

Modelling The Voronoi Diagram Into The Graph Neural Network of Geo-Topologic Building Layout (Part I)

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Abstract

The GNN machine learning supposes the architectural space design of human form and function layers combined. The introductory GNN model adapts the geo-topologic layers of VD/DG property at embedded resolutions of spatial layout. The model objective overlays the spatial distributiveness of relative asymmetry and centrality with the universal distance of space integration. The demonstration on the original layout of the Ashanti Shrine at Ghana explores the functional topology graph with its geometry of nodal distribution. The room adjacency determines the ordinal ranks per spatial measurement at each graph layer with their resulted summation. The clustered results of the adjacency matrix represent the abstract pixel plan realization with the updated GNN connectivity. The essence of the Shrine Room symbolized dual structure of deep integral space in hybrid form and function. The spatial model may extend to complex network computing in universal prospects.

Keywords: Voronoi; graph; topology; geodesic distance; space integration; Ashanti Shrine

1. Introduction

The interdisciplinary Graph Neural Network (GNN) fosters the artificial intelligence of machine learning. The input data of nodes, edges and global layout constitute the GNN layers of parametric output [1]. Nevertheless, the GNN application on the discipline of building design confronts the 'form follows function' criterion of spatial layers [2,3,4]. The dependent form geometry on the independent function topology set the GNN layers in sequence of varied networking, which hinders their merge [5,6,7,8]. The intangible connectedness of functional space comprises multilayers such as visibility, circulation,

environmental and cultural topologies [9]. The following tangible connectivity of tensor form includes the spatial layers of shape, material and coordinate geometry [10,11,12]. However, the form and function share the same location of space in geo-topologic dimensions [13]. The locational criterion of space design addresses the intangible Voronoi Diagram (VD) of extensive informatics, which may extend to the detailed level of building rooms [14,15,16,17]. The VD's objective application on the building scale approaches the GNN model of combining form and function layers at their common geodesic location [18].

The convex grid of VD for building spaces verifies the dual Delaunay graph (DG) in GNN modelling [19]. In this model, the DG geometry identifies the relative location of each space to all others with their function [20,21]. The resultant adjacency matrix enables the geo-topologic measurements of integral dimensions such as relative asymmetry (RA), betweenness centrality (BC), closeness centrality (CC) and universal distance (UD) [22]. The same graph explores circulative criteria of Eulerian and Hamiltonian algorithms [23]. Meanwhile, the DG embeds further graphs of Pitteway (PG), Relative Neighbor (RNG), Euclidean-Minimum-Spanning-Tree (EMST) and the directed Nearest Neighbor (NNG) where the same matrix format resets the spatial adjacency [24]. In this regard, the GNN model aggregates the VD layers of spatial formation at DG resolutions. The geospatial results identify the influential rooms of integrated layout structure [25]. The limitations of the applied conceptual method discuss the future issues of the GNN in architectural space design.

2. Methods

The graph representation of space adapts different connectivity of functional and geometrical objectives. The method of spatial tessellation defines the convex hull and the graph elements of different layouts [26,27,28]. On one, the profound method of space syntax tessellates the functional space into the largest-possible convex cells for axial visibility of social integration [29]. On the other, the shape grammar adjoins the polyhedral or other convex unit to form the geometrical layout [30,31,32,33,34]. The unmatched graph layout of form and function dissatisfies the GNN process. In this regard, one spatial approach determines the dual VD/DG of geo-topologic interface [35].

The parametric VD/DG satisfies the GNN process to run parallel layers along the constant nodal distribution [36]. In corollary to the geometrical DG nodes, the VD traces the geodesic convex hull of intangible boundaries out of the tangible solid. As a result, the VD/DG geometry shifts from the shaped space to the geodesic distancing of the generative points amid the convex boundaries [37]. The transformed 'form follows function' into 'form corresponds function' emphasizes the locational criteria of space with the data attributes [38,39,40].

The statistical depth measurement of spatial distributiveness represents the RA of adjacency flows at the topologic layout [29]. The different centrality measure of geodesic connectivity determines the BC shortest path and the CC relative depth at the geodesic locational layout [41,42,43]. Meanwhile, the DG unfolds into subsequent graph embeds where each of which represents a standalone layer of tuned connectivity [44,45]. Overview of the spatial networks overlay varies connectivity layers of different

objectives at the same DG nodes. This is new to the GNN of interactive connectedness at multiple networks of the same nodal layout. Not only the GNN overwrites the output of connective weighs, but sieves the connectivity per layer as well.

In another front, the graph algorithms resolve the network trajectories of Euler and Hamilton cycles where the DG applies [46,47,48]. In addition, the VD/DG holds geometrical moments such as the equidistant tri-point connectedness and the circumcircle triangulation of the generative point process [35]. In output, the GNN model enumerates the edge overwrite of clustered encounters. The more encounter the higher the rank of the edge for the global structure of the geo-topologic layout. The pooling strategy transfers from the edge encounter to the DG nodes at all layers. The node receiving more edge encounter denotes the higher geo-topologic rank in comparison to others.

Nevertheless, the model supposes the equal weight of edge connectivity at any spatial layer and among layers without bias. The following chart summarizes the geo-topologic data structure of VD/DG spatial layers with the GNN processing towards the aggregative output (Figure 1). The clarity of the proposed GNN model explores one of the demonstrated building layouts in the space syntax originality (Figure 2). The topology graph extracts the functional space structure of the Ashanti shrine at Ghana, whereas the building plan updates the geometrical logic of the VD/DG layers in the GNN aggregate.

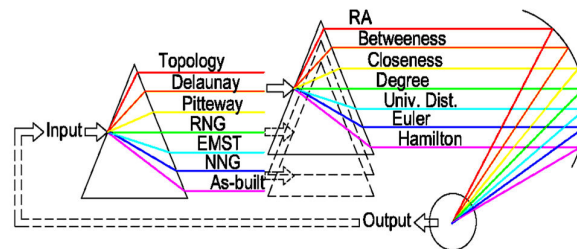


Fig 1: The proposed GNN process of spatial layers

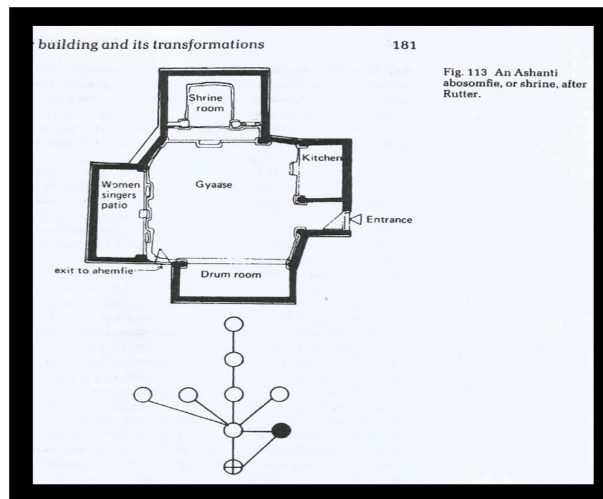


Fig 2: The topology graph of space syntax [29, p.181]

3. Results

The room shape of the simple Ashanti shrine at Ghana filters the functional permeability apart from the visible convexity of space organization. In particular, the u-shaped room privacy of deep asymmetrical permeability behind the shrine room subdivides into three consecutive convex spaces. The infinite space of the functional outdoor considers the topological graph network but the geodesic correspondence

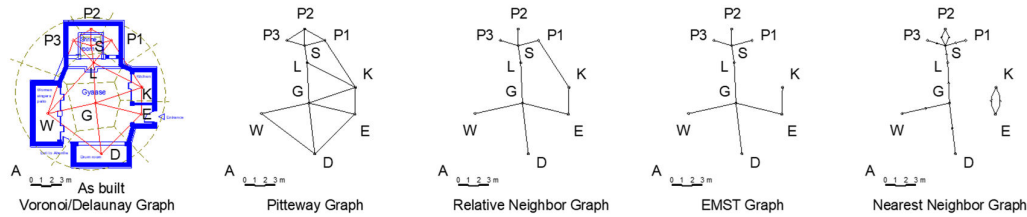


Fig 3: The GNN layers

Table 1. The spatial measurements

Graph Layers	CENTRALITY of VD/DG Layout																UNIVERSAL DISTANCE				SPATIAL INTEGRATION												
	GRAPH BETWEENNESS								GRAPH CLOSENESS								GRAPH DEGREE				GRAPH GEODESY												
	DELAUNAY		PITTEWAY		RNG		EMST		DELAUNAY		PITTEWAY		RNG		EMST		DELAUNAY		PITTEWAY		RNG		EMST		DELAUNAY		PITTEWAY		RNG		EMST		
Rm.	Geodesy	Topology	Geodesy	Topology	Geodesy	Topology	Geodesy	Topology	Closeness	standardize	Closeness	standardize	Closeness	standardize	Closeness	standardize	Degree	DELAUNAY	PITTEWAY	RNG	EMST	DELAUNAY	PITTEWAY	RNG	EMST	DELAUNAY	PITTEWAY	RNG	EMST	DELAUNAY	PITTEWAY	RNG	EMST
E	0.034722	0.020833	0.055556	0.055556	0.138889	0.138889	0.222222	0.222222	0.578947	0.526316	0.55	0.5	0.454545	0.458333	0.416667	0.3	0.3	0.2	0.2	68.87	68.42	77.45	82.01	0.45	0.257143	0.285714	0.342857	0.4	0.4	0.45	0.257143		
G	0.156667	0.074074	0.37963	0.337963	0.472222	0.444444	0.694444	0.694444	0.733333	0.666667	0.6875	0.625	0.611111	0.555556	0.611111	0.555556	0.5	0.5	0.4	0.4	49.4	50.9	53.32	53.32	0.15	0.142857	0.171429	0.228571	0.228571	0.15	0.142857		
A	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.478261	0.434783	0.44	0.4	0.458333	0.416667	0.44	0.4	0.2	0.2	0.2	0.2	—	—	—	—	0.45	0.371429	0.4	0.4	0.4	0.4	0.428571		
D	0.013889	0.013889	0.013889	0.013889	0	0	0	0	0.55	0.5	0.478261	0.434783	0.407407	0.37037	0.407407	0.37037	0.3	0.3	0.1	0.1	78.27	79.77	92.92	92.92	0.5	0.285714	0.371429	0.485714	0.485714	0.5	0.285714		
W	0.060185	0.060185	0	0	0	0	0	0	0.6875	0.625	0.458333	0.416667	0.407407	0.37037	0.407407	0.37037	0.4	0.2	0.1	0.1	70.09	84.11	92.33	91.48	0.65	0.171429	0.4	0.485714	0.485714	0.65	0.171429		
L	0.360648	0.234259	0.444444	0.37963	0.541667	0.472222	0.555556	0.555556	0.785714	0.714286	0.647059	0.588235	0.578947	0.526316	0.578947	0.526316	0.6	0.3	0.2	0.2	43.33	51.82	51.33	54.17	0.3	0.114286	0.2	0.257143	0.257143	0.3	0.114286		
K	0.148148	0.097222	0.180815	0.150463	0.152778	0.111111	0	0	0.6875	0.625	0.647059	0.588235	0.478261	0.434783	0.333333	0.30303	0.4	0.4	0.2	0.2	58.83	59.38	74.61	102	0.5	0.171429	0.2	0.340206	0.657143	0.5	0.171429		
S	0.111111	0.037963	0.444444	0.3125	0.555556	0.5	0.5	0.5	0.578947	0.526316	0.55	0.5	0.5	0.5	0.454545	0.4	0.4	0.4	0.4	48.15	50.16	51.22	56.64	0.55	0.257143	0.285714	0.342857	0.342857	0.55	0.257143			
P1	0.000259	0.000259	0.148148	0.148148	0.111111	0.097222	0	0	0.611111	0.555556	0.55	0.5	0.458333	0.416667	0.354839	0.322581	0.4	0.3	0.2	0.1	53.38	59.74	57.4	73.12	0.228571	0.285714	0.428571	0.6	0.6	0.228571			
P2	0.041667	0.041667	0.013889	0.013889	0	0	0	0	0.478261	0.434783	0.44	0.4	0.37931	0.344828	0.354839	0.322581	0.3	0.3	0.1	0.1	56.14	60.18	61.96	69.44	0.9	0.342857	0.428571	0.542857	0.5	0.5	0.342857		
P3	0.078704	0.078704	0	0	0	0	0	0	0.611111	0.555556	0.302857	0.37143	0.37931	0.344828	0.354839	0.322581	0.4	0.2	0.1	0.1	57.06	65.02	65.78	71.2	0.4	0.228571	0.514286	0.542857	0.5	0.5	0.514286		
AVER.	0.1025	0.066806	0.168981	0.137037	0.197222	0.176389	0.197222	0.197222	0.616426	0.560387	0.531006	0.482733	0.473493	0.430448	0.43646	0.396781	0.38	0.31	0.2	0.2	58.35	63.05	67.83	74.63	0.48	0.233766	0.322078	0.395382	0.462338	0.48	0.233766		
SUM	1.025	0.668056	1.689815	1.37037	1.972222	1.763889	1.972222	1.972222	6.780686	6.16426	5.841069	5.310063	5.208422	4.734929	4.801056	4.364596	4.2	3.4	2.2	2	583.5	630.5	678.3	746.3	4.3	2.571429	3.542857	4.340206	5.085714	4.3	2.571429		

dissolves to infinity. The articulated space defines the geo-topologic layers with their spatial measurements (Figure 3 & Table 1). The classical space syntax represents the topology graph of permeable connectivity per room function. The resulted graph determines the RA of spatial integration per room depth and the global average.

The highest graph integrity of the central courtyard (G) extends a spatial ring with the entrance (E) and the exit to the palace “Ahenfie” (A). The radial room nodes of the kitchen (K) drumming (D) and singing (W) share the same layout. Nevertheless, the landing terrace (L) at the front of the shrine room (S) forms a graph-tail of deeper private rooms (P1), (P2) and (P3) in sequence. The most disintegrated RA of P3 is screened from the public vision as well. The global network satisfies the Eulerian path {EA, AG, GL, LS, SP} of Hamiltonian nodes as well. The sacred rooms emphasize the axial spatial experience versus the radial public domain. The VD property transforms the hard room boundary to the soft geometry of geodesic distance space. The geometrical centroids correspond to the dual DG nodes where the geo-topologic layers proceed in GNN blocks. The integral analysis extends the graph topology to the geometrical layers of DG, RNG, EMST, NNG resolutions of BC, CC and UD connectivity measurements in GNN aggregate.

3.1. The DG Resolution

The DG network traces the node connectivity of shared VD edge. The space syntax developed a comparable approach of room subdivision into convex cells as per the shaped spaces [49, p.41 & p.199]. However, the space connectivity kept topologic for the social objective. On the contrary, the VD/DG approach sets the spatial connectivity upon the geometrical convex hull and not only topology. This difference of network conception extends the permeable criterion of spatial integration from the visible topology to the territorial geometry. The resulted RA of DG network observes the L-space on top layout integrity between the public and the shrine spaces. The external A-node holds the most disintegrated space. Overview of the geometrical DG optimizes the global RA in comparison to the topological distributiveness.

The BC measures the shorter path of the topologic depth or the geodesic distance of room pairs. The shrine room has the topological fourth rank, but the seventh geodesic distance. The wide difference indicates the integral topology while being distant. Nevertheless, the L-space sustains the top rank at both topologic and geodesic measurements in contrast to the bottommost P3 privacy. The logic of the former integrates the public domain with the shrine zone, while the latter represents the shrine's deepest exclusiveness. The CC determines the efficiency of nodal depth at the global layout topology. The most efficient L, G and K nodes exclude the shrine domain at the bottom rank. The average rank composes the exposed shrine room with the drumming and singing effects at one interactive background.

Nevertheless, the nodal degree of connectivity converts the constraint P3 of end node into linkage opportunity on the layout level of social integrity. The universal distance of the geodesic network determines the geometrical location of each node in relationship to all others of the layout [50,51]. The result observes the L and S nodes in shortest reach of the layout integrity. The symbolic structure of the shrine room becomes the shallowest to all other domains. Meanwhile, the Hamiltonian algorithm satisfies the layout cycle of all nodes up to the start point. The dissatisfied Eulerian cycle or path without edge overwrite emphasizes the spatial encounter in sequence of nodes along optional connectedness.

The deep structure of room adjacency explores the scatter-plot matrix of spatial measurements. The resulted plot of the DG resolution (Figure 4) indicates the strongest correlations at the deep zone of shrine rooms (bottom-right corner of the plot). Therefore, the most disintegrated rooms prove integral within a specific functional block. The correlative matrix extends to the L-space, but proves scattered elsewhere. This proves the pivotal role of the shrine's terrace to integrate the public with the shrine domains. Meanwhile, the plot correlates the courtyard (G) to all rooms more than the way back. Thus, the asymmetrical adjacencies filters the flow of spatial integration at the global and the local contexts of the layout. Although the D, W and K spaces occupy similar pavilions of the layout, the plot shows a different structure for each. In particular, the D- space proves correlative of more layout integrity, whereas the K-correlations are scattered all over except with the E-node. However, the plot of the A-node has no specific correlation with the layout in accordance with its exterior location.

3.2. The PG Resolution

The PG fine-tunes the DG network into the connectivity of only direct crossing over the shared VD edges. The RA shifts to the G space on top rank, whereas the P3 node secludes at bottom. The open courtyard structures the layout integrity in connection to all social domains, except the screened-out shrine space. Nevertheless, the geodesic BC sets the S node at the highest rank, with distinct decrease from P1 to P3 nodes. The topological layout resets the G and L nodes back on top rank, with the P3 kept at

minimal. Meanwhile, the CC network preserves the same ranking of nodes. Besides, the degree of node connectivity conforms to the CC result. However, the UD locates the transitional L-node in shortest layout with Hamiltonian cycles. The Euler property satisfies portions of the network, such as the shrine domain in one path. Therefore, the PG overlays several nodes of layout integration in geo-topologic structure.

The scatter-plot of the PG resolution (Figure 5) clarifies the layout correlations per room adjacency. The E- and K-nodes improve the layout correlation in contrast to the predecessor DG resolute. In addition, the G-space becomes correlative along the asymmetrical two-way adjacency. The ring spaces, which spans the various domains of the layout, reflects the homogenous correlations of the matrix plot with high integrity. Moreover, the shrine zone sharpens the room correlations among each other, regardless of being hold disintegrated at the global layout structure. Meanwhile, the L-node decreases the correlations in comparison to the now more integrated G courtyard. Therefore, the less degree of node connectivity from the DG to the PG resolutions contributed to more integral layout correlations of direct refined adjacencies.

3.3. The RNG Resolution

The RNG selects the PG adjacency of exclusive spindles at each pair of connected nodes. The resulted RNG emphasizes the star-graph structure with a single cycle space. The RA top integrity shares the public G with the L and S nodes of various social domains. Meanwhile, the integral open shrine contrasts with the three disintegrated P nodes at the back. In addition, the central G integration diminishes at W and D terminals of only sound effect. This integral logic differentiates the functions per social domain and not against the domains. The BC qualifies the S node on top layout integrity against the W and D at bottom. Nevertheless, the CC changes the integrity of the G node on top and the P nodes at bottom. In this regard, the centrality swaps the layout integrity from one social domain to the other at different network layers. The UD allocates the transiting L node at the shortest layout of integrating the social domains. In addition, the long cycle component increases the social encounter with potential Euler and Hamilton networks.

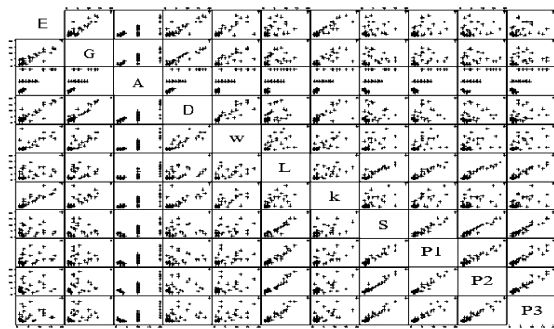


Fig 4: The scatter-plot of the Delaunay adjacencies

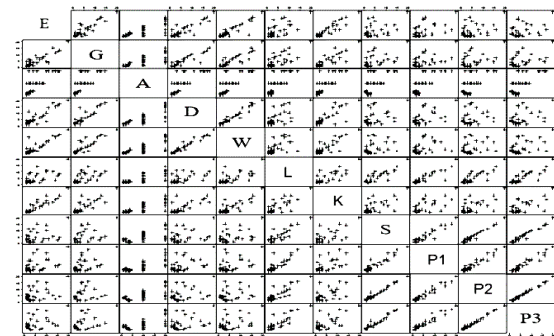


Fig 5: The scatter-plot of the Pitteway adjacencies

The RNG scatter-plot reveals more structure of the layout adjacencies (Figure 6). The W- and D-nodes correlate each other apart from their disintegrational layout structure. Nevertheless, the G-node and the S-node share the correlation of layout integrity, the former with the public rooms and the latter with the private domain. This corresponds to the swap between the two nodes at the topologic and geodesic top layout integration respectively. Meanwhile, the E- and K-nodes exhibit more correlation of layout integrity against the previous plot resolutions. In this regard, the located two nodes at the single cycle space of the RNG resolution restructures the adjacency measurements of the global layout. The common theme of plot correlation defines two separate zones of the public domain (top-left) and the private shrine (bottom-right) simultaneously. Further finding observes the K-node of mutual correlation along both asymmetry directions of the two-zoned social domains. Therefore, not only the adjacency structure correlates the layout integrity according to the highest spatial measurements, but through geo-topologic shortcuts as well.

3.4. The EMST Resolution

The EMST sub-graph identifies the shortest distance of spanning the layout nodes. The same G, L and S nodes maintain the top RA integration. Meanwhile, the choir nodes gain integral momentum of average RA values. This result conforms to the topology graph of space syntax, except the geodesic integrity of the S node. Nevertheless, the BC network specifies the G node at the highest integration. This node of cut-vertex minimizes the layout to the shrine rooms of graph component. Also cutting the S nodes limits the layout to the ordinary public component. Thus, the two cut-vertices represent the graph-minor of the whole layout structure. The UD network continues the layout integration of the G-node. However, the K node of longest tail-graph indicates the least layout integration in both topologic and geodesic measurements. The longest paths of satisfying the Euler or Hamilton properties pass through the G, L and S nodes to integrate the layout's various social domains.

The scatter-plot of the EMST resolution redefines the layout structure of correlated adjacencies (Figure 7). Despite the minimal connectedness of spanning the layout, the matrix plot indicates stable correlations throughout the nodal structure. The pivotal L-

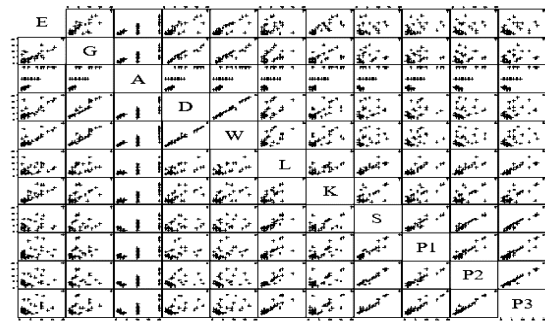


Fig 6: The scatter-plot of the RNG adjacencies



Fig 7: The scatter-plot of the EMST adjacencies

node correlates with the other sub-pivotal G- and S-nodes. The latter two correlates with the zoned-up social domains respectively. Thus, the domino-effect of correlative adjacencies maintain the global layout integrity in compensation of the least integral tree-graph measurements. The plot is scattered elsewhere, except for the distinctive mutual correlation between the E- and K-nodes as never before this plot resolution. In this regard, the geo-topologic adjacency measures with their correlations at each layered resolution are complementary to each another in one layout structure.

3.5. The NNG Digraph Resolution

The unique resolution of the NNG network forms the directed graph from each vertex to the nearest found. The resulted digraph disconnects the K and E nodes from the rest. All of the directed connectivity

run towards the S node of symbolic structure. The spatial experience extends from the public domain to the shrine room in axial orientation. Nevertheless, the as-built network

measures the UD from every room to all others with their ordinal ranks. The asymmetrical connectivity considers both the one- and two-way circulation among the rooms. The S node has the relative longest destination to the global plan, but nearer on the way back. The comparable D node keeps afar at the two-way directions. Meanwhile, the shrine's P nodes are the farthest to reach-out or to be-reached at the global plan. Therefore, the shrine zone of double-fold structure justifies the visible and closer location, while keeping distance at fading depths. Nevertheless, the G node is set nearest to go-and-return or pass-through the whole layout. The functional overlap of the open courtyard represents the social union of the layout in universal dimensions.

The digraph scatter-plot reveals the wide difference of asymmetrical correlative adjacencies at each paired vertices (Figure 8). Among all nodes, the G- and L-nodes represent a dense correlation towards other nodes of the layout. However, the back-way subdivides the correlation into the public domain towards the G-node and the private shrine domain towards the L-node. Nevertheless, the private shrine rooms maintain homogenous correlation of some extent towards the public G, D, W and K nodes in common. Meanwhile, the public rooms correlate within the same domain, but scatter towards the shrine privacy. Therefore, the integral logic of digraph adjacency allows the private domain to correlate with the public, but the public diversity is restricted from the correlation in return.

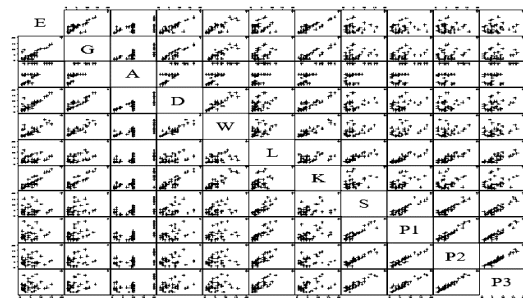


Fig 8: The scatter-plot of the digraph adjacencies

4. The GNN Model

The asymmetry of the total space connectivity differentiates the one-way (R1) from the two-way (R2) destination. The top integral nodes of G, L and S sustain the overall high rank along both directions. Nevertheless, the average integrity observes some variance, especially the P2 node of disintegrated R1 but integrated R2. This emphasizes the symbolic structure of the shrine space being distant but within reach. The other D, W and K nodes of utility functions exchange their average integrity at both directions without altering the layout structure. However, the least integral A node keeps the palace away from all nodes in any direction. This may explain the separate exit door from the shrine layout to form a different spatial structure. The pixel representation of the GNN model translates the output matrix to the grid of node adjacencies (Figure 9). The process adjoins the high rank of paired matrix nodes along the grid of pixel functions. In this process, any value (<45) which is below the overall average supposes the high bond of adjacent pixels. The E with the K nodes score the highest adjacency bond. The E node extends another pixel bond with the G node. Nevertheless, the G node attracts more bonds with the W, D and L nodes as well. This shapes the G node as a central pixel surrounded by the bonding nodes. In addition, the L node bonds with the

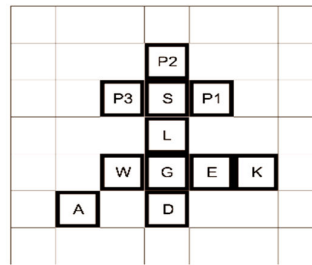
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Fig 10: The GNN clusters

S node, which in turn forms other starring bonds with the P nodes. The resulted pixel bonds sketch the abstract morphology of archetype model. In this regard, the GNN matrix and pixel grid laydown the building plan to be sized and shaped as suit.

Further GNN representation updates the graph itself of clustered connectivity (Figure 10). The same adjacency matrix subdivides the nodal ranks into intervals of similar connectivity. The interval of the highest integral value (<45) defines the cluster of edge connectively of the same line type. Similarly, the moderate interval ($>45<85$) defines the second cluster of edges with a different line type. The interval (>85) indicates the least integral value of the graph connectivity. The resulted clusters of graph edges represent the GNN model along the DG mother network with all the embedded resolutions combined. The resulted GNN influences the pooling strategy from the typified links to the functional nodes. The integral cluster emphasizes the G and S nodes of radial connectivity, which overlap at the L node. The disintegrated cluster excludes the P nodes from the public domain.

5. Discussion

The spatial constraint of different form from function layers rethinks the GNN model of regular connectivity. The introduction of the VD/DG property shifts the tangible form and the intangible function to the geodesic opportunity of common node position in geo-topologic space. The argument combines the form and function networks along the positioned spatial nodes of different connectivity though. The application to the GNN computing satisfies the regular node layout of interactive connectivity. This approach adds a new weighing dimension to the GNN model of new pooling strategy. Not only the edge weighing depends on the fed informatics but the frequency of edge overwrite as well. Meanwhile, the VD/DG embeds subsequent resolutions of the same layout. Thus, the GNN combines the space informatics of the geo-topologic layers and the embedded resolutions of correlated spatial measurements as well. The working application of the method on the Ashanti Shrine summed up the integral adjacency of rooms in matrix, pixel and graph output. The value of the output represents the archetype of space organization for the machine learning to retrieve. However, the limited scope of graph size and layers may explore higher complexity without loss of model generality.

6. Conclusions

The interdisciplinary GNN model of architectural space design sheds a new light on the combined form and function possibility. The resolve of the different geometry from topology layout concludes the abstract pixel format of space adjacency. The position of each pixel determines the correlative degree of adjacency in relationship to all others. The proposed method defines three main steps of model formulation, which are; 1) defining the layers of geo-topologic dimensions; 2) setting the adjacency matrix of spatial measurements in ordinal ranks and 3) summing-up the ranks in matrix, pixel and graph adjacencies. The process involves the GNN structure, which adopts the ontological DG of space. The graph's geodesic location concludes new dimensions of machine learning, which allows the interchange of connectivity along the sub-graph resolutions of the stationed nodes. The rank of each measurement of space determines

the additive unit of cluster process in the GNN output. The pilot study on the Ghanaian Ashanti Shrine concludes deep learning of spiritual model between the public and the shrine integrity beyond the non-distributive space topology. The introduced application may extend to train a message-passing model of genuine GNN on a dataset of many plans or a defined task of layout retrieval. In another front, the concluded GNN of space adjacency addresses the software-aided design. The spatial breakdown into the correlative geodesic distance and topology layers targets the architectural toolbox in AI's packaging perspective.

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The research has no competing interests.

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