Disturbance in a Generalized Thermoviscoelastic Half-Space with Voids with Microtemperature without Energy Dissipation Due to Thermal Source

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Abstract

In the present paper I investigate the effects of thermal excitations on a generalized thermoviscoelastic half-space with voids with microtemperature without energy dissipation. The exact expressions for displacement components, components of the microtemperature vector, stresses, temperature distribution, change in volume fraction field, and heat flux moment vector are derived using the normal mode analysis approach on non-dimensional field equations. The viscosity effect and effect of microtemperature and voids on field variables has been depicted graphically for temperature gradient boundary.

Keywords: Thermoviscoelasticity with voids, normal-mode analysis, microtemperature.

1. INTRODUCTION

Nunziato and Cowin (1979) were the first to examine the theory of voids in thermoelastic materials. Iesan (1986) investigated the theory of voids in thermoelastic materials. Dhaliwal and Wang (1995) proposed a thermoelasticity theory for elastic materials with voids that incorporates the heat-flux as one of the constitutive variables and assumes a heat-flux evolution equation. Kumar and Rani (2004) examined at how mechanical and thermal stimuli affected the response of a generalised thermoelastic half-space with voids. Kumar and Rani (2007) investigated axisymmetric deformation in a thermoelastic material with voids due to mechanical and thermal causes. The asymptotic spatial behaviour was investigated by Pompei and Scalía (2011).


The theory of thermoelasticity for bodies with microstructures shows importance in recent years. Grot invented the concept of microtemperatures and a thermodynamics theory for elastic materials with microstructure (1969). Riha (1975, 1977) investigated a heat-conducting micropolar fluid with microtemperatures theory. Casas and Quintanilla (2005) established the exponential stability of solutions of the equation in this theory.

Ieşan (2007) develops a microstretch elastic solids with microtemperatures linear theory. The theory of thermoelasticity with microtemperatures was investigated by Ieşan and Quintanilla (2010). Scalía et al. (2010) investigated basic theorems in thermoelasticity equilibrium theory using microtemperatures. Bitsadze and Jaiani (2013) discussed some basic plane thermoelasticity boundary value problems using microtemperatures. The influence of initial stress on a porous thermoelastic media using micro-temperatures was investigated by Othman et al. (2016). For the half-space, George and Bitsadze (2018) investigated basic thermoelasticity problems with microtemperatures. Marin et al. (2020) discussed microtemperatures and thermoelastic materials with a dipolar structure.

In a homogeneous, isotropic, thermoviscoelastic half-space with voids and microtemperature due to thermal source, the components of displacement, components of the microtemperatures vector, stresses, temperature distribution, change in volume fraction field, and first heat flux moment vector are proposed. The Lord-Shulman, Green-Lindsay, Green and Naghdi theories of types II and III have all been used to demonstrate the concept. To derive exact formulas for physical values, the normal mode analysis is performed. The
viscosity effect, influence of microtemperatures, and voids are all depicted visually.

2. Basic equations

The constitutive relations for a homogeneous and isotropic thermoelastic medium with voids with microtemperature, according to Iesan and Quintanilla (2000), are

\[ t_{ij} = \lambda u_{k,k} \delta_{ij} + \mu (u_{ij} + u_{ji}) + b \eta \delta_{ij} - \beta T_0 (1 + \delta_{2k1} \frac{\partial \eta}{\partial \tau}) \delta_{ij}, \quad (i,j) = 1, 2, 3. \]

\[ q_i = KT_i + k_i w_i, \]

\[ q_{ij} = -k_i w_{ir} \delta_{ij} - k_i w_{ji} - k_i w_{ij}, \]

\[ \rho \eta = \beta e_{rr} + a T + m^* \phi, \]

\[ \rho \mathbf{e}_{ij} = -b^* w_{ij} \mathbf{\delta}_{ij} = a \phi_{ij}. \]

The linear theory of thermoelasticity with voids with microtemperatures in the context of (G-N) theory of type III, in the absence of body force, body couple, equilibrated force, and \( \kappa = 2 \) for G-L theory. The thermal relaxations have their usual meaning. \( \epsilon^* \) is specific heat at constant strain. \( K^* \) is the material constant characteristic of the theory. When \( K^* \rightarrow 0 \) then (11) reduces to the heat conduction equation in (G-N) theory (of type II).

(iii) First moment of energy balance equation is

\[ \rho \epsilon_j = q_{j,i} + q_0. \]

The derivative temporal is represented by the superposed dot, while the other symbols are as explained previously. Using Eqs (1)-(5) in Eqs (6)-(8), and following Iesan (2011), Lord-Shulman (1967), Green-Lindsay (1972), Green and Naghdi (1993), the equations in a homogeneous, isotropic, thermoviscoelastic medium with voids with microtemperature, in the context of (G-N) theory of type III, as follows

\[ \alpha_1 \mathbf{V}^2 \mathbf{v} - q_1 (\mathbf{V}, \mathbf{v}) + \mathbf{v} (m^* + \tau^* \mathbf{v}^2) T = \rho \chi \mathbf{\phi}, \]

\[ K^2 \mathbf{V}^2 + K^* \mathbf{V}^2 \mathbf{\ddot{v}} - \beta T_0 \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \mathbf{V} \mathbf{\ddot{v}} - \left( (\gamma \mathbf{V}^2 - m^* T_0) \frac{\partial}{\partial x} + K \mathbf{V} \mathbf{\ddot{w}} = \rho_{c_e} (T + \tau_0 \mathbf{\ddot{v}}) + \mathbf{aT}_0 \mathbf{\ddot{v}}, \right. \]

where

\[ \alpha_1 = \alpha + \alpha^* \] and \( \tau_0 \) are thermal relaxation times. For L-S theory, \( \tau_1 = 0, \delta_{1k} = 1 \) and for G-L theory \( \tau_1 > 0, \delta_{1k} = 0 \) (i.e., \( k = 1 \) for L-S theory and \( k = 2 \) for G-L theory). The thermal relaxations \( \tau_0 \) and \( \tau_1 \) satisfy the inequality \( \tau_1 \geq \tau_0 > 0 \) for the G-L theory only.\( \mathbf{V} = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \) and other symbols have their usual meaning. \( \mu^*, \lambda^*, b^*, \alpha^*, \gamma^*, \xi^* \) are constitutive coefficients. \( \epsilon^* \) is specific heat at constant strain. \( K^* \) is the material constant characteristic of the theory. When \( K^* \rightarrow 0 \) then (11) reduces to the heat conduction equation in (G-N) theory (of type II).

3. Formulation and Solution of the problem

In the undeformed state at uniform temperature, we study a homogeneous, isotropic, thermally conducting generalised thermoviscoelastic half-space with voids with microtemperature. To. The rectangular Cartesian co-ordinate system \( (x,y,z) \) is introduced, with the origin at \( z = 0 \) and the \( z \)-axis pointing normally into the medium. At the origin of the rectangular Cartesian co-ordinates, a thermal source is supposed to be acting.

All quantities studied in the two-dimensional issue are functions of the time variable \( t \) and the coordinates \( x \) and \( z \).
As a result, the displacement vector \( u \) and the microtemperature vector \( w \) are used.

\[
u = (u_1(x, z, t), 0, u_3(x, z, t)), \quad w = (w_1(x, z, t), 0, w_3(x, z, t))
\]

To simplify the algebra, only problems with zero initial conditions are considered.

Introducing dimensionless quantities

\[
x' = \frac{\omega_1^* x}{c_2}, \quad z' = \frac{\omega_1^* z}{c_2}, \quad t' = \omega_1^* t, \quad (u_1', u_3') = \frac{\omega_1^*}{c_2} (u_1, u_3),
\]

\[
(w_1', w_3') = \frac{\omega_1^*}{c_2} (w_1, w_3), \quad T' = \frac{T}{T_0}, \quad \varphi' = \frac{\omega_1^*}{c_2} \varphi, \quad \epsilon_1 = \frac{\beta c_2^2}{K \omega_1^*}, \quad \tau_0 = \omega_1^* \tau_0,
\]

\[
\tau_1 = \omega_1^* \tau_1, \quad \alpha' = \frac{\omega_1^*}{c_2} \alpha,
\]

\[
q_{ij}' = \frac{\omega_1^*}{\beta T_0 c_2} q_{ij}, \quad \tau_{zz}' = \frac{t_{zz}}{\beta T_0}, \quad \tau_{zx}' = \frac{t_{zx}}{\beta T_0},
\]

\[
\square' = \frac{c_2}{\omega_1^*},
\]

where

\[
c_2 = (\frac{\rho}{c})^{\frac{1}{2}} \quad \text{and} \quad \omega_1^* = \frac{nc_2^2}{\kappa}.
\]

After suppressing the primes, equations (9) – (12) can be recast into the dimensionless form as:

\[
\left(1 + Y_1 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial z^2}\right) + \left(\frac{\partial}{\partial t}\right) \left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_3}{\partial x \partial z}\right) + \left(\frac{\partial}{\partial t}\right) \left(\frac{\partial \varphi}{\partial x} - \gamma_6 \left(1 + \tau_1 \delta_{2k} \frac{\partial T}{\partial x}\right) \right) = \gamma_7 \frac{\partial^2 u_1}{\partial t^2},
\]

\[
Y_1 = \frac{\mu^* \omega_1^*}{\mu}, \quad Y_2 = \frac{\lambda + \mu^* \omega_1^*}{\mu c_2}, \quad Y_3 = \frac{\lambda^* + \mu^* \omega_1^2}{\mu c_2}, \quad Y_4 = \frac{bc_2^2}{\omega_1^* \mu \chi}, \quad Y_5 = \frac{b^* c_2}{\mu \chi}, \quad Y_6 = \frac{\beta T_0}{\mu}, \quad Y_7 = \frac{\rho \omega_1^*}{\mu}, \quad Y_8 = \frac{b \chi}{\alpha}, \quad Y_9 = \frac{\gamma^* \omega_1^*}{\alpha}.
\]
\[ \gamma_{11} = \frac{\xi c_2^2}{\omega_1^2 \alpha}, \gamma_{12} = \frac{\xi c_2^2}{\omega_1^2 \alpha}, \gamma_{13} = \frac{m^* \chi T_0}{\alpha}, \gamma_{14} = \frac{m^* \chi^2 T_0}{\alpha^2}, \gamma_{15} = \frac{K \omega_1^*}{K}, \gamma_{16} = \frac{\xi c_2^2}{\omega_1^4 K T_0 \chi}, \gamma_{17} = \frac{m^* c_2^4}{\alpha^3 K \chi}, \gamma_{18} = \frac{K_1 c_2^2}{\omega_1^2 K T_0}, \gamma_{19} = \frac{\rho c_e c_2^2}{\alpha K}, \gamma_{20} = \frac{a T_0 c_2^2}{\alpha^2 K}, \gamma_{21} = \frac{K_4 + K_6}{K_6}, \gamma_{22} = \frac{K_3 T_0 c_2^2}{K_6 \omega_1^2}, \gamma_{23} = \frac{K_2 c_2^2}{K_6 \omega_1^2}, \gamma_{24} = \frac{b^* c_2^2}{\alpha K \omega_1^*}. \]

Using the dimensional form of the expression relating displacement components and microtemperature components \( u(x, z, t) \) and \( w(x, z, t) \) as well as the scalar potential function \( \varphi_1(x, z, t), \varphi_2(x, z, t) \) and \( \psi_2(x, z, t) \) in equations (16) – (21), we obtain

\[
\begin{align*}
\mathbf{u} &= \frac{\partial \varphi_1}{\partial x} - \frac{\partial \psi_1}{\partial z}, \quad u_3 = \frac{\partial \varphi_1}{\partial z} + \frac{\partial \psi_1}{\partial x}, \\
w_1 &= \frac{\partial \varphi_2}{\partial x} - \frac{\partial \psi_2}{\partial z}, \\
w_3 &= \frac{\partial \varphi_2}{\partial z} + \frac{\partial \psi_2}{\partial x}. 
\end{align*}
\]

In equations (16) – (21), we obtain

\[
\begin{align*}
\left[ (1 + \gamma_2) + \frac{\partial}{\partial t}(\gamma_1 + \gamma_3) - \gamma_7 \frac{\partial^2}{\partial t^2} \right] \varphi_1 &= 0, \\
-(\gamma_4 + \gamma_5 \frac{\partial}{\partial t}) \varphi_1 &= 0, \\
-(\gamma_6(1 + \tau_1 \delta_{2k} \frac{\partial}{\partial t}) T &= 0, \\
\left[ (1 + \gamma_{15}) \frac{\partial}{\partial t} \right] \varphi_1 &= 0, \\
\left[ (1 + \gamma_{10}) \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right] \varphi_1 &= 0, \\
-\varepsilon_1 \left( \frac{\partial}{\partial t} + \tau_0 \delta_{1k} \frac{\partial^2}{\partial t^2} \right) \varphi_1 &= 0, \\
-(\gamma_{16} \varphi_1 - \gamma_{17}) \frac{\partial}{\partial t} \varphi_1 &= 0, \\
+\gamma_{18} \varphi_1 &= 0. 
\end{align*}
\]

\section*{4. Normal mode analysis}

The following form can be used to decompose the solution of the considered physical variable in terms of normal modes

\[
(u_1, v_1, \varphi_1, \psi_1, u_3, w_3, \varphi_2, \psi_2, \varphi, \sigma_{ij}, q_{ij}, T)(x, z, t) = (u_1 *, v_1 *, \varphi_1 *, \psi_1 *, u_3 *, w_3 *, \varphi_2 *, \psi_2 *, \varphi *, \sigma_{ij} *, q_{ij} *, T *) \exp(\omega t + \text{im} x),
\]

where \( (u_1 *, v_1 *, \varphi_1 *, \psi_1 *, u_3 *, w_3 *, \varphi_2 *, \psi_2 *, \varphi *, \sigma_{ij} *, q_{ij} *, T *) \) are the magnitude of the functions, The complex time constant is \( \omega \) and The wave number in the \( x \)-direction is denoted by \( \alpha \).

In equations (23)-(28), we use normal mode analysis technique and obtain

\[
\begin{align*}
(1 + \gamma_8 \frac{\partial}{\partial t}) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \varphi_1 &= 0, \\
-(\gamma_9 + \gamma_{10} \frac{\partial}{\partial t}) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) \psi_1 &= 0, \\
+(\gamma_{12} + \gamma_{13} \frac{\partial}{\partial t}) \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) T &= 0.
\end{align*}
\]

\( \varepsilon_1 (\frac{\partial}{\partial t} + \tau_0 \delta_{1k} \frac{\partial^2}{\partial t^2} \right) \varphi_1 &= 0, \\
-(\gamma_{25} \frac{\partial}{\partial z^2} - \gamma_{26} \varphi_1 *) &= 0, \\
+\gamma_{27} \varphi_1 * - \gamma_{28} T * &= 0, \\
(\gamma_{29} \frac{\partial}{\partial z^2} - \gamma_{30} \varphi_1 *) &= 0, \\
-\gamma_{31} (\frac{\partial^2}{\partial z^2} - m^2 \varphi_1 *) &= 0, \\
+\gamma_{13} \frac{\partial^2}{\partial z^2} + \gamma_{32} T * &= 0.
\]
\[\begin{align*}
\gamma_{33}\left(\frac{\partial^2}{\partial z^2} - m^2\right)\varphi_1 + \gamma_{18}\left(\frac{\partial^2}{\partial z^2} - m^2\right)\varphi_2^* \\
- \left(\gamma_{16}\frac{\partial^2}{\partial z^2} \omega - \gamma_{35}\right)\varphi^* \\
+ \left(\gamma_{27}\frac{\partial^2}{\partial z^2} - \gamma_{34}\right)T^* \\
= 0, \\
(\gamma_{38}\frac{\partial^2}{\partial z^2} - \gamma_{39})\varphi_2^* - \gamma_{24}T^* \\
= 0, \\
\left(\frac{\partial^2}{\partial z^2} - \lambda_2^2\right)\psi_1 *= 0, \\
\left(\frac{\partial^2}{\partial z^2} - \lambda_3^2\right)\psi_2 *= 0,
\end{align*}\]

Where

\[\begin{align*}
\gamma_{25} &= m^2(1 + \gamma_1 \omega) + \gamma_{11} \gamma_2 \omega^2 m^2, \\
\gamma_{26} &= (1 + \gamma_6 \omega) + \gamma_{16} \omega^2, \\
\gamma_{27} &= -m^2(1 + \gamma_9), \\
-(\gamma_{11} + \gamma_{12} - \gamma_{14}) - \gamma_{15} \omega^2 - m^2 \gamma_{16} \omega^2, \\
\gamma_{28} &= m^2(\gamma_9 + \omega \gamma_{10}), \\
\gamma_{29} &= (\gamma_9 + \omega \gamma_{10}), \\
\gamma_{30} &= -m^2 \xi_1 \omega^2 (1 + \tau_q \omega + \frac{\tau_q^2}{2} \omega^2), \\
\gamma_{31} &= \frac{\gamma_{30}}{-m^2}, \\
\gamma_{32} &= -m^2 \gamma_{33}, \\
\gamma_{33} &= \left[(\tau_T \omega^2 + \omega) + \gamma_{17}(1 + \tau_V \omega)\right]
\end{align*}\]

Eliminating \(\varphi_1^*, \varphi_2^*, \varphi^*\) and \(T^*\) from the resulting expressions, we obtain

\[
\left(\frac{\partial^8}{\partial z^8} + M \frac{\partial^6}{\partial z^6} + N \frac{\partial^4}{\partial z^4} + O \frac{\partial^2}{\partial z^2} + P\right)\left(\varphi_1^*, \varphi_2^*, \varphi^*, \psi_1^*, T^*\right) = 0, \tag{36}
\]

where

\[M = \left\{-\gamma_{39}(\gamma_{25}\gamma_{29}\gamma_{27} + \gamma_{25}\gamma_{13}\gamma_{16}) - \gamma_{38}(\gamma_{25}\gamma_{29}\gamma_{34} + \gamma_{25}\gamma_{30}\gamma_{27} + \gamma_{25}\gamma_{13}\gamma_{35} - \omega \gamma_{25}\gamma_{16}\gamma_{32} + \gamma_{26}\gamma_{29}\gamma_{27} + \omega \gamma_{26}\gamma_{13}\gamma_{16} + \gamma_{18}\gamma_{24}\gamma_{25}\gamma_{29})/a_1, \right. \]
\[N = \left\{\gamma_{38}(\gamma_{26}(\gamma_{34}\gamma_{29} + \gamma_{27}\gamma_{30} + \gamma_{13}\gamma_{35} - \omega \gamma_{16}\gamma_{26})) - \gamma_{39}(\gamma_{25}\gamma_{29}\gamma_{34} + \gamma_{25}\gamma_{30}\gamma_{27} + \gamma_{25}\gamma_{13}\gamma_{35} - \omega \gamma_{25}\gamma_{16}\gamma_{32} + \gamma_{26}\gamma_{29}\gamma_{27} + \omega \gamma_{26}\gamma_{13}\gamma_{16}) + \gamma_{18}\gamma_{36}(\gamma_{19}\gamma_{30} + \gamma_{20}\gamma_{29} + \gamma_{27}\gamma_{31}) + m^2 \gamma_{18}\gamma_{24} + \gamma_{25}\gamma_{29})/a_1, \right. \]
\[O = \left\{\gamma_{38}(\gamma_{26}\gamma_{32}\gamma_{35} - \gamma_{26}\gamma_{30}\gamma_{34}) - \gamma_{39}(\gamma_{26}(\gamma_{34}\gamma_{29} + \gamma_{27}\gamma_{30} + \gamma_{13}\gamma_{35} - \omega \gamma_{16}\gamma_{26})) \right. \]
\[+ m^2 \gamma_{18}\gamma_{24}(\gamma_{25}\gamma_{30} + \gamma_{26}\gamma_{29} + \gamma_{27}\gamma_{31})/a_1, \right. \]
\[P = \left\{-\gamma_{39}(-\gamma_{26}\gamma_{30}\gamma_{34} + \gamma_{26}\gamma_{32}\gamma_{35}) - m^2 \gamma_{18}\gamma_{24}(\gamma_{25}\gamma_{30} + \gamma_{26}\gamma_{29} + \gamma_{27}\gamma_{31})/a_1 \right. \]
\[a_1 = (\gamma_{25}\gamma_{29}\gamma_{27} + \gamma_{25}\gamma_{13}\gamma_{16})\gamma_{38} \right. \]

The roots of equations (30) - (35) are \(\pm \lambda_\ell (\ell = 1, 2, 3, 4)\). We use the regularity condition at \(z = \infty\), the solutions of equations (30) - (35) may be written as

\[
\varphi_1^* = A_1 e^{\lambda_1 z} + A_2 e^{\lambda_2 z} + A_3 e^{\lambda_3 z} + A_4 e^{\lambda_4 z}, \tag{37}
\]
\[
\varphi_2^* = b_1 A_5 e^{\lambda_4 z} + b_2 A_6 e^{\lambda_2 z} + b_3 A_7 e^{\lambda_3 z} + b_4 A_8 e^{\lambda_4 z}, \tag{38}
\]
\[
\varphi^* = g_1 A_9 e^{\lambda_1 z} + g_2 A_10 e^{\lambda_2 z} + g_3 A_11 e^{\lambda_3 z} + g_4 A_12 e^{\lambda_4 z}, \tag{39}
\]
\[
T^* = h_1 A_13 e^{\lambda_1 z} + h_2 A_14 e^{\lambda_2 z} + h_3 A_15 e^{\lambda_3 z} + h_4 A_16 e^{\lambda_4 z}, \tag{40}
\]
\[
\psi_1^* = A_6 e^{-\lambda_6 z}. \tag{41}
\]
\[
\varphi_5^* = A_5 e^{-\lambda_5} \tag{42}
\]

Where

\[
b_\ell = \frac{\lambda_1^4 y_{54} + \lambda_2^4 y_{55} + \gamma_{56}}{\lambda_3^4 y_{51} + \lambda_4^4 y_{52} + y_{53}},
\]
\[
g_\ell = -\frac{\lambda_1^4 y_{41} + \lambda_2^4 y_{42} + y_{43}}{\lambda_3^4 y_{44} + \lambda_4^4 y_{45}},
\]

374
5. Applications

Thermal source

In this case the boundary conditions are

\[ t_{33}(x, z, t) = 0, \]
\[ t_{31}(x, z, t) = 0, \]
\[ \frac{\partial \varphi}{\partial z} = 0, \]
\[ q_{33} = 0, \]
\[ q_{31} = 0 \text{ at } z = 0, \]
\[ \frac{\partial T}{\partial z}(x, z = 0) = Pe^{\omega t + i\mu x} \text{ at } z = 0, \]
\[ T(x, z = 0) = Pe^{\omega t + i\mu x} \text{ at } z = 0, \]

The magnitude of constant temperature applied on the boundary is denoted by \( P \).

We obtain the expressions for displacement components, components of the microtemperatures vector, stresses, temperature distribution, change in volume fraction field, and components of the first heat flux moment vector by using equations (1), (3), (13)-(15), (22) and substituting the values of \( \varphi_1^*, \varphi_2^*, \psi_1^*, \psi_2^*, T^*, \varphi^* \) from equations (37)-(42) in the boundary conditions (43).

\[ u_1^* = \frac{P}{\Delta} \left\{ \text{Im} \left( \Delta_1^i \hat{e}^{\lambda_1 z} + \Delta_2^i \hat{e}^{\lambda_2 z} + \Delta_3^i \hat{e}^{\lambda_3 z} \right) \right\} e^{\omega t + i\mu x}, \]
\[ u_3^* = -\frac{P}{\Delta} \left\{ (\lambda_1 \Delta_1^i \hat{e}^{\lambda_1 z} + \lambda_2 \Delta_2^i \hat{e}^{\lambda_2 z} + \lambda_3 \Delta_3^i \hat{e}^{\lambda_3 z}) + \Delta_4^i \hat{e}^{\lambda_4 z} \right\} e^{\omega t + i\mu x}, \]
\[ w_1^* = \frac{P}{\Delta} \left\{ \text{Im} \left( b_1 \Delta_1^i \hat{e}^{\lambda_1 z} + b_2 \Delta_2^i \hat{e}^{\lambda_2 z} \right) \right\} e^{\omega t + i\mu x}, \]
\[ w_3^* = \frac{P}{\Delta} \left\{ (\lambda_1 b_1 \Delta_1^i \hat{e}^{\lambda_1 z} + \lambda_2 b_2 \Delta_2^i \hat{e}^{\lambda_2 z} + \lambda_3 b_3 \Delta_3^i \hat{e}^{\lambda_3 z} + \lambda_4 b_4 \Delta_4^i \hat{e}^{\lambda_4 z}) + \mu \Delta_5^i \hat{e}^{\lambda_5 z} \right\} e^{\omega t + i\mu x}, \]
\[ \sigma_{33}^* = \frac{P}{\Delta} \left\{ (n_1 \Delta_1^i \hat{e}^{\lambda_1 z} + n_2 \Delta_2^i \hat{e}^{\lambda_2 z} + n_3 \Delta_3^i \hat{e}^{\lambda_3 z}) + \mu n_4 \Delta_4^i \hat{e}^{\lambda_4 z} + \mu n_5 \Delta_5^i \hat{e}^{\lambda_5 z} \right\} e^{\omega t + i\mu x}, \]
\[ \sigma_{31}^* = \frac{P}{\Delta} \left\{ (n_6 \Delta_1^i \hat{e}^{\lambda_1 z} + n_7 \Delta_2^i \hat{e}^{\lambda_2 z} + n_8 \Delta_3^i \hat{e}^{\lambda_3 z} + n_9 \Delta_4^i \hat{e}^{\lambda_4 z} + n_10 \Delta_5^i \hat{e}^{\lambda_5 z}) \right\} e^{\omega t + i\mu x}, \]
\[ \varphi^* = \frac{P}{\Delta} \left\{ (g_1 \Delta_1^i \hat{e}^{\lambda_1 z} + g_2 \Delta_2^i \hat{e}^{\lambda_2 z} + g_3 \Delta_3^i \hat{e}^{\lambda_3 z} + g_4 \Delta_4^i \hat{e}^{\lambda_4 z}) \right\} e^{\omega t + i\mu x}, \]
\[ T^* = \frac{P}{\Delta} \left\{ (h_1 \Delta_1^i \hat{e}^{\lambda_1 z} + h_2 \Delta_2^i \hat{e}^{\lambda_2 z} + h_3 \Delta_3^i \hat{e}^{\lambda_3 z} + h_4 \Delta_4^i \hat{e}^{\lambda_4 z}) \right\} e^{\omega t + i\mu x}, \]
\[
q_{33} = \frac{P}{\Delta} \left\{ (n_{11} b_1 \Delta_1 \tilde{e}^{\lambda_1 z} + n_{12} b_2 \Delta_2 \tilde{e}^{\lambda_2 z}) + n_{13} b_3 \Delta_3 \tilde{e}^{\lambda_3 z} + n_{14} b_4 \Delta_4 \tilde{e}^{\lambda_4 z}) + n_{15} \Delta_5 \tilde{e}^{\lambda_5 z} \right\} e^{\omega t + i \omega x},
\]

\[
q^*_3 = \frac{P}{\Delta} \left\{ 2(n_{16} \Delta_1 \tilde{e}^{\lambda_1 z} + n_{17} \Delta_2 \tilde{e}^{\lambda_2 z}) + n_{18} \Delta_3 \tilde{e}^{\lambda_3 z} + n_{19} \Delta_4 \tilde{e}^{\lambda_4 z}) + n_{20} \Delta_5 \tilde{e}^{\lambda_5 z} \right\} e^{\omega t + i \omega x},
\]

(44)

where

\[
\Delta = \Delta_1 * + \Delta_2 *
\]

\[
\Delta_1 * = n_{15}(n_{5} n_{9} + n_{10} n_{4})[\lambda_1 g_1 (n_{17}(-\lambda_3 + h)h_3 - n_{18}(-\lambda_2 + h)h_2) - \lambda_2 g_2 (n_{16}(-\lambda_3 + h)h_3 - n_{18}(-\lambda_1 + h)h_1)] + \lambda_3 g_3 (n_{16}(-\lambda_2 + h)h_2 - n_{17}(-\lambda_1 + h)h_1)]
\]

\[
\Delta_2^* = n_{20}(n_{5} n_{9} + n_{10} n_{4})[\lambda_1 g_1 (n_{12} d_2 (-\lambda_3 + h)h_3 - n_{13} d_3 (-\lambda_2 + h)h_2 - n_{14} d_4 (-\lambda_1 + h)h_1)]
\]

\[
\Delta'_1 = n_5 \Delta''_1 - \Delta''_1 n_{10}
\]

\[
\Delta''_1 = -n_{15}[n_7 \lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}] - n_8[\lambda_2 g_2 n_{19} - \lambda_4 g_4 n_{17}] + n_9[\lambda_2 g_2 n_{18} - \lambda_3 g_3 n_{17}] + n_{20}[n_7 \lambda_3 g_3 n_{14} b_4 - \lambda_4 g_4 n_{13} b_3] - n_8[\lambda_2 g_2 n_{13} b_3 - \lambda_3 g_3 n_{12} b_2] + n_9[\lambda_2 g_2 n_{12} b_2 - \lambda_3 g_3 n_{13} b_3]
\]

\[
\Delta''_2 = -n_{15}[n_6 \lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}]
\]

\[
\Delta''_3 = -n_{15}[n_6 \lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}] - n_7[\lambda_1 g_1 n_{19} - \lambda_4 g_4 n_{16}] + n_8[\lambda_1 g_1 n_{18} - \lambda_3 g_3 n_{17}] + n_{20}[n_1 \lambda_2 g_2 n_{14} b_4 - \lambda_4 g_4 n_{12} b_2] - n_2[\lambda_1 g_1 n_{14} b_4 - \lambda_4 g_4 n_{12} b_2] - n_3[\lambda_1 g_1 n_{12} b_2 - \lambda_2 g_2 n_{11} b_1]
\]

\[
\Delta''_4 = -n_{15}[n_6 \lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}]
\]

\[
\Delta''_5 = -n_{15}[n_6 \lambda_3 g_3 n_{19} - \lambda_4 g_4 n_{18}]
\]
\[ \Delta_4 = -n_5\{ \lambda_2 g_2 n_1 b_3 - \lambda_3 g_3 n_1 b_2 \} \\
- n_7\{ \lambda_1 g_1 n_1 b_3 \\
- \lambda_3 g_3 n_1 b_1 \} \\
+ n_8\{ \lambda_1 g_1 n_1 b_2 \\
- \lambda_2 g_2 n_1 b_1 \} \]

\[ +n_{10}\{ n_1\{ \lambda_2 g_2 n_1 b_4 - \lambda_3 g_3 n_1 b_2 \} \\
- n_2\{ \lambda_1 g_1 n_1 b_3 \\
- \lambda_3 g_3 n_1 b_1 \} \\
+ n_3\{ \lambda_1 g_1 n_1 b_2 \\
- \lambda_2 g_2 n_1 b_1 \} \} \]

\[ \Delta'_5 = -n_5\Delta''_5 + \Delta'''_5 n_{10} \]

\[ \Delta''_5 = n_6\{ \lambda_2 g_2 \{ n_1 b_3 n_1 b_9 - n_1 b_4 n_1 b_8 \} \\
- \lambda_3 g_3 \{ n_1 b_2 n_1 b_9 \\
- n_1 b_4 n_1 b_6 \} \\
+ \lambda_4 g_4 \{ n_1 b_2 n_1 b_8 \\
- n_1 b_3 n_1 b_7 \} \}

\[ -n_7\{ \lambda_1 g_1 \{ n_1 b_3 n_1 b_9 - n_1 b_4 n_1 b_8 \} \\
- \lambda_3 g_3 \{ n_1 b_2 n_1 b_9 \\
- n_1 b_4 n_1 b_6 \} \\
+ \lambda_4 g_4 \{ n_1 b_2 n_1 b_8 \\
- n_1 b_3 n_1 b_7 \} \} \]

\[ +n_8\{ \lambda_1 g_1 \{ n_1 b_2 n_1 b_9 - n_1 b_4 n_1 b_8 \} \\
- \lambda_2 g_2 \{ n_1 b_1 n_1 b_9 \\
- n_1 b_4 b_1 n_6 \} \\
+ \lambda_4 g_4 \{ n_1 b_1 n_1 b_8 \\
- n_1 b_2 n_1 b_6 \} \} \]

\[ -n_9\{ \lambda_1 g_1 \{ n_1 b_2 n_1 b_9 - n_1 b_3 n_1 b_7 \} \\
- \lambda_2 g_2 \{ n_1 b_1 n_1 b_9 \\
- n_1 b_3 n_1 b_6 \} \\
+ \lambda_3 g_3 \{ n_1 b_1 n_1 b_7 \\
- n_1 b_2 n_1 b_6 \} \} \]

\[ \Delta'''_5 = n_1\{ \lambda_2 g_2 \{ n_1 b_3 n_1 b_9 - n_1 b_4 n_1 b_8 \} \\
- \lambda_3 g_3 \{ n_1 b_2 n_1 b_9 \\
- n_1 b_4 n_1 b_6 \} \\
+ \lambda_4 g_4 \{ n_1 b_2 n_1 b_8 \\
- n_1 b_3 n_1 b_7 \} \}

\[ -n_2\{ \lambda_1 g_1 \{ n_1 b_4 n_1 b_9 - n_1 b_4 n_1 b_8 \} \\
- \lambda_3 g_3 \{ n_1 b_1 n_1 b_9 \\
- n_1 b_4 n_1 b_6 \} \\
+ \lambda_4 g_4 \{ n_1 b_1 n_1 b_8 \\
- n_1 b_3 n_1 b_7 \} \} \]
\[ +n_3[n_6(\lambda_2 g_2 n_{14} b_4 - \lambda_4 g_4 n_{12} b_2) - n_7(\lambda_1 g_1 n_{14} b_4 - \lambda_4 g_4 n_{11} b_1) + n_9(\lambda_1 g_1 n_{12} b_2 - \lambda_2 g_2 n_{11} b_1)] +n_4[n_6(\lambda_2 g_2 n_{13} b_3 - \lambda_3 g_3 n_{12} b_2) - n_7(\lambda_1 g_1 n_{13} b_3 - \lambda_3 g_3 n_{11} b_1) + n_8(\lambda_1 g_1 n_{12} b_2 - \lambda_2 g_2 n_{11} b_1)] \]

\[ n_1 = -[(\lambda + \omega \lambda) m^2 + ((\lambda + \omega \lambda) + 2(\mu + \omega \mu)) \lambda_1^2 + (b + \omega b) \frac{c_2^2 g_1}{\omega^2 K} - \beta T_0 (1 + \tau_1 \delta_{2k} \omega) h_1], \]

\[ n_2 = -[(\lambda + \omega \lambda) m^2 + ((\lambda + \omega \lambda) + 2(\mu + \omega \mu)) \lambda_2^2 + (b + \omega b) \frac{c_2^2 g_1}{\omega^2 K} - \beta T_0 (1 + \tau_1 \delta_{2k} \omega) h_1], \]

\[ n_3 = -[(\lambda + \omega \lambda) m^2 + ((\lambda + \omega \lambda) + 2(\mu + \omega \mu)) \lambda_3^2 + (b + \omega b) \frac{c_2^2 g_1}{\omega^2 K} - \beta T_0 (1 + \tau_1 \delta_{2k} \omega) h_1], \]

\[ n_4 = -[(\lambda + \omega \lambda) m^2 + ((\lambda + \omega \lambda) + 2(\mu + \omega \mu)) \lambda_4^2 + (b + \omega b) \frac{c_2^2 g_1}{\omega^2 K} - \beta T_0 (1 + \tau_1 \delta_{2k} \omega) h_1], \]

\[ n_5 = -2m_i \lambda_5(\mu + \omega \mu), \]

\[ n_6 = -2m_i \lambda_6, n_7 = -2m_i \lambda_7, n_8 = -2m_i \lambda_8, \]

\[ n_9 = -2m_i \lambda_9, n_{10} = -(m_i \lambda_6 + m^2), \]

\[ n_{11} = -(s_1 \lambda_4 m^2 + s_2 + \lambda_2^2), \]

\[ n_{12} = -(s_1 \lambda_4 m^2 + s_2 + \lambda_3^2), \]

\[ n_{13} = -(s_1 \lambda_4 m^2 + s_2 + \lambda_4^2), \]

\[ n_{14} = -(s_1 \lambda_4 m^2 + s_2 + \lambda_5^2), \]

\[ n_{15} = m s_1 \lambda_4 \lambda_5 - m s_2, n_{16} = m s_1 \lambda_1 \lambda_6 = s_1 \lambda_2 \lambda_3, \]

\[ n_{18} = s_1 b_2 \lambda_3, n_{19} = s_1 b_2 \lambda_4, \]

\[ n_{20} = s_1 \lambda_6 + s_1 \lambda_5, s_1 = \frac{\omega_2 \lambda_2^2}{\beta T g^2 d^2}, s_2 = s_1 + (K_4 + K_5 + K_6), \]

The expressions for temperature gradient boundary and temperature input boundary are obtained by replacing \( \Delta \) by \( \Delta_1 \) and \( \Delta_2 \), respectively.

**PARTICULAR CASE:** On Neglecting the microtemperature, voids and viscosity effect i.e., \( g_1 = \alpha = b = \xi_1 = m^* = \chi = 0 \), \( \mu^* = \lambda^* = b^* = a^* = \gamma^* = \xi^* = K^* = 0 \) in equation (44), the corresponding expressions of stresses, displacement and temperature distribution for thermoelastic half-space are obtained.

**Special case 1:** For L–S theory, we obtain the corresponding expressions of thermoelastic half-space with voids and microtemperature by taking \( k = 1 \), \( \tau_1 = \mu^* = \lambda^* = b^* = a^* = \gamma^* = \xi^* = K^* = 0 \) in equation (44), respectively.

**Special case 2:** The expressions of thermoelastic half-space with voids and microtemperature, are obtained by taking \( k = 2 \), \( \mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* = K^* = 0 \) in equation (44), for G–L theory.

**Special case 3:** In case of coupled theory of thermoelasticity, the expressions of thermoelastic half-space with voids and microtemperature are obtained by taking \( \tau_0 = \tau_1 = 0 \), \( \mu^* = \lambda^* = b^* = \alpha^* = \gamma^* = \xi^* = K^* = 0 \) in equation (44), and obtain the expressions of thermoelastic half-space with voids and microtemperature, respectively.

6. **Numerical results and discussion**

Following Tomar et al. (2013) the hypothetical values of the relevant parameters are

\[ \lambda = 1.5 \times 10^3 \text{ Nm}^{-2}, \quad \mu = 2.5 \times 10^4 \text{ Nm}^{-2}, \quad a = 2 \times 10^8 \text{ Nm}^{-2} \text{ K}^{-1}, \]

\[ b = 2.1 \times 10^8 \text{ Nm}^{-2}, \quad \alpha = 4 \times 10^3 \text{ N}, \quad \beta = 4 \times 10^3 \text{ Nm}^{-2} \text{ K}, \quad \xi = 40 \text{ Nm}^{-2}, \quad m^* = 5 \times 10^{-3} \text{ N/m}^2 \text{ K}, \]

\[ K = 0.016 \times 10^{-3} \text{ Ns}^{-1} \text{ K}, \quad \rho = 2.6 \times 10^6 \text{ kgm}^{-3}, \quad \lambda^* = 2.6 \times 10^3 \text{ Ns}^{-1} \text{ m}, \quad \mu^* = 1.0 \times 10^2 \text{ Ns}^{-1} \text{ m}, \]

\[ b^* = 1.2 \times 10^4 \text{ Ns}^{-1} \text{ m}, \quad \gamma^* = 1.6 \times 10^4 \text{ Ns}^{-1} \text{ m}, \quad \alpha^* = 1.6 \text{ Ns}, \quad \chi = 0.2 \times 10^6 \text{ m}^2, \quad \tau_0 = 0, \quad \nu = 0.3 \times 10^8 \text{ N}, \quad T_0 = 300 \text{ K}, \]

and \( c_e = 1.04 \times 10^7 \text{ J kg}^{-1} \text{ degree}^{-1} \). For \( P=1 \), \( K^* = c_e \left( \frac{\lambda^* + 2\mu^*}{4} \right) \) and other physical constants are (Steeb et al. (2013))

\[ K_1 = 2 \times 10^{10} \text{ Wm}^{-1}, K_2 = 0.1 \times 10^{10} \text{ Wm}^{-1}, \]

\[ K_3 = 0.4 \times 10^{10} \text{ Wm}^{-1}, K_4 = 0.3 \times 10^{10} \text{ Wm}^{-1}, \]

\[ K_5 = 0.5 \times 10^{10} \text{ Wm}^{-1}, K_6 = 0.7 \times 10^9 \text{ Wm}^{-1}, \]

\[ b^* = 1.3849 \times 10^{10} \text{ N}, \]

Figures 1–14 graphically compare the values of components of the microtemperatures vectors \( (w_1^* \text{ and } w_2^*) \), tangential stress \( \sigma_{21}^* \), normal stress \( \sigma_{33}^* \), boundary temperature field \( T^* \) and change in volume fraction field \( \phi^* \), the components of the first heat flux moment vector \( q_{33}^* \) with distance \( x \), for G-N.
L-S and G-L theories, for non-dimensional relaxation times $\tau_0 = 0.02, \tau_1 = 0.05$
and $\omega = \omega_0 + \eta t$, $\omega_0 = 2.0, \eta = 0.1, m = 0.9$. For time $t = 1.0$ and $t = 2.0$, in the range $0 \leq x \leq 10$ at $z=1$, the computation has been done. The variation for G-N theory with viscous effect is shown by black lines with and without centre symbols in figures 1-7, whereas the variation for G-N theory without viscous effect is represented by red lines with and without centre symbols in figures 1-7. The variation for L-S theory is represented by black lines with and without centre symbols in figures 8-14, whereas the variation for G-L theory is represented by red lines with and without centre symbols. The temperature gradient boundary is represented in the figures.

**Thermal source (Temperature gradient boundary)**

Figure 1. depicts the variation of microtemperatures vector $w_1^*$ with distance $x$. Initially, the values $w_1^*$ remain the same in the range $0 \leq x \leq 5$ for viscous and without viscous effect. The values of $w_1^*$ for the case without viscous effect are more than with viscous effect in the range $5 \leq x \leq 10$ for time $t=1.0$ and $t=2.0$. Figure 2. displays the variation of microtemperatures vector $w_2^*$ with distance $x$. For viscous effect and without viscous effect the values of $w_2^*$ at time $t=1$ are more than that at time $t=2$ in the whole range $0 \leq x \leq 10$. Figure 3. shows the variation of temperature distribution $T^*$ with distance $x$. For viscous and without viscous effect the value of $T^*$ shows the opposite oscillatory pattern in the range $0 \leq x \leq 10$. The variation of tangential stress $\sigma_{31}^*$ with distance $x$ has been shown in Figure 4.

For viscous and without viscous effect the values of $\sigma_{31}^*$ start from zero and decrease with an increase in distance $x$.

Figure 5 depicts the variation of $\sigma_{33}^*$ with distance $x$. For viscous effect the values of $\sigma_{33}^*$ at time $t=3$ are more than at time $t=1$ and for without viscous effect the values of $\sigma_{33}^*$ at time $t=1$ are more than at time $t=3$ in the whole range. Figure 6 shows the variation of change in volume fraction field $\varphi^*$ with distance $x$. The values of $\varphi^*$ are the same for viscous and without viscous effect in the range $0 \leq x \leq 3$ and increase with an increase in distance $x$ in the range $3.1 \leq x \leq 10$ for viscous and without viscous effect. The variation of $q_{33}^*$ has been shown in Figure 7. The values of $q_{33}^*$ increase as time decreases for viscous and without viscous effect in the whole range.

Figure 8. shows the variation of microtemperatures vector $w_1^*$ with distance $x$. Near the point of application of source, in the range, $0 \leq x \leq 3.5$, the values of $w_1^*$ for L-S and G-L theories show very small differences and in the range $3.6 \leq x \leq 10$ an appreciable difference is observed, i.e., the values of $w_1^*$ for L-S theory are more than G-L theory for time $t=1$ and $t=2$. Figure 9 displays the variation of microtemperatures vector $w_3^*$ with distance $x$. The values of $w_3^*$ for time $t=2$ lies between the time $t=1$ for L-S and G-L theories. Figure 10 depicts the variation of temperature distribution $T^*$ with distance $x$. The value of $T^*$ for L-S and G-L theories start from zero and decreases with an increase in distance $x$ for both values of time in the whole range. The variation of tangential stress $\sigma_{31}^*$ with distance $x$ has been shown in Figure 11. Initially, the values of tangential stress for both the theories are the same and the values of L-S theory are less than G-L theory for both values of time in the range $2 \leq x \leq 10$. Figure 12 depicts the variation of $\sigma_{33}^*$ with distance $x$. The values of $\sigma_{33}^*$ start from zero and decrease with an increase in distance $x$ for both the theories and for both values of the time. Figure 13 shows the variation of $\varphi^*$ with distance $x$. The values of $\varphi^*$ for L-S theory are more than those of G-L theory for both values of time in the whole range. The variation of the first heat flux moment vector $q_{33}^*$ has been shown in Figure 14. The values of $q_{33}^*$ decrease with an increase in distance $x$ in the range $0 \leq x \leq 10$ for both L-S and G-L theories and for time $t=1$ and $t=2$.

**CONCLUSIONS**

1. The thermo-viscoelastic materials with voids has application in the distribution of field quantities.
2. For finding the solution of the problem the normal mode analysis technique has been used.
3. The comparison G-N theory, L-S, G-L theories with viscous and without viscous effect has been depicted graphically for temperature gradient boundary.
4. It is noticed that the viscous effect plays an important role in all considered physical quantities.
5. It is observed that the deformation of a body depends on viscous effect, the nature of the applied force as well as the type of boundary conditions. The problem investigated here is applicable in the field of earthquake engineering, seismology and geophysics.

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Figure 1. Variation of microtemperatures vector $w_i$ with distance $x$. 
Figure 2. Variation of microtemperatures vector $w_3$ with distance $x$. 
Figure 3. Variation of temperature distribution $T'$ with distance $x$. 
Figure 4. The variation of tangential stress $\sigma_{31}^t$ with distance $x$. 
Figure 5. The variation of normal stress $\sigma_{33}$ with distance x.
Figure 6. Variation of change in volume fraction field $\varphi^*$ with distance $x$. 
Figure 7. Variation of the first heat flux moment vector $q_{33}^*$ with distance $x$. 

$q_{33}^*$
Figure 8. Variation of microtemperatures vector $w^*_i$ with distance $x$. 
Figure 9. Variation of microtemperatures vector $w_3$ with distance $x$. 
Figure 10. Variation of temperature distribution $T^*$ with distance $x$. 
Figure 11. The variation of tangential stress $\sigma_{31}^*$ with distance $x$. 

\[ \sigma_{31}^* \]
Figure 12. The variation of normal stress $\sigma_{33}$ with distance $x$. 
Figure 13. Variation of change in volume fraction field $\varphi^*$ with distance $x$. 
Figure 14. Variation of the first heat flux moment vector $q_{33}$ with distance $x$. 