Transient Analysis Of Variable Profile Longitudinal Fin Using Meshless Local Petrov Galerking Method (MLPG)

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Abstract
Transient heat transfer through a longitudinal fin of 3-different profiles is studied. Meshless Local Petrov Galerkin Method (MLPG) is used to perform a nonlinear analysis to predict temperature distribution and heat transfer rate of 1-dimensional heat transfer equation. A longitudinal fin of 3-different profiles, such as rectangular, triangular and concave parabolic are analyzed. Transient analysis of temperature variation along with the fin length for different time range till achievement of steady state has been studied and explained. Moving least square (MLS) approximants are used to approximate the unknown function of temperature $T(x)$ with $T_h(x)$. Essential boundary conditions are imposed using penalty method. An iterative predictor-corrector scheme is employed to handle nonlinearity. Modelling fin in a convective-radiative environment removes the assumption of no radiation condition. Accuracy of MLPG is established then the method is used to generate results for transient condition with constant thermal conductivity and convection along with radiation effect. Variation of temperature along the fin length of variable profile and temperature-time history has been carried out and gives closer values to real problems for respective fin surfaces.

Keywords – Transient analysis, longitudinal fins; MLPG; Variable profile fin

Introduction
FEM is largely used for analysis of problems having arbitrary shapes. But usually it has been seen that in FEM, mesh generation is time consuming & costly. So, it becomes necessary to find methods which consume less time to prepare data. Many meshless has been proposed for number of engineering heat transfer problems. But, to the best of authors’ knowledge, meshless local Petrov- Galerkin (MLPG) method is not used to study the behavior of fins of varying cross-sectional area. MLPG method was developed by Atluri and Zhu [1-2]. Unlike FEM and most of the other meshfree methods, MLPG method works on local weak form and performs integration over overlapping simple local domains, and thus has removed the need of mesh at any stage of analysis. Hence, it is truly a meshfree method. The method was further elaborated and developed by Sladek et al. [3], Atluri and Shen [4], Qian and Batra [5], Xue–Hong et al. [6], Baradaren and Mahmoodebadi [7], Thakur et al. [8-10], Dai et al. [11] and Zhang et al. [12] concluded that MLPG has a very high rate of convergence and it does not need any post processing technique and does not exhibit any volumetric locking. MLPG method works on Petrov–Galerkin formulation i.e. trial and test functions are selected from different spaces. This provides a large number of possible combinations to formulate MLPG method.

In the present article, the MLPG method is used to compare extended surfaces of longitudinal fins of variable profile fin under transient, linear and radiative conditions. The analysis is structured in two sections - first section establishes the validity of MLPG method by solving fin under transient state condition for constant thermal properties. Second section performs solving transient state condition for nonlinear heat transfer analysis on fin of all three profiles with constant conductivity & convection with variable radiative conditions. Fins are extended surfaces, in longitudinal or radial fins to increase heat transfer from a surface. Fins of variable geometries and material properties are used in wide range of general as well as sophisticated engineering applications. It includes air conditioning, aerospace, automobile, chemical processing equipment, from large industrial heat exchangers to small systems such as transistors and other electronic components.

Exact analysis of heat transfer over fin surfaces is very complicated problem. Study of various fin profiles, such as triangular and concave and its comparison with rectangular profile using MLPG method is a topic current research
interest. Rectangular fins are simple in shape and construction and in spite of higher relative weight compared to triangular and convex and thus the results are needed to be compared with triangular and convex. Kraus et al. [13] presented and extensive review to this topic. Kosarev [14] examined steady state problems of longitudinal rectangular fin of temperature dependent thermal conductivity. Snider [15] described precise, unambiguous and accurate mathematical model of extended surfaces for design evaluation. Xiang [16] studied on optimization of circular fins of parabolic profile with variable thermal parameters and presented design guidelines for engineering practice. Nguyen and Aziz [17] used FEM to predict performance of convecting-radiating fins of rectangular, triangular and concave parabolic shapes and showed that effect of profile shape is most pronounced when Biot number and radiation number are small compared to unity. Al-Sanea and Mujahi [18] constructed finite volume model to evaluate time dependent heat transfer characteristics of longitudinal fins with rectangular profile. Zubair et al. [19] studied on circular fin of parabolic profile with different thermal conductivity and found that heat transfer rate is 20% higher than constant thickness fin. Singh et. al. [20] studied and proposed a hyperbolic function for transient and steady state solutions of two-dimensional heat transfer through the fins using a meshless element free Galerkin method. Lai et al. [21] studied and introduced recursive formula for calculating thermal performance of singular annular fin with variable thermal properties with and without heat transfer on fin tip. Moitsheki and Harley [22] studied transient heat transfer through a longitudinal fin of various profiles, employed Classical Lie point symmetry methods. Harley and Moitsheki [23] studied a model describing the temperature profile of rectangular fin in MATLAB and found out that the thermo-geometric parameter is proportional to the length of fin and inversely proportional to the fin thickness and the temperature at the tip of fin will be lower for either longer or thinner fins. Sadri et al. [24] studied on straight fin with variable thermal conductivity and heat transfer co-efficient using differential transformation method (DTM) and found results are in good agreement with numerical solutions. Khan and Aziz [25] investigated transient thermal performance of constant area longitudinal fin of functionally graded material and found that transient and steady state performance is affected by fin material. Pirompugd and Wongwises [26] studied on partially wet fins efficiencies for longitudinal fin of rectangular, triangular, concave parabolic and convex parabolic profile and found that the fin with larger cross-section has a higher conduction heat transfer and more fin efficiency. Pashah et. al. [27] studied on four fin profiles using non-dimensional finite element formulation and concluded that polymer composite material result in increase in performance of a fin, Rusagara and Harley [28] investigated and proposed new index to measure fin performance for non-linear thermal conductivity. Mosayebidorcheh et al. [29] studied the transient thermal analysis of longitudinal fins with variable cross section considering internal heat generation using power-law temperature-dependent (DTM & FDM) model and concluded that concave is the best profile out of rectangular, convex, triangular and concave and also showed that fin surface temperature is greatly depended on type of heat transfer. Lin and Chen [30] used differential transform method (DTM) and double-decomposition method (DDM) to solve the annular hyperbolic profile fins with variable thermal conductivity and concluded that numerical solutions of DDM are not correct and thus DTM is more precise. Sobamowo [31] studied and analyzed heat transfer in longitudinal rectangular fin with temperature-dependent thermal properties and internal heat generation using Galerkin’s weighted residual method and showed that fin temperature, the total heat transfer, the fin effectiveness, the fin efficiency are significantly affected by thermo-geometric and thermal parameters of the fin. Kim [32] studied thermal optimization of a tube with internal fins with variable thickness and concluded that thermal resistance reduced by up to 12% using concave fin, Babaealahi and Eshraghi [33] studied on convex profile fin with variable thermal conductivity and mass transfer and concluded that generalized differential transformation method (GDTM) is having high accuracy. Mosayebidorcheh et. al. [34] focused on thermal behaviour of radial fins of rectangular, triangular and hyperbolic profiles with temperature-depended properties using DTM and FDM. Roy et al. [35] numerically studied rectangular, convex, triangular profile with all variable thermal conductivity, heat transfer co-efficient, surface emissivity and heat generation using Adomain Decomposition Method (ADM) and found that results obtained by modified ADM are more accurate, Singh et al. [36] studied on cross-flow type fin and tube heat exchanger, characterized by a dimensionless design variable named aspect ratio which was varied parametrically to obtain different profiles in furnishing enhanced overall performance along with the net material cost.

Methods

Fin profile & geometrical parameters

A longitudinal fins are investigated in current analysis with three different profile as shown in the Fig 1.
The different parameters used for the transient, linear and nonlinear analysis are listed in Table 1.

<table>
<thead>
<tr>
<th>Thermo-Geometrical Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference heat transfer coefficient, ( h_0 )</td>
<td>10 W/m(^2)K</td>
</tr>
<tr>
<td>Reference thermal conductivity, ( k )</td>
<td>400 W/m.K</td>
</tr>
<tr>
<td>Density, ( \rho )</td>
<td>7800 kg/m(^3)</td>
</tr>
<tr>
<td>Specific heat, ( C )</td>
<td>400 J/kg.(^\circ)C</td>
</tr>
<tr>
<td>Effective sink temperature for radiation, ( T_s )</td>
<td>20(^\circ)C</td>
</tr>
<tr>
<td>Base temperature, ( T_b )</td>
<td>200(^\circ)C</td>
</tr>
<tr>
<td>Initial temperature, ( T_{init} )</td>
<td>200(^\circ)C</td>
</tr>
<tr>
<td>Thickness of the fin at base, ( \delta_b )</td>
<td>0.008 m</td>
</tr>
<tr>
<td>Fin height over base, ( b )</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Cross-sectional area, ( A )</td>
<td>0.0008 m(^2)</td>
</tr>
<tr>
<td>Thermal conductivity, ( k(T) )</td>
<td>( k_0[1+\beta T] )</td>
</tr>
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</table>

Heat transfer coefficient, \( h(T) \)

\[
h_0, \left[ \frac{T - T_{\infty}}{T_s - T_{\infty}} \right] \quad n = 1
\]

2.2. Governing Equation and Boundary Conditions

One dimensional heat transfer equation is considered. Heat transfer coefficient follows the power law and is temperature-dependent. Thermal conductivity of the fin material is also assumed to temperature-dependent. The fin is exposed for conjugate convection and radiation conditions. There is no heat generation in the solid. The governing differential equation \( \Omega \) for such a case is given by

\[
\frac{d}{dx} \left[ (\delta L)k(T) \frac{dT}{dx} \right] - 2Lh(T)(T - T_{\infty}) - h_r(2L)(T - T_s) - \rho c \frac{\partial T}{\partial t} = 0 \quad (2)
\]

Boundary Conditions: Following boundary conditions are assumed, in general,

\[
\begin{align*}
T(x) &= \bar{T} \quad \text{on } \Gamma_1 \\
q(x) &= \bar{q} \quad \text{on } \Gamma_2 \\
q(x) &= h(T_{\infty} - T) + h_r(T_s - T) \quad \text{on } \Gamma_3
\end{align*}
\]

If, \( h_r = \varepsilon \sigma (2L)(T^2 - T_s^2)(T + T_{\infty}) \) then eq. (1) becomes

\[
\frac{d}{dx} \left[ (\delta L)k(T) \frac{dT}{dx} \right] - 2Lh(T)(T - T_{\infty}) - h_r(2L)(T - T_s) - \rho c \frac{\partial T}{\partial t} = 0 \quad (2)
\]

Where, \( v \) is the test function. Using divergence theorem, Eq. (4) yields the desired weak form as:

\[
\int_{\Omega} \left[ \frac{d}{dx} \left( \rho L \frac{dT}{dx} \right) - 2Lh(T)(T - T_{\infty}) \right] d\Omega = 0
\]

Where, \( \Gamma_{\Omega} \) is the boundary of the local domain, \( \Omega_{\Omega} \). In case of 1-D problem, boundary integrals turn to be a point value on boundaries. Taking advantage of MLPG method’s flexibility, the test function \( v \) is selected such that it vanishes at the boundary of local domain. Hence, boundary integral remains non-zero only when local domain intersects the global boundary. The essential boundary conditions are imposed by
penalty function method, developed by Zhu and Atluri. Therefore, equation (5) can be written as:
\[ [q^v]_{\Gamma_iq} + [q^v]_{\Gamma_{2q}} + [v_T(T(T_{\inf} - T))]_{\Gamma_{1q}} \]
\[ - \int_{\Omega_q} \frac{dv}{dx} \delta_k(T) \frac{dT}{dx} d\Omega - \int_{\Omega_q} 2h(T)vT d\Omega \]
\[ + \int_{\Omega_q} 2h(T)vT_{\inf} d\Omega - \int_{\Omega_q} 2h_T v d\Omega + \int_{\Omega_q} 2h_T s v d\Omega \]
\[ + \int_{\Omega_q} \rho c T v d\Omega - \alpha[(T - \tilde{T})v]_{\Gamma_{1q}} = 0 \] (6)
Where \( \Gamma_{1q} = \Gamma_1 \cap \Gamma_q \), \( \Gamma_{2q} = \Gamma_2 \cap \Gamma_q \), \( \Gamma_3 = \Gamma_3 \cap \Gamma_q \) and \( \alpha \) is the penalty parameter = 1 x 10^10
The unknown function, \( T \), is approximated by moving least square scheme (MLS)as follows:
\[ T_h(x) = \sum_{r=1}^{n_t} \Phi_i T_i = \Phi \mathbf{T} \] (7)
where \( \Phi \) is the vector of meshfree shape functions \( \Phi_i \), \( \mathbf{T} \) represents the vector of nodal parameters \( T_i \) and \( n_t \) is the number of nodes in the support domain at point \( x \). Substituting the approximation (8) in Eq. (7) and performing integration over all local domains corresponding to all field nodes, the discrete system can be obtained as follows:
\[ CT + KT = F \] (8)
The typical matrix elements are expressed as
\[ K_{ij} = \left[ \int_{\Omega_j} \frac{dv_i}{dx} \delta_k(T) \frac{dT}{dx} d\Omega + \int_{\Omega_j} 2h(T)v_i \phi_j d\Omega \right] \]
\[ + \left[ \int_{\Omega_j} 2h_T v_i d\Omega + \int_{\Omega_j} \rho c T v_i d\Omega \right] \] (9)
\[ C_{ij} = \left[ \int_{\Omega_j} \rho c T v d\Omega \right] \] (10)
\[ F_i = \left[ \int_{\Omega_j} 2h(T)T_{\inf} v_i d\Omega + \int_{\Omega_j} 2h_T s v_i d\Omega \right] \] (11)
Consideration of variable thermal conductivity and variable heat transfer coefficient yields non-linear set of algebraic equations. For such nonlinear problems, an iterative solution procedure is required. A predictor-corrector scheme based on direct substitution iteration has been applied in the current analysis which has the following form:
- **Predictor:**
  \[ [C(X^*) + \theta \Delta A(X^*)]X^{*+1} = \]
  \[ [C(X^*) + (1 - \theta)\Delta A(X^*)]X^* + \Delta B(X^*) \] (12)
- **Corrector:**
\[ [C(X^*) + \theta \Delta A(X^*)]X^{*+1} = \]
\[ [C(X^*) + (1 - \theta)\Delta A(X^*)]X^* + \Delta B(X^*) \] (13)
Where \( \theta = 0, 1, 2, 3 \ldots \) up to convergence and \( X^{*+1} = wX^* + (1 - w)X^* \quad 0 \leq w \leq 1 \)
\[ X^{*+1} = X^{*+1} \] (14)

**Results**

In this category, the MLPG method is applied on a problem solved initially by Singh et al. Transient analysis of rectangular profile at distance of \( x = 0.25m \) with constant thermal properties i.e. thermal conductivity and convection coefficient are considered. This is to check the accuracy of MLPG result and to validate it by comparing with that of obtained. A comparison of the results are shown in Table 3 and found to be in good agreement.

<table>
<thead>
<tr>
<th>Table 3. Validation of results [20]</th>
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<tbody>
<tr>
<td><strong>RECTANGULAR PROFILE (x=0.25)</strong></td>
</tr>
<tr>
<td><strong>Temperature °C</strong></td>
</tr>
<tr>
<td><strong>Time t, (s)</strong></td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>100</td>
</tr>
<tr>
<td>200</td>
</tr>
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<td>300</td>
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<td>900</td>
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<td>1000</td>
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The maximum relative error with EFG solution is found to be 3.25% at initial time of 100 seconds but immediately as time proceeds to 200 seconds relative maximum error with EFG is less than 1%. 

**Case 1 : Comparison of temperature variation with constant conductivity, convection with & without radiation**

In this section transient analysis is performed to observe fin response for temperature variation along the length of rectangular, triangular and concave profile with constant conductivity & convection with radiation and without radiation are plotted in figure 2, figure 3 & figure 4 respectively.
Case 2: Comparison of temperature time history for constant conductivity & convection with & without radiation

In this section, fin response is observed for temperature time history along the length of all rectangular profiles, triangular profile and concave profile with constant conductivity, convection with & without radiation effects are plotted in figure 5, figure 6 & figure 7 respectively.
Discussion

From the above plots of fig. 2, fig. 3 and fig. 4, we can see that temperature distribution along fin length on the three profiles shows that in rectangular profile steady state is achieved gradually after much higher period of time, whereas compared to triangular and concave it gets in steady state almost after few time i.e. almost 200 seconds. Secondly it can be observed that temperature drop is high in rectangular profile compared to triangular and concave, which shows that temperature difference is function of cross-sectional area.

Initially all the curves are very close but apart when moves away towards the end of fin profile. Also radiation effect is considerable on all profiles and drop in temperature is more when radiation effect is considered.

Above plots of fig. 5, fig. 6 and fig. 7 shows that drop in temperature along the fin length initially is maximum in rectangular profile, whereas very less in triangular and concave profile as later steady state is achieved.

Conclusion

Numerical simulation of radiative fin of variable profile is performed using MLPG method. The MLPG is a truly meshfree method as it does not require any mesh either at the stage of interpolation or at the stage of integration. Fins are very common to increase the rate of heat transfer but it is very essential to predict results with high accuracy. MLPG method can solve nonlinear fin problems and predict accurate solution.

Also, it can be concluded that while selecting of fin profile is depended upon its tip fin temperature, application, feasibility, heat transfer rate, material, economic factors etc. Using MLPG method, the developed code has a potential to obtain solution very close to real situations. It can also be concluded that the effect of radiation significantly affects output parameters.

References

[14] D. A. Kosarev, Steady state heat transfer through a wall with longitudinal rectangular fins and variable thermal


