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Algebra of Morphological Filter on Intuitionistic Fuzzy Hypergraphs

With application in Summarization of Documents

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***Abstract*— Graph morphology has been an area of interest over many years and has its applications in text and image processing. Later came hypergraphs with more number of nodes in a hyperedge. Intuitionistic Fuzzy hypergraphs (IFHG) are further developments in graph theory where a membership degree and non membership degree is given for every node and hyperedge. Further many morphological operations can be applied on the sub graphs of this IFHG. Morphological operations include dilation, erosion, adjunction, opening, closing and filtering. The purpose of this paper is to define union, intersection, complement operations on morphological filter designed on IFHG. An application of this type of modeling in the field of document summarization is also shown in this paper.**

Keywords—Intuitionistic Fuzzy; hypergraph; filter; summarization;

I. INTRODUCTION

Let $[H_{IF}, (\mu_n, \gamma_n), (\mu_e, \gamma_e), H^n, H^e]$ be an intuitionistic fuzzy hypergraph with membership degree μ_n and non membership degree γ_n defined on the set of nodes H^n and membership degree μ_e and non membership degree γ_e defined on a set of hyperedges H^e. Depending on the membership degree μ_n, the node can be treated as high priority, medium priority and low priority. The non membership degree $\gamma_n <= 1 - \mu_n$. Similarly depending on the membership degree μ_e, the hyperedge can be treated as high priority, medium priority and low priority. The sum of the membership degree and non membership degree of the node is less than or equal to 1 [1]. ie, $\mu_n + \gamma_n <= 1$.So also, the sum of the membership degree and non membership degree of the hyperedge is less than or equal to 1 [1]. ie, $\mu_e + \gamma_e <= 1$. If all the nodes in a hyperedge have $\mu_n > 0.5$, then μ_e is the supremum of all μ_n in that edge. In such a case $\gamma_e <= 1 - \mu_e$. If there is at least one node with $\gamma_n > 0.5$, then the γ_e of that edge is the supremum of all γ_n in that edge. In such a case $\mu_e <= 1 - \gamma_e$. Morphological operations like opening, closing, half opening, half closing of hypergraphs [2] are defined which can be extended to weighted hypergraphs also. Morphological operations like dilation, erosion, adjunction are already defined on hypergraphs [3] and morphological dilation is defined on Intuitionistic Fuzzy Hypergraphs [4]. Morphological Erosion of IFHG [5] is also done, where erosion with respect to nodes and hyperedges are considered. This paper introduces algebraic operations on morphological filtering of IFHG. In order to apply morphological filter, we define sub IFHGs X_{IF} and Y_{IF} which are obtained from $\alpha - \beta$ cut on H_{IF}. Here δ is the dilation operator and ε is the erosion operator.

II. RELATED WORKS

Structure of Intuitionistic Fuzzy graphs (IFG) [6], strong arc, weak arc, strong path, strongest path are studied earlier, where the authors introduced the concept of arc being called as a bridge. Some properties of complete IFG are also discussed. Complement of an IFG , operations like union , join, cartesian product, composition are analyzed ,where the authors also studied about the properties of self complementary IFG [7]. Cardinality of IFG, the concepts like bipartite, complete bipartite, strong arc, strength of connectedness, dominating set, number, independent set, number are studied. The authors studied some properties of minimal independent, minimal dominating IFG [8], where these concepts can be applied to network applications. A method to find the shortest hyperpath [9] in an Intuitionistic fuzzy weighted hypergraph is proposed where ranking is done using scores and accuracy. Now coming to the field of Intuitionistic Fuzzy hypergraphs, concepts of IFHG [1], strength of an edge, (α, β) cut, incidence matrix, adjacent level between two vertices , stronger edge, dual IFHG, its incidence matrix are already defined . These (α, β) cuts can be used for hypergraph partitioning. Operations like complement, join, union, intersection, ringsum, cartesian product, composition are defined for intuitionistic fuzzy graphs [10], where the authors further proposed to apply these operations in clustering techniques. Isomorphism between two intuitionistic fuzzy

directed hypergraphs (IFDHG) is also discussed [11], where the authors have introduced IFDHG, its order, in-degree, out-degree, homomorphism, weak isomorphism and co-weak isomorphism between two IFDHGs. Application of IFHG in radio coverage network is also suggested by [12], in which their model can be used to determine station programming and develop marketing strategies. The authors also suggested the use of IFHG for clustering computer networks. An application with Intuitionistic Fuzzy sets for career choice[13] which is a decision making system was developed, where the system represented the performance of students using membership μ, non membership υ and hesitation margin π. They applied normalized Euclidean distance to determine the apt career choice.

Now let us see some fuzzy based works in Text processing. A lot of methods have been developed for text summarization using fuzzy based systems. A number of parameters like sentence position in paragraph, sentence length, similarity to title, similarity of keyword, similarity to text concept, proper noun, and sentence cohesion are used in fuzzy systems. The authors [14] have compared the score of vector based method and fuzzy method given by five judges and the fuzzy based summary gave a summary which rejects 77% of the concepts as opposed to 66% performance by the vector based method. In another method, the vector features [15] created for each sentence in the document include title feature, sentence length, term weight, sentence position, sentence to sentence similarity, numerical data etc. The results compared with word summarizer, copernic summarizer has shown a better result. Almost the same set of features are used by a triangular membership function [16] which fuzzifies each score to three values: low, medium and high. A parallel summary using latent semantic analysis is also taken and both are merged to get the final summary. Experimental results have shown an average precision of 89%. A comparison of fuzzy system is done with neural network with features like cue phrases, legal vocabulary, paragraph structure, citation, term weight, named entity, similarity to neighboring sentences, absolute location etc and a better result is demonstrated by fuzzy based system [17], [18]. A fuzzy logic based inference system computes the score of each sentence from highest to value above a threshold. The results compared with word summarizer show a better output for fuzzy system [19].

III. PRELIMINARY DEFINITIONS

Let us define IFHG = [H_{IF} , (μ_n , γ_n), (μ_e , γ_e), H^n, H^e], where H^n = {n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8, n_9, n_{10}, n_{11}, n_{12}, n_{13}, n_{14}, n_{15}, n_{16}, n_{17}, n_{18}, n_{19}, n_{20}, n_{21}, n_{22}, n_{23}, n_{24}, n_{25}, n_{26} , n_{27}, n_{28}, n_{29}, n_{30}, n_{31}, n_{32}, n_{33}, n_{34}, n_{35}, n_{36}, n_{37}, n_{38}, n_{39}, n_{40}, n_{41}} and H^e = {e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12} , e_{13}, e_{14} , e_{15}, e_{16}} as given in Fig.1. Some nodes are of low priority (μ_n < 0:5), some nodes are of medium priority (μ_n = 0:5) and few others are of high priority (μ_n > 0:5). Let X_{IF} be obtained by (α, β) cut on H_{IF} such that, 0.5 < α <= 0.7; {β <= 1 – α}∩ {β <= 0.4}. Let Y_{IF} be obtained by (α, β) cut on H_{IF} such that, α >= 0:7; {β <= 1 – α }∩ {β <= 0.3} . Here α corresponds to membership degree and β corresponds to non membership degree. The details of the hypergraphs H_{IF}, X_{IF} and Y_{IF} are given in Fig .1. Having given IFHGs H, X and Y, let us define dilation $\delta^n(Y^e)$ as the set of all nodes within the hyperedges in Y . Now $\delta^e(Y^n)$ is the set of hyperedges in H which consists of at least one node in Y . Let us define erosion with the help of the operator ε. Now $\varepsilon^e(Y^n)$ is the set of all hyperedges in H which consists of nodes of Y only. Similarly $\varepsilon^n(Y^e)$ is the set of all nodes not in $Y^{e\prime}$.

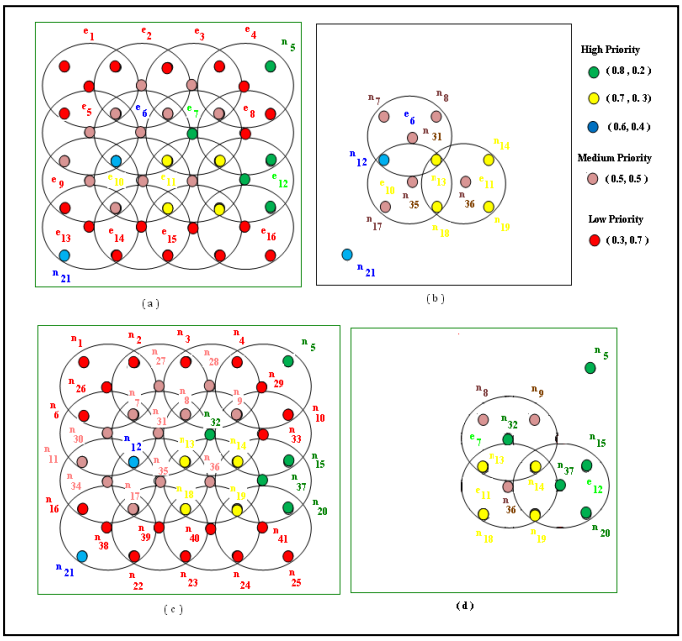

Fig .1 (a) H (b) X (c) H-with Node numbers (d) Y

IV. ALGEBRA OF FILTER

A morphological filter is nothing but an opening filter (γ) which is obtained as (δ o ε) or a closing filter (Φ) which is obtained as (ε o δ) . Opening filter can again be classified as i) opening filter w.r.to nodes $\delta^n(\varepsilon^e(Y^n))$ for which the filtrate are nodes ii) opening filter w.r.to hyperedges $\delta^e(\varepsilon^n(Y^e))$ for which the filtrate are hyperedges. Closing filter can be classified as i) closing filter w.r.to nodes $\varepsilon^n(\delta^e(Y^n))$ for which the results are nodes ,ii) closing filter w.r.to hyperedges $\varepsilon^e(\delta^n(Y^e))$ for which the results are hyperedges. Now let us apply algebraic operations like union, intersection, complement operations on these filters.

A. Proposition A: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\delta^e(\varepsilon^n(X \cup Y)^e) = \delta^e(\varepsilon^n(X^e)) \cup \delta^e(\varepsilon^n(Y^e)) \qquad (1)$$

where $\delta^e(\varepsilon^n(X \cup Y)^e$ is an opening filter w.r.to hyperedges.

Proof : Consider L.H.S of eq(1) . Let v be an arbitrary node in $\varepsilon^n (X \cup Y)^e$ .ie,

$$v \notin (X \cup Y)^{e\prime} ;\ v \in (X \cup Y)^e \qquad (2)$$

Let e be the edge which contains this v. ie,

$$e \subseteq \delta^e(\varepsilon^n (X \cup Y)^e) \qquad (3)$$

Since $v \in (X \cup Y)^e$ this e may be present either in X or Y . Consider R.H.S of eq(1) . Let v be a node in $\varepsilon^n (X^e)$. ie,

$$v \notin X^{e\prime} ; v \in X^e \qquad (4)$$

Now $\delta^e(\varepsilon^n (X^e)) = \{e_1, e_2 \ldots e_p\}$ which contain v . Let u be a node in $\varepsilon^n (Y^e)$.
ie,

$$u \notin Y^{e\prime} ; u \in Y^e \qquad (5)$$

Now $\delta^e(\varepsilon^n (Y^e)) = \{e_{p+1}, \ldots e_k\}$. let e be an arbitrary edge in $\{e_1, \ldots e_k\}$.
This e can be either a member of $\delta^e(\varepsilon^n (X^e))$ or $\delta^e(\varepsilon^n (Y^e))$. ie,

$$e \subseteq \delta^e(\varepsilon^n (X^e)) \cup \delta^e(\varepsilon^n (Y^e)) \qquad (6)$$

Hence Eq(1) is implied from eq(3) and eq(6).

Example 4.1 Consider the IFHGs given in Fig 1. Let us find the R.H.S of eq(1) ,where we get $\varepsilon^n (X^e)= \{n_{31}, n_{35}; n_{36}\}$. Thus we get $\delta^e(\varepsilon^n (X^e)) = \{e_6, e_{10}, e_{11}\}$. Again in R.H.S of eq(1) we can find $\varepsilon^n (X^e)= \{n_{32}, n_{36}, n_{37}\}$. Thus we get $\delta^e(\varepsilon^n (Y^e)) = \{e_7, e_{11}, e_{12}\}$. Now $\delta^e(\varepsilon^n (X^e))$ U $\delta^e(\varepsilon^n (Y^e)) = \{e_6, e_7, e_{10}, e_{11}, e_{12}\}$. Let us consider the L.H.S of eq(1) , where $\delta^e(\varepsilon^n (X \text{ U } Y)^e) = \{e_6, e_7, e_{10}, e_{11}, e_{12}\}$. The results are shown in Fig.2(a).

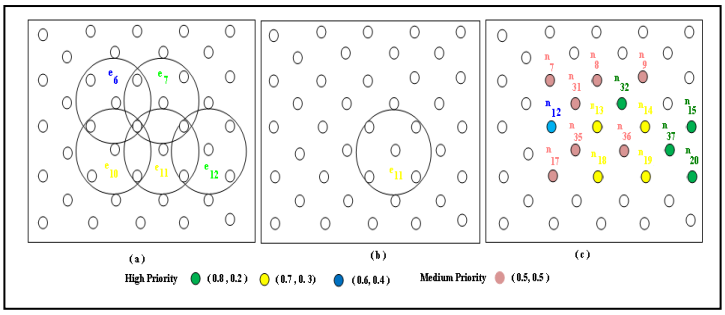

Fig 2 :Results of (a) Proposition A (b) Proposition B (c) Proposition C

B. Proposition B: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\delta^e(\varepsilon^n (X \cap Y)^e) = \delta^e(\varepsilon^n (X^e)) \cap \delta^e(\varepsilon^n (Y^e)) \qquad (7)$$

where $\delta^e (\varepsilon^n (X \cap Y)^e$ is an opening filter w.r.to hyperedges.

Proof : Consider L.H.S of eq (7). Let v be an arbitrary node in $\varepsilon^n (X \cap Y)^e$ ie,

$$v \notin (X \cap Y)^{e\prime} ;\ v \in (X \cap Y)^e \qquad (8)$$

Let e be the edge which contains this v. Now we can say that

$$e \subseteq \delta^e(\varepsilon^n (X \cap Y)^e) \qquad (9)$$

Consider R.H.S of eq(7). Let $\delta^e(\varepsilon^n (X^e)) = \{e_1, e_2 \ldots e_p\}$. Let $\delta^e(\varepsilon^n (Y^e)) = \{e_{p+1}, \ldots e_k\}$. let e be an arbitrary edge in $\{e_1, \ldots e_k\}$.
This e will be a member of $\delta^e(\varepsilon^n (X^e))$ and $\delta^e(\varepsilon^n (Y^e))$. ie,

$$e \subseteq \delta^e(\varepsilon^n (X^e)) \cap \delta^e(\varepsilon^n (Y^e)) \qquad (10)$$

Eq(7) is implied from eq(9) and eq(10).

Example 4.2 Consider the IFHGs given in Fig 1. Let us find the R.H.S of eq(7) ,where we get $\varepsilon^n (X^e)= \{n_{31}, n_{35}; n_{36}\}$. Thus we get $\delta^e(\varepsilon^n (X^e)) = \{e_6, e_{10}, e_{11}\}$. Again in R.H.S of eq(7) we can find $\varepsilon^n (Y^e)= \{n_{32}, n_{36}, n_{37}\}$. Thus we get $\delta^e(\varepsilon^n (Y^e)) = \{e_7, e_{11}, e_{12}\}$. Now $\delta^e(\varepsilon^n (X^e)) \cap \delta^e(\varepsilon^n (Y^e)) = \{e_{11}\}$. Let us consider the L.H.S of eq(7) , where $\delta^e(\varepsilon^n (X \cap Y)^e) = \{e_{11}\}$. The results are shown in Fig.2(b).

C. Proposition C: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\delta^n(\varepsilon^e (X \cup Y)^n) = \delta^n(\varepsilon^e (X^n)) \cup \delta^n(\varepsilon^e (Y^n)) \qquad (11)$$

where $\delta^n(\varepsilon^e (X \cup Y)^n)$ is an opening filter w.r.to nodes.

Proof : Consider L.H.S of eq(11).Let e be an arbitrary edge in $\varepsilon^e (X \cup Y)^n$. Let v be an arbitrary node in e. By definition of δ^n , this v can be a node either of X or Y . ie, $v \in X$ or $v \in Y$. Consider R.H.S of eq(11). Let $\{v_1, v_2 \ldots v_p\}$ be the nodes in $\delta^n(\varepsilon^e (X^n))$ .By definition of δ^n , a node in the above set belongs to X. ie $v \in X$. Let $\{v_{p+1}, v_{p+2} \ldots v_k\}$ be the nodes in $\delta^n(\varepsilon^e (Y^n))$. By definition of δ^n, a node in the above set belongs to Y. ie $v \in Y$. A node v in $\{v_1, v_2 \ldots v_k\}$ is either an element of X or Y .Thus $v \in X$ or $v \in Y$. This implies eq(11).

Example 4.3 Consider IFHGs in Fig.1 . Let us find the R.H.S of eq(11),where we get $\varepsilon^e (X^n) = \{e_6, e_{10}, e_{11}\}$. Thus $\delta^n(\varepsilon^e (X^n)) = \{n_7, n_8, n_{12}, n_{13}, n_{14}, n_{17}, n_{18}, n_{19}, n_{31}, n_{35}, n_{36}\}$. Again $\varepsilon^e(Y^n) = \{e_7, e_{11}, e_{12}\}$. Thus we get $\delta^n(\varepsilon^e (Y^n)) = \{n_8, n_9, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{32}, n_{36}, n_{37}\}$. Thus $\delta^n(\varepsilon^e(X^n)) \cup \delta^n(\varepsilon^e (Y^n)) = \{n_7, n_8, n_9, n_{12}, n_{13}, n_{14}, n_{15}, n_{17}, n_{18}, n_{19}, n_{20}, n_{31}, n_{32}, n_{35}, n_{36}, n_{37}\}$.

Considering L.H.S , we get $\varepsilon^e (X \cup Y)^n = \{ e_6, e_7, e_{10}, e_{11}, e_{12}\}$. Thus $\delta^n(\varepsilon^e (X \cup Y)^n) = \{ n_7, n_8, n_9, n_{12}, n_{13}, n_{14}, n_{15}, n_{17}, n_{18}, n_{19}, n_{20}, n_{31}, n_{32}, n_{35}, n_{36}, n_{37}\}$. The results are shown in Fig.2(c).

D. Proposition D: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\delta^n(\varepsilon^e (X \cap Y)^n) \subset \delta^n(\varepsilon^e(X^n)) \cap \delta^n(\varepsilon^e(Y^n)) \qquad (12)$$

where $\delta^n(\varepsilon^e (X \cap Y)^n)$ is an opening filter w.r.to nodes.

Example 4.4: Consider the IFHGs given in Fig.1 . Let us find the L.H.S of eq(12). Here $\varepsilon^e (X \cap Y)^n$ is the set of edges which contains $(X \cap Y)^n$ only. ie, $\varepsilon^e (X \cap Y)^n = \{ e_{11}\}$. Now $\delta^n(\varepsilon^e (X \cap Y)^n)$ is the set of nodes within $\{e_{11}\}$. Thus $\delta^n(\varepsilon^e (X \cap Y)^n) = \{n_{13}, n_{14}, n_{18}, n_{19}\}$. Now consider R.H.S of eq(12). Here $\delta^n(\varepsilon^e(X^n)) = \{ n_7, n_8, n_{12}, n_{13}, n_{14}, n_{17}, n_{18}, n_{19}, n_{26}, n_{27}, n_{28}\}$. Also $\delta^n(\varepsilon^e(Y^n)) = \{ n_8, n_9, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{28}, n_{29}, n_{30} \}$.Thus $\delta^n(\varepsilon^e(X^n)) \cap \delta^n(\varepsilon^e(Y^n)) = \{ n_8, n_{13}, n_{14}, n_{18}, n_{19}, n_{28}\}$. Thus we can see that L.H.S ⊂ R.H.S. The results are given in Fig 3(a) and 3(b).

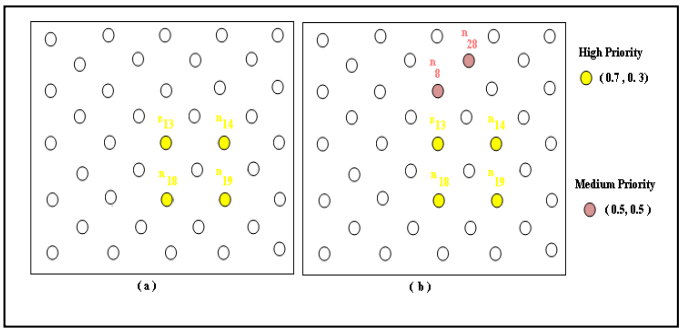

Fig 3 : (a) L.H.S of Proposition D , 3(b) R.H.S. of Proposition D

So far we have seen opening filters. Now let us see closing filters, which are of four types.

E. Proposition E: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\varepsilon^e(\delta^n(X \cup Y)^e) = \varepsilon^e(\delta^n(X^e)) \cup \varepsilon^e(\delta^n(Y^e)) \qquad (13)$$

where $\varepsilon^e(\delta^n(X \cup Y)^e)$ is a closing filter w.r.to hyperedges.

Proof : Consider the L.H.S of eq(13) .Let *v* be an arbitrary node in $\delta^n(X \cup Y)^e$. According to the definition of δ^n, $v \in X$ or $v \in Y$. Let *h* be an edge which contains this *v*. By definition of ε^e, this $h \in X$ or $h \in Y$. Consider R.H.S of eq(13). Let $\{h_1, h_2,\ldots. h_p\}$ be the edges in $\varepsilon^e(\delta^n(X^e))$. By definition of ε^e, an edge *h* in $\{ h_1, h_2,\ldots\ldots h_p \}$ is an element of X. ie, $h \in X$. Let $\{ h_{p+1}, h_{p+2},\ldots. h_k \}$ be edges in $\varepsilon^e(\delta^n(Y^e))$. By definition of ε^e, an edge *h* in $\{ h_1, h_2,\ldots. h_k\}$ will be edge of X or Y . ie $h \in X$ or $h \in Y$. Eq(13) is implied from this.

Example 4.5: Consider the IFHGs given in Fig.1. In L.H.S of eq(13), $\delta^n(X \cup Y)^e$ are the nodes within $(X \cup Y)^e$. ie, $\delta^n(X \cup Y)^e = \{ n_7, n_8, n_9, n_{12}, n_{13}, n_{14}, n_{15}, n_{17}, n_{18}, n_{19}, n_{20}, n_{31}, n_{32}, n_{35}, n_{36}, n_{37}\}$. Now $\varepsilon^e(\delta^n(X \cup Y)^e) = \{ e_6, e_7, e_{10}, e_{11}, e_{12}\}$. Consider R.H.S of eq(13). Here $\delta^n(X^e)$ are the nodes within X^e. ie, $\delta^n(X^e) = \{ n_7, n_8, n_9, n_{12}, n_{13}, n_{14}, n_{15}, n_{17}, n_{18}, n_{19}, n_{31}, n_{35}, n_{36}\}$. Thus $\varepsilon^e(\delta^n(X^e))$ is the set of edges which consists of these nodes only. ie $\varepsilon^e(\delta^n(X^e)) = \{ e_6, e_{10}, e_{11}\}$. Now $\delta^n(Y^e)$ are the nodes in Y^e. ie, $(\delta^n(Y^e) = \{ n_8, n_9, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{32}, n_{36}, n_{37}\}$. Thus $\varepsilon^e(\delta^n(Y^e))$ is the set of edges which contain the above nodes only. ie, $\varepsilon^e(\delta^n(Y^e)) = \{ e_7, e_{11}, e_{12}\}$. Therefore $\varepsilon^e(\delta^n(X^e)) \cup \varepsilon^e(\delta^n(Y^e)) = \{ e_6, e_{10}, e_{11}\} \cup \{ e_7, e_{11}, e_{12}\} = \{ e_6, e_7, e_{10}, e_{11}, e_{12}\}$. The results are shown in Fig 4(a).

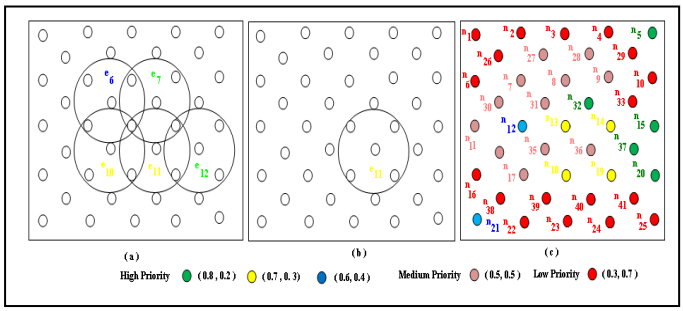

Fig 4 : Results of (a) Proposition E (b) Proposition F (c) Proposition G

F. Proposition F: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\varepsilon^e(\delta^n(X \cap Y)^e) = \varepsilon^e(\delta^n(X^e)) \cap \varepsilon^e(\delta^n(Y^e)) \qquad (14)$$

where $\varepsilon^e(\delta^n(X \cap Y)^e)$ is a closing filter w.r.to hyperedges.

Proof : Consider L.H.S of eq(14). Let *v* be a node in $\delta^n(X \cap Y)^e$. By definition of δ^n, it is a node in both X and Y. Let *e* be an edge which contains this *v*. By definition of ε^e, this is an edge in both X and Y. ie, $e \in X$; $e \in Y$ .Consider R.H.S of eq(14). Let $\{u_1, u_2, \ldots. u_p \}$ be nodes in $\delta^n(X^e)$. By definition of δ^n, these nodes are in X. Let $\{e_1, e_2\ldots.e_p\}$ be the edges which consist of these nodes. By definition of ε^e, these edges are in X. Let $\{u_{p+1}, u_{p+2}\ldots. u_k\}$ be nodes in $\delta^n(Y^e)$. By definition of δ^n, these nodes are in Y . Let $\{e_{p+1}, e_{p+2}\ldots. e_k\}$ be the edges which consist of these nodes. By definition of ε^e, these edge is in Y .Now an edge *e* in $\{e_1, e_2\ldots.e_k\}$ is present both in X and Y. ie $e \in X$, $e \in Y$.

Example 4.6 Consider L.H.S of eq(14). There $\delta^n(X \cap Y)^e = \{n_{13}, n_{14}, n_{18}, n_{19}, n_{36}\}$. Now $\varepsilon^e(\delta^n(X \cap Y)^e)$ is the set of edges which contain the above nodes only. ie , $\varepsilon^e(\delta^n(X \cap Y)^e) = \{e_{11}\}$. Consider R.H.S of eq(14). $\delta^n(X^e)$is the set of nodes in X^e. ie , $\delta^n(X^e) = \{n_7, n_8, n_{12}, n_{13}, n_{14}, n_{17}, n_{18}, n_{19}, n_{31}, n_{35}, n_{36}\}$.Now $\varepsilon^e(\delta^n(X^e))$ is the edges which contain the above nodes only. ie , $\varepsilon^e(\delta^n(X^e)) = \{e_6, e_{10}, e_{11}\}$. Likewise $\delta^n(Y^e)$ is the set of nodes in Y^e only = $\{n_8, n_9, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{32}, n_{36}, n_{37}\}$. Now $\varepsilon^e(\delta^n(Y^e))$ is the set of edges which contains these nodes only. ie , $\varepsilon^e(\delta^n(Y^e)) = \{e_7, e_{11}, e_{12}\}$. Now $\varepsilon^e(\delta^n(X^e)) \cap \varepsilon^e(\delta^n(Y^e)) = \{e_{11}\}$. The results are shown in Fig 4(b).

G. Proposition G: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\varepsilon^n(\delta^e(X \cup Y)^n) = \varepsilon^n(\delta^e(X^n)) \cup \varepsilon^n(\delta^e(Y^n)) \qquad (15)$$

where $\varepsilon^n(\delta^e(X \cup Y)^n)$ is a closing filter w.r.to nodes.

Proof : Consider L.H.S of eq(15). Let $\{e_1, e_2 \ldots e_k\}$ be the edges in $\delta^e(X \cup Y)^n$. Let *v* be an arbitrary node in any edge in $\{e_1, e_2 \ldots e_k\}$. According to the definition of $\varepsilon^n(\delta^e(X \cup Y)^n)$, this *v* should be contained in $\{e_1, e_2 \ldots e_k\}$ but not in $\{e_1, e_2 \ldots e_k\}'$. Consider R.H.S of eq(15). Let $\{e_1, e_2 \ldots e_p\}$ be the edges in $\delta^e(X^n)$. Let $\{n_1, n_2 \ldots n_p\}$ be the nodes in $\{e_1, e_2 \ldots e_p\}$. According to the definition of $\varepsilon^n(\delta^e(X^n))$ these nodes are present in $\{e_1, e_2 \ldots e_p\}$ but not in $\{e_1, e_2 \ldots e_p\}'$. Let $\{e_{p+1}, e_{p+2} \ldots e_k\}$. be the edges in $\delta^e(Y^n)$. Let $\{n_{p+1}, n_{p+2} \ldots n_k\}$ be the nodes in $\{e_{p+1}, e_{p+2} \ldots e_k\}$. According to the definition of $\varepsilon^n(\delta^e(Y^n))$ these nodes are present in $\{e_{p+1}, e_{p+2} \ldots e_k\}$ but not in $\{e_{p+1}, e_{p+2} \ldots e_k\}'$. Ie any node *v* in $\{e_1, e_2 \ldots e_k\}$.is present in $\{e_1, e_2 \ldots e_k\}$ but not in $\{e_1, e_2 \ldots e_k\}'$.

Example 4.7 Consider L.H.S of eq(15). Here $\delta^e(X \cup Y)^n$ is the set of edges which contains nodes in $(X \cup Y)^n = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}$. Thus $\varepsilon^n(\delta^e(X \cup Y)^n)$ = nodes in the above edges but not in their complement = All nodes in H. Consider R.H.S of eq(15). Here $\delta^e(X^n)$ is the set of edges which contain nodes in X^n = All edges in H. Now $\varepsilon^n(\delta^e(X^n))$ is the set of nodes in above edges but not in their complement = All nodes in H. Now $\delta^e(Y^n)$ is the set of edges which contain nodes in $Y^n = \{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}$. Hence $\varepsilon^n(\delta^e(Y^n))$ is the nodes in the above edges , but not in their complement = $\{n_3, n_4, n_5, n_8, n_9, n_{10}, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{21}, n_{22}, n_{23}, n_{24}, n_{25}, n_{27}, n_{28}, n_{29}, n_{31}, n_{32}, n_{33}, n_{35}, n_{36}, n_{37}, n_{39}, n_{40}, n_{41}\}$. Hence $\varepsilon^n(\delta^e(X^n)) \cup \varepsilon^n(\delta^e(Y^n))$ = All nodes in H. The results are shown in Fig 4(c).

H. Proposition H: Let H be a parent IFHG, X and Y be sub IFHGs, δ be the dilation operator and ε be the erosion operator, then

$$\varepsilon^n(\delta^e(X \cap Y)^n) \subset \varepsilon^n(\delta^e(X^n)) \cap \varepsilon^n(\delta^e(Y^n)) \qquad (16)$$

where $\varepsilon^n(\delta^e(X \cap Y)^n)$ is a closing filter w.r.to nodes.

Example 4.8 Consider L.H.S of eq(16). Here $\delta^e(X \cap Y)^n$ is the set of edges which consists of any element in $(X \cap Y)^n$. = $\{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{11}, e_{12}\}$. Now $\varepsilon^n(\delta^e(X \cap Y)^n)$ is the set of nodes in the above edges but not in their complement . ie $\varepsilon^n(\delta^e(X \cap Y)^n) = \{n_3, n_4, n_5, n_8, n_9, n_{10}, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{27}, n_{28}, n_{29}, n_{31}, n_{32}, n_{33}, n_{35}, n_{36}, n_{37}\}$. Now consider R.H.S of eq(16). $\delta^e(X^n)$ is the set of edges which contain any node in X^n . ie, $\delta^e(X^n)$ = All edges in H. Now $\varepsilon^n(\delta^e(X^n))$ = All nodes in H. Now $\delta^e(Y^n)$ is the set of edges which contain nodes in Y^n . ie, $\delta^e(Y^n) = \{e_2, e_3, e_4, e_6, e_7, e_8, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}\}$. Hence $\varepsilon^n(\delta^e(Y^n))$ are the nodes in the above edges, but not in their complement = $\{n_3, n_4, n_5, n_8, n_9, n_{10}, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{21}, n_{22}, n_{23}, n_{24}, n_{25}, n_{27}, n_{28}, n_{29}, n_{31}, n_{32}, n_{33}, n_{35}, n_{36}, n_{37}, n_{39}, n_{40}, n_{41}\}$. Hence $\varepsilon^n(\delta^e(X^n)) \cap \varepsilon^n(\delta^e(Y^n)) = \{n_3, n_4, n_5, n_8, n_9, n_{10}, n_{13}, n_{14}, n_{15}, n_{18}, n_{19}, n_{20}, n_{21}, n_{22}, n_{23}, n_{24}, n_{25}, n_{27}, n_{28}, n_{29}, n_{31}, n_{32}, n_{33}, n_{35}, n_{36}, n_{37}, n_{39}, n_{40}, n_{41}\}$. Thus L.H.S ⊂ R.H.S. The results are shown in Fig 5.

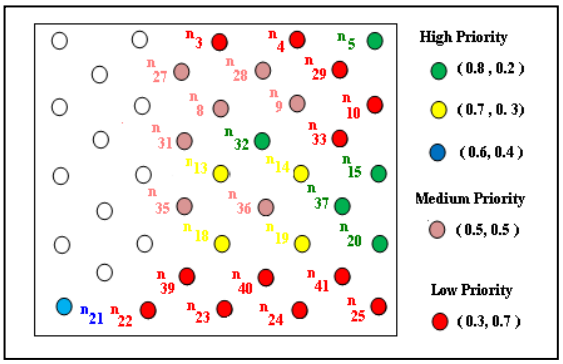

Fig 5: Results of Proposition H

V. APPLICATION OF FILTER IN MULTI DOCUMENT SUMMARY

Multi document summarization is an important area in the field of Natural Language processing. There are many summarization methods developed so far. In this section let us see how multiple documents can be represented using an IFHG assigning membership and non membership values to the nodes and hyperedges.

A. Modeling Multiple documents using IFHG

Let $[H_{IF}, (\mu_n, \gamma_n), (\mu_e, \gamma_e), H^n, H^e]$ be an Intuitionistic Fuzzy hypergraph as mentioned in Section III. Let H^e be the

hyperedges, where a hyperedge e_i represents a document D_i. Let H^n be the nodes where a node v_j represents a keyword w_j in the document. The same is shown in Fig 6 (a). Therefore the number of hyperedges in the H_{IF} will be same as the number of documents considered for summarization and the number of nodes in a hyperedge will be same as the number of keywords in that document .The membership value for a node v_j depends on the normalized TF - IDF value and the priority of that word. Irrespective of the domain there are words which are having high priority. Eg:- Words like important, famous , beautiful, attractive, relevant etc are given high priority. So these words will have $\mu_n > 0.5$. For such words $\gamma_n <= 1 - \mu_n$. There are words which are having very low priority. Some sample words include notorious, expensive, least, badly etc. These words are having $\gamma_n > 0.5$. So for such words $\mu_n <= 1 - \gamma_n$. Rest of the words are given $\mu_n = \gamma_n = 0.5$ as they are medium priority words. Once all these nodes (words) are assigned with both membership degree (μ_n) and non membership degree (γ_n), the hyperedges are also assigned membership degree (μ_e) and non membership degree (γ_e) as per the rules in Section I.

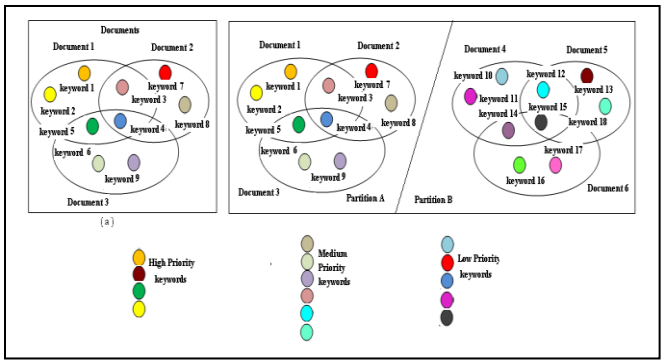

Fig 6 : (a) IFHG of Documents (b) Partitions

B. System Architecture

For the formation of IFHG, all the documents are subjected to preprocessing steps like stop word removal and stemming. Keywords in a document are found out by considering the *tf - idf* of the words. Only words with *tf - idf* above a threshold θ are considered for the construction of the IFHG. Once an IFHG is formed as mentioned in Section *V.A*, it can be partitioned in to sub IFHGs based on the absence of overlapping nodes between the hyperedges as shown in Fig 6 (b). Such partitions are now document clusters. For each cluster, sub IFHG X_i is created by applying an (α, β) cut. This X_i is subjected to opening and closing filters as mentioned in Section IV. The results of filter w.r.to hyperedges will yield good documents and results of filter w.r.to nodes will yield priority keywords. These priority words combined with

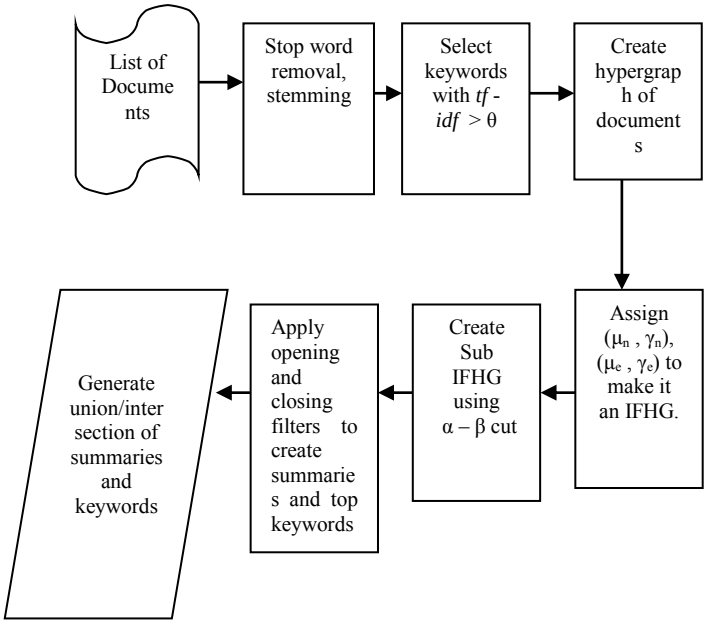

Fig 7. System Architecture

priority documents yield good summaries. If needed, the number of parameters considered for assigning membership degree can be increased .The architecture of the system is shown in Fig 7. The algorithm is shown in Fig.8.

Algorithm 1 Multidocument summary using IFHG

Input : n documents, θ, α_n
Output : Summary
for each document D_i; $i = 1$ to n do
  Remove stop words
  Perform Stemming of words
  Find $tf - idf$
  Find all words w_j in D_i with $tf - idf > \theta$
end for
P = number of hypernodes = Total Number of words in all the D_i
J = number of hyperedges = n;
for each $k = 1$ to P do
  Assign μ_{n_k}, γ_{n_k} of all nodes as per rules in section V.A
end for
for each $m = 1$ to J do
  Assign μ_{e_k}, γ_{e_k} for all edges as per rules in section V.A
end for
t Clusters = Find sub IFHGs with non overlapping hypernodes
for each cluster C_i, $i = 1$ to t do
  Find sub IFHG X_i with (α, β) cut where $\alpha >= \alpha_n$
  Apply opening filter w.r.to nodes $\delta^n(\varepsilon^e(X_i)^n)$ to find keywords in summary.
  Apply opening filter w.r.to hyperedges $\delta^e(\varepsilon^n(X_i)^e)$ to find relevant documents.
  Find $Opening_Summary_Filter(C_i) = Combine(\delta^e(\varepsilon^n(X_i)^e), \delta^n(\varepsilon^e(X_i)^n))$
  Apply closing filter w.r.to nodes $\varepsilon^n(\delta^e(X_i)^n)$ to find keywords in summary.
  Apply closing filter w.r.to edges $\varepsilon^e(\delta^n(X_i)^e)$ to find relevant documents.
  Find $Closing_Summary_Filter(C_i) = Combine(\varepsilon^e(\delta^n(X_i)^e), \varepsilon^n(\delta^e(X_i)^n)$.
end for
for two clusters C_i, C_j do
  Find $Opening_Summary_Filter(C_i) \cup Opening_Summary_Filter(C_j)$ to create union of two summaries
  Find $\delta^n(\varepsilon^e(X_i)^n) \cup \delta^n(\varepsilon^e(X_j)^n)$ to create union of keywords in summaries
  Find $Closing_Summary_Filter(C_i) \cup Closing_Summary_Filter(C_j)$ to create union of two summaries
  Find $\varepsilon^n(\delta^e(X_i)^n) \cup \varepsilon^n(\delta^e(X_j)^n)$ to find all the keywords of two summaries
end for

Fig . 8 Algorithm for Multi document Summary Filter

In the above algorithm we can see that summary can be generated either by combining an opening filter with respect to hyperedges along with opening filter with respect to nodes or

by using a closing filter with respect to the hyperedges along with closing filter with respect to the nodes. Similarly keywords can be found by using an opening filter or by using a closing filter.

VI. RESULT ANALYSIS

The proposed system with multiple documents modeled as IFHG was tested with different number of input documents. The system developed in python, and the hypergraph created using python pygraph works for both Malayalam documents and English documents. The difference between two systems lies in the stemming phase, where in Malayalam, a stemmer developed using Tree based method [20] is used. Porter stemmer is being used for English documents. Rests of the modules are common for Malayalam and English summarization system. The results obtained with various test cases are given in Table I. The results of the IFHG system is compared with results of 50 human summarizers and Rouge-L, Rouge-2, Rouge-1 scores are calculated and tabulated in Table II.

TABLE I. RESULTS OF FILTER

Sl No	Results of Multi document Summarization					
	No of Docs	*Total No of Lines*	*Total No of Words*	*No of nodes created*	*No of Summary Lines*	*Percentage of Summary*
1	3	43	1975	14	16	37
2	3	125	4453	18	32	26
3	4	135	5749	20	34	25
4	4	257	9497	27	76	30
5	5	393	13,372	48	110	27
6	6	444	25,852	50	125	28

Advantages of IFHG based Summary system:

- Even though the number of words in the document may be large, the IFHG created will be having only less number of nodes as we are considering only keywords for constructing the nodes. This reduces the space required for storing the data structure.
- Priority levels are not limited to high, medium and low. Different levels of priorities like very high, high, medium high, low, very low can be set with the help of the membership and non membership degrees of the words as well as the documents.
- Varying the threshold of (α, β) cut results in different summaries with different priority levels. Increasing the threshold of (α, β) cut results in a very short summary, where by reducing it results in a lengthy summary.
- Range oriented (α, β) cuts like $value_1 < \alpha < value_2$ are also possible which results in different sets of summaries.
- Since filters w.r.to nodes result in keyword list of documents, union/intersection of filters result in union/intersection of keywords. Since filters w.r.to hyperedges result in selection of good documents, they combined with node filters result in summary. Union /intersection of summaries are also possible.

TABLE II. ROUGE SCORES

Rouge- Scores of Multi Document Summarization										
No of docs	*size*	*Rouge-L*			*Rouge-2*			*Rouge-1*		
		P	*R*	*F*	*P*	*R*	*F*	*P*	*R*	*F*
3	2k	0.87	0.93	0.90	0.88	0.93	0.91	0.90	0.94	0.91
3	4.5k	0.85	0.97	0.90	0.84	0.96	0.89	0.87	0.96	0.91
4	5.7k	0.91	0.97	0.93	0.90	0.96	0.93	0.91	0.96	0.94
4	9.5k	0.85	0.97	0.89	0.85	0.98	0.91	0.88	0.99	0.93
5	13k	0.88	0.99	0.93	0.88	0.99	0.93	0.89	0.99	0.94
6	25k	0.87	0.99	0.91	0.88	0.96	0.92	0.91	0.96	0.93

VII. CONCLUSION

The proposed work is a novel method which models multiple documents using Intuitionistic Fuzzy hypergraphs. The system is tested with documents of varying size and has shown better results when compared to human summaries. Documents in various domains are considered for summarization. The priority levels, (α, β) cuts, range oriented (α, β) cuts and union / intersection operations applied on the filters give different types of summaries suitable for different applications. Here IFHG takes only less space since care is taken to reduce the number of nodes. We have also developed a system which models sentences as hyperedges and words as nodes. Such IFHG modeling and filtering can be done in other areas like mobile networking, social networking, image processing etc. Modeling medical reports as IFHG and creating medical report summary is a future enhancement of this work.

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