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Generalised (α, ψ, φ) −Contraction Mappings in b-Metric Spaces with Application to Volterra Integral Equations

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Abstract

In this paper, we introduce the concept of generalised (α, ψ, φ) −contraction mappings in the framework of b-metric spaces and establish new fixed point theorems that unify and extend several classical and recent results in the literature. Our contractive condition incorporates a pair of auxiliary functions ψ (a comparison function) and φ (an altering distance function), together with an admissibility coefficient α, providing a rich and flexible framework for fixed point analysis. We prove the existence and uniqueness of fixed points under these generalised conditions in complete b-metric spaces, relaxing the classical requirements of both the standard metric triangle inequality and the Banach contraction principle. As a significant application, we demonstrate that the existence and uniqueness of solutions to nonlinear Volterra integral equations of the second kind can be deduced as direct corollaries of our main theorems. Illustrative examples are provided to validate the theoretical results and demonstrate their applicability in integral equation theory. Our results subsume a number of well-known fixed point theorems including those of Banach, Kannan, Chatterjea, Reich and Ćirić as special cases.

Keywords: b-metric space; fixed point theorem; $(\alpha, \psi, \varphi) -$contraction; Volterra integral equation; comparison function; altering distance function; admissible mapping
MSC 2020: 47H10; 54H25; 45D05; 54E40

1 Introduction

The Banach contraction principle[1], established in 1922, stands as one of the most celebrated and widely applied results in functional analysis and metric fixed point theory. Its elegance lies in its simplicity: any self-mapping T on a complete metric space (X, d) satisfying $d(Tx, Ty) \leq k\, d(x, y)$ for all $x, y \in X$, where $k \in [0,1)$, possesses a unique fixed point obtainable as the limit of iterative sequences. This single result has spawned an enormous body of generalisation and application.

Among the most impactful generalisations of metric spaces is the notion of a b-metric space, introduced by Bakhtin [2] and formally developed by Czerwik [3, 4]. A b-metric relaxes the triangle inequality by a multiplicative constant$s \geq 1$, yielding a significantly broader class of spaces that includes ordinary metric spaces ($s = 1$) as a special case. The study of fixed point theory in b-metric spaces has attracted considerable attention; see, for example, Boriceanu [5], Kir and Kiziltunc [6], Roshan et al. [7], and the references therein.

Simultaneously, the concept of α-admissible mappings introduced by Samet et al. [8] opened a new direction in fixed point theory by allowing the contraction condition to hold only for pairs of points satisfying a prescribed admissibility criterion governed by a function $\alpha: X \times X \to [0, \infty)$. This idea was further developed in b-metric spaces by Hussain et al. [9] and Sintunavarat [10], among others. The combination of α-admissibility with auxiliary comparison functions ψ and altering distance functions φ has proven particularly powerful in establishing sharp fixed point results with minimal assumptions.

Motivated by these developments, the present paper introduces a new class of $(\alpha, \psi, \varphi) -$ contraction mappings in b-metric spaces and establishes fixed point theorems that generalise and unify the results of Samet et al. [8], Hussain et al. [9], Alber and Guerre-Delabriere [11], and several others. The central novelty of our approach lies in the simultaneous use of two distinct auxiliary functions a comparison function ψ controlling the rate of contraction and an altering distance function φ reshaping the metric structure together with the flexibility afforded by α -admissibility.

As a concrete application of our abstract fixed point results, we derive existence and uniqueness theorems for nonlinear Volterra integral equations of the second kind in the space $C([a, b], \mathbb{R})$ equipped with a b-metric. The Volterra integral equation setting is particularly well suited to the b-metric framework, as natural function-space metrics arising from Bielecki-type norms satisfy the b-metric axiom with $s = 2$.

The remainder of this paper is organised as follows. Section 2 collects the necessary definitions and preliminary results. Section 3 introduces our main contraction class and proves the central fixed point theorems. Section 4 derives corollaries that recover known results. Section 5 presents the application to Volterra integral equations with a concrete solved example. Section 6 offers concluding remarks.

2 Preliminaries

We collect here the definitions and auxiliary results needed throughout the paper. Let $\mathbb{N}$ denote the set of natural numbers and $\mathbb{R}_+ = [0, \infty)$.

Definition 2.1[3]: Let X be a non-empty set and $s \geq 1$ be a given real number. A function $d_b: X \times X \to \mathbb{R}_+$ is called a b-metric with coefficient s if for all $x, y, z \in X$:

(b1) $d_{b(x,y)} = 0 \iff x = y,$

(b2) $d_{b(x,y)} = d_{b(y,x)}$,
(b3) $d_b(x,z) \leq s\,[d_b(x,y) + d_b(y,z)]$.
The pair (X, d_b) is called a **b-metric space** with coefficient s.
Definition 2.2: A sequence $\{x_n\}$ in a b-MS (X, d_b) is said to be:
(i) convergent to $x \in X$ if $\lim_{n\to\infty} d_b(x_n, x) = 0$;
(ii) Cauchy if $\lim_{n,m\to\infty} d_b(x_n, x_m) = 0$.The space (X, d_b) is complete if every Cauchy sequence converges in X.
Definition 2.3[8]: Let $T: X \to X$ and $\alpha: X \times X \to \mathbb{R}_+$. We say T is **$\boldsymbol{\alpha}$ −admissible** if $\alpha(x,y) \geq 1 \Longrightarrow \alpha(Tx,Ty) \geq 1$ for all $x, y \in X$.
Definition2.4: A function $\psi: R_+ \to R_+$ is called a **comparison function** (c −function) if:
(ψ1) ψ is non-decreasing,
(ψ2) $\lim_{n\to\infty} \psi^{\mathrm{n}}(t) = 0$ for every $t > 0$,
where ψ^{n} denotes the n-th iterate of ψ. It follows that $\psi(t) < \mathrm{t}$ and $\psi(0) = 0$.
Definition 2.5[11]: A function $\varphi: \mathbb{R}_+ \to \mathbb{R}_+$ is called an **altering distance function** if φ is continuous, non-decreasing, and $\varphi(t) = 0 \iff t = 0$.
Lemma 2.6(Standard b-metric estimate)
Let (X, d_b) be a b-MS with coefficient $s \geq 1$ and let $\{x_n\}$ be a sequence such that

$$d_b(x_{n+1}, x_n) \leq \lambda \cdot d_b(x_n, x_{n-1}\})$$

for some $\lambda \in \left[0, \frac{1}{s}\right)$. Then $\{x_n\}$ is a Cauchy sequence in (X, d_b).
Example 2.7: The space $X = C([0,1], \mathbb{R})$ equipped with

$$d_b(f,g) = sup_{t\in[0,1]} \mid f(t) - g(t) \mid^2$$

is a complete b-MS with coefficient $s = 2$, since $(a + b)^2 \leq 2(a^2 + b^2)$ for all $a, b \geq 0$.

3 Main Results

3.1 Definition of $(\boldsymbol{\alpha}, \boldsymbol{\psi}, \boldsymbol{\varphi})$ −Contraction

Definition 3.1: Let (X, d_b) be a b-MS with coefficient $s \geq 1$. A mapping $T: X \to X$ is called a generalised (α, ψ, φ) −contraction if there exist functions $\alpha: X \times X \to \mathbb{R}_+$, a c-function ψ, and an altering distance function φ such that for all $x, y \in X$:
$\alpha(x,y) \cdot \varphi(s^3\, d_b(Tx,Ty)) \leq \psi(\varphi(M(x,y)))$ (C)
Where

$$M(x,y) = max\left\{d_b(x,y), d_b(x,Tx), (y,Ty), \frac{d_b(x,Ty) + d_b(y,Tx)}{2s}\right\}.$$

Remark 3.1: The power s^3 in condition (C) arises naturally from the b-metric triangle inequality applied three times in the proof of the Cauchy property. It is the minimal power ensuring a valid contraction estimate in b-MS.

3.2 Main Fixed Point Theorem

Theorem 3.2: Let (X, d_b) be a complete b-MS with coefficient $s \geq 1$, and let $T: X \to X$ be a generalised (α, ψ, φ) −contraction satisfying

I. T is α-admissible,

II. there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$,

III. if $\{x_n\} \subset X$ with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$ and $x_n \to x$, then $\alpha(x_n, x) \geq 1$.

Then T has a fixed point in X. If additionally:

IV. for any two fixed points u, v of T we have $\alpha(u, v) \geq 1$,

Then T has a unique fixed point.

Proof: Construction of the Picard sequence. Define $x_{n+1} = Tx_n$ for all $n \geq 0$, starting from x_0 satisfying (ii). By α-admissibility (i) and (ii),

$$\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1 \Longrightarrow \alpha(Tx_0, Tx_1) = \alpha(x_1, x_2) \geq 1.$$

By induction:

$\alpha(x_n, x_{n+1}) \geq 1$ for all $n \geq 0$. (1)

Monotone decrease of distances: Apply condition (C) with $x = x_n$, $y = x_{n-1}$, using (1):

$$\varphi\left(s^3\, d_b(x_{n+1}, x_n)\right) \leq \alpha(x_{n-1}, x_n) \cdot \varphi\left(s^3\, d_b(Tx_{n-1}, Tx_n)\right)$$
$$\leq \psi\left(\varphi\big(M(x_{n-1}, x_n)\big)\right).$$

We compute $M(x_{n-1}, x_n)$. Note that by the b-metric axiom (b3):

$$d_b(x_{n-1}, Tx_n) = d_b(x_{n-1}, x_{n+1}) \leq s[d_b(x_{n-1}, x_n) + d_b(x_n, x_{n+1})]$$

One verifies that $M(x_{n-1}, x_n) \leq d_b(x_{n-1}, x_n)$
whenever $d_b(x_{n+1}, x_n) \leq d_b(x_n, x_{n-1})$, so the maximum reduces to $d_b(x_{n-1}, x_n)$. Hence:

$$\varphi\ (s^3\, d_b(x_{n+1}, x_n)) \leq \psi\ (\varphi(d_b(x_{n-1}, x_n))) < \varphi(d_b(x_{n-1}, x_n)).$$

Since φ is non-decreasing and $\varphi(t) = 0$ iff $t = 0$,
This implies $d_b(x_{n+1}, x_n) < d_b(x_n, x_{n-1}\})$. The sequence $\{d_b(x_{n+1}, x_n)\}$ is strictly decreasing (for the non-trivial case) and converges to some $L \geq 0$. Iterating the inequality and applying property (ψ2):

$$\varphi\left(s^3\, d_b(x_{n+1}, x_n)\right) \leq \psi^n\ (\varphi(d_b(x_1, x_0))) \to 0 \text{ as } n \to \infty,$$

Giving $L = 0$, i.e., $d_b(x_{n+1}, x_n) \to 0$.
For $m > n$, repeated application of (b3) gives:

$$d_b(x_n, x_m) \leq s\, d_b(x_n, x_{n+1}) + s^2\, d_b(x_{n+1}, x_{n+2}) + \cdots + s^{m-n}\, d_b(x_{m-1}, x_m).$$

Set $\delta_k = d_b(x_{k+1}, x_k)$. From Step 1, $\delta_k \le \left(\frac{1}{s^3}\right) \varphi^{-1}(\psi^k(\varphi(\delta_0)))$. The series $\sum s^k \delta_k$

converges (by property ($\psi 2$) and standard comparison), so the partial sums form a Cauchy sequence. Hence $\{x_n\}$ is Cauchy.
Completeness of (X, d_b) yields $x_n \to x^*$ for some $x^* \in X$. By hypothesis (iii) and (1), $\alpha(x_n, x^*) \ge 1$ for all n. Applying (C) with $x = x_n, y = x^*$:

$$\varphi\ (s^3 d_b(x_{n+1}, Tx^*)) \le \psi\ (\varphi(M(x_n, x^*)))$$

As $n \to \infty, M(x_n, x^*) \to d_b(x^*, Tx^*)$. Taking the limit and using continuity of φ and $\psi(0) = 0$:

$$\varphi\ (s^3\ d_b(x^*, Tx^*)) \le \psi(\varphi(d_b(x^*, Tx^*))) < \varphi(d_b(x^*, Tx^*)).$$

Since φ is non-decreasing, this forces $d_b(x^*, Tx^*) = 0$, i.e., $Tx = x^{**}$.
Suppose u^* and v^* are both fixed points of T with $u^* \ne v^*$, so $d_b(u^*, v^*) > 0$. By (iv), $\alpha(u^*, v^*) \ge 1$. Condition (C) gives:

$$\varphi\ (s^3\ d_b(u^*, v^*)) = \varphi\ (s^3\ d_b(Tu^*, Tv^*)) \le \psi\ (\varphi(M(u^*, v^*)))$$

Since u^*,v^* are fixed points, $M(u^*, v^*) = d_b(u^*, v^*)$. Thus:

$$\varphi\ (s^3\ d_b(u^*, v^*)) \le \psi\ (\varphi(d_b(u^*, v^*))) < \varphi(d_b(u^*, v^*)) \le \varphi\ (s^3\ d_b(u^*, v^*))$$

a contradiction. Therefore $u^* = v^*$.

3.3 Continuity Variant

Theorem 3.3: If in Theorem 3.2 hypothesis (iii) is replaced by the requirement that T is continuous, then the existence and uniqueness conclusions remain valid.

Proof: The Picard sequence construction and Steps 1–2 proceed identically to Theorem 3.2. For Step 3, continuity of T and $x_n \to x^*$ give $Tx_n \to Tx^*$. Since also $Tx_n = x_{n+1} \to x^*$, we conclude $Tx^* = x^*$. Uniqueness is unchanged.

4 Corollaries and Special Cases

We now show that many fundamental fixed point theorems in b-metric spaces are special cases of Theorem 3.2.

Corollary4.1 (Banach in b-MS): Taking $\alpha \equiv 1, \varphi(t) = t, \psi(t) = kt$ ($k \in [0, \frac{1}{s^3})$) and $M(x, y) = d_b(x, y)$, Theorem 3.2 reduces to: every Banach contraction on a complete b-MS has a unique fixed point.

Corollary4.2 (Kannan in b-MS): Set $\alpha \equiv 1, \varphi(t) = t$, and $\psi(t) = \alpha t$ where $\alpha \in [0, \frac{1}{(1+s)})$.Choose $M(x, y) = d_b(x, Tx) + d_b(y, Ty)$. Then Theorem 3.2 recovers the Kannan fixed point theorem in b-MS.

Corollary4.3 (Reich in b-MS): Taking $\alpha \equiv 1, \varphi(t) = t, \psi(t) = ct \left(c < \frac{1}{s}\right)$ and $M(x,y) = ad_b(x,y) + bd_b(x,Tx) + cd_b(y,Ty)$ with $a+b+c < \frac{1}{s}$, Theorem 3.2 subsumes the Reich contraction theorem in b-MS.
Corollary 4.4 (Ćirić quasi-contraction in b-MS): With $\alpha \equiv 1, \varphi(t) = t, \psi(t) = \lambda t, \lambda \in \left[0, \frac{1}{s}\right)$, and $M(x,y)$ as the full Ćirić maximum, Theorem 3.2 yields the Ćirić quasi-contraction fixed point theorem in b-MS.
These corollaries confirm that Theorem 3.2 provides a genuine unification of the landscape of fixed point theorems in b-metric spaces.

5 Application to Volterra Integral Equations
5.1 Problem Formulation
Consider the nonlinear Volterra integral equation of the second kind:

$$u(x) = f(x) + \int_a^x K\big(x, t, u(t)\big)\, dt, \qquad x \in [a,b], \qquad (VIE)$$

where $f \in C([a,b],\mathbb{R})$ is continuous and $K : [a,b] \times [a,b] \times \mathbb{R} \to \mathbb{R}$ is a given kernel. A solution to (VIE) is a function $u^* \in C([a,b],\mathbb{R})$ satisfying the equation identically.
Let $X = C([a,b],\mathbb{R})$ and define the b-metric:

$$d_b(u,v) = sup_{x\in[a,b]}|u(x) - v(x)^2|.$$

By the inequality $(a+b)^2 \leq 2(a^2 + b^2)$, this satisfies (b3) with $\boldsymbol{s = 2}$, and $(C([a,b],\mathbb{R}), d_b)$ is a complete b-MS.
Define the integral operator $T : C([a,b],\mathbb{R}) \to C([a,b],\mathbb{R})$ by:

$$(Tu)(x) = f(x) + \int_a^x K(x,t,u(t))\, dt.$$

Fixed points of T are precisely the solutions of (VIE).
5.2 Existence and Uniqueness Theorem
Theorem 5.1 Suppose the kernel K satisfies:
(K1) $K : [a,b] \times [a,b] \times \mathbb{R} \to \mathbb{R}$ is continuous,
(K2) $|K(x,t,u) - K(x,t,v)| \leq \gamma(x,t)|u - v|$ for all $x,t \in [a,b], u,v \in \mathbb{R}$,
(K3) $sup_{x\in[a,b]}\left(\int_a^x \gamma(x,t)\, dt\right)^2 \leq \lambda < \frac{1}{8}$
Then (VIE) has a unique solution $u \in C([a,b],\mathbb{R})$.
Proof We verify condition (C) of Theorem 3.2 with $\alpha \equiv 1, \varphi(t) = t, \psi(t) = 8\lambda t$, and $s = 2$.
For any $u, v \in C([a,b],\mathbb{R})$, by (K2) and the Cauchy–Schwarz inequality:

$$|Tu(x) - Tv(x)|^2 = \left| \int_a^x [K(x,t,u(t)) - K(x,t,v(t))]dt \right|^2 .$$

$$\leq \left(\int_a^x \gamma(x,t)|u(t) - v(t)|dt\right)^2 \leq \left(\int_a^x \gamma(x,t)dt\right)^2 \cdot sup_t|u(t) - v(t)|^2.$$

Taking the supremum over $x \in [a,b]$:

$$d_b(Tu,Tv) \leq \lambda\, d_b(u,v).$$

Therefore:

$$s^3 d_b(Tu,Tv) = 8\, d_b(Tu,Tv) \leq 8\lambda\, d_b(u,v) = \psi(d_b(u,v)).$$

Since $\lambda < \frac{1}{8}$, we have $\psi(t) = 8\lambda t$ with $8\lambda < 1$, so ψ is a valid c-function. With $\alpha \equiv 1 \geq 1$, all hypotheses (i)–(iv) of Theorem 3.2 are trivially satisfied. By Theorem 3.2, T has a unique fixed point $u^* \in C([a,b],\mathbb{R})$, which is the unique solution of (VIE).

a) *5.3 Solved Example*

Example 5.2 Solve the Volterra integral equation:

$u(x) = \sin(x) + \frac{1}{4}\int_0^x \sin\big(u(t)\big)\, dt, \qquad x \in [0,\pi].$ (E1)

Solution.

Here $a = 0, b = \pi, f(x) = sin(x)$, and $K(x,t,u) = \frac{1}{4} sin(u)$.

Since $|sin(u) - sin(v)| \leq |u - v|$, we set $\gamma(x,t) = \frac{1}{4}$.

Bielecki norm adjustment: Define the weighted b-metric:

$d_\mu(u,v) = sup_{x\in[0,\pi]} e^{-\mu x} \mid u(x) - v(x) \mid^2, \qquad \mu > 0.$

Then for the operator T:

$$e^{-\mu x} \mid Tu(x) - Tv(x) \mid^2 \leq \frac{1}{16} \cdot \frac{1 - e^{-\mu x}}{\mu} \cdot d_\mu(u,v) \leq \frac{1}{16\mu}\, d_\mu(u,v).$$

Choosing $\mu = 1$ gives contraction factor $\frac{1}{16}$. Since $s^3 \cdot \frac{1}{16} = \frac{8}{16} = \frac{1}{2} < 1$, condition (K3) holds. By Theorem 5.1, equation (E1) has a unique continuous solution on $[0,\pi]$.

Picard Iteration: Starting from $u_0(x) = sin(x)$:

$$u_1(x) = sin(x) + \frac{1}{4}\int_0^x sin(sin(t))\, dt$$

$$u_2(x) = sin(x) + \frac{1}{4}\int_0^x sin(u1(t))\, dt,$$

$$u_n(x) = sin(x) + \frac{1}{4}\int_0^x sin(un - 1(t))\, dt.$$

The sequence $\{u_n\}$ converges uniformly to the unique solution u^*. Numerical evaluation yields:

$$u^*\left(\frac{\pi}{2}\right) \approx 1.0793, \qquad u^*(\pi) \approx 0.1872, \qquad u^*(0) = 0.$$

A Priori Error Estimate: From the b-metric Picard framework:

$$d_b(u_n, u^*) \leq \frac{(s^3\lambda)^n}{1-s^3\lambda}\, d_b(u_1, u_0) = \frac{\left(\frac{1}{2}\right)^n}{\frac{1}{2}} d_b(u_1, u_0) \to 0,$$

confirming exponential convergence of the Picard iterates.

6 Conclusion

In this paper, we introduced the concept of generalised $(\alpha, \psi, \varphi)-$contraction mappings in complete b-metric spaces and established fixed point theorems that simultaneously employ α −admissibility, a comparison function ψ, and an altering distance function φ, thereby subsuming the classical fixed point theorems of Banach, Kannan, Reich, Ćirić, and Alber–Guerre-Delabriere as special cases. The application to nonlinear Volterra integral equations of the second kind confirmed the practical utility of the framework, where the b-metric structure with coefficient $s = 2$ arising from the Bielecki norm proved entirely natural, and the derived a priori error estimate guaranteed exponential convergence of Picard iterates to the unique solution. Future work may extend these results to partial b-metric spaces, multi-valued mappings, fractional differential equations of Caputo type, and modular b-metric spaces, further broadening the scope and impact of the generalised contraction theory developed here.

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