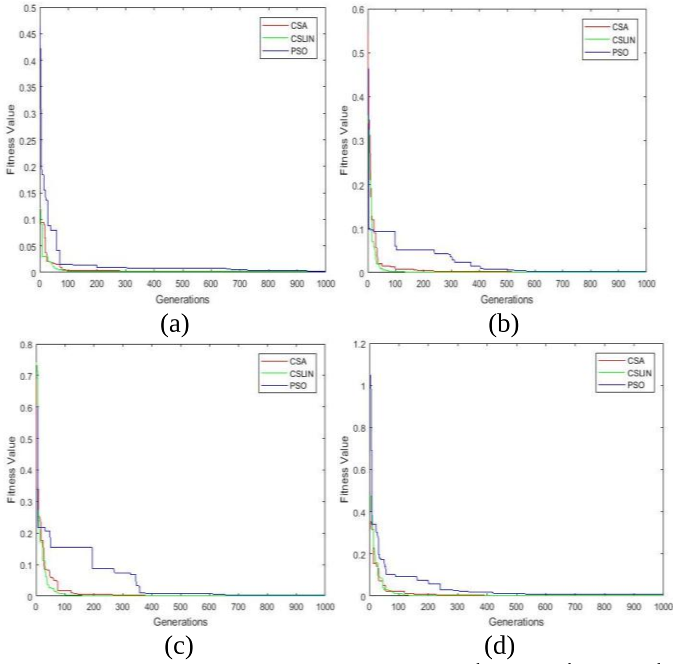

Figure 7: Fitness curves for FIR-FOD (a) 8th, (b) 10th, (c) 12th, (d) 14th orders using PSO, CS and CSLIN algorithm

Conclusion

A new updated form of CS algorithm is utilized in this paper in the designing of FIR-FOD system. The objective fitness function is taken as WLS, which improves the response of FIR-FOD. The simulated performance analysis of the updated cuckoo search algorithm is compared with both PSO and standard CSA for order 8th, 10th, 12th and14th. After performing extensive simulations, it is concluded that the updated algorithm outperforms both PSO and standard CSA in terms of various performance parameters like fitness value, magnitude error, phase error and execution time. The updated algorithm is fast and provides better exploration in search space than the other two used in this paper. Further, the algorithm updated here can be applied in the designing of two-dimensional filters and differentiators.

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