Simulation of COVID 19 with Control

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Abstract
Coronavirus disease (COVID-19) is an infectious disease caused by the severe acute respiratory syndrome corona virus 2 (SARS-CoV-2). Utmost people infected with the contagion will witness mild to moderate respiratory illness and recover without taking special treatment. Still, some will become seriously ill and require medical attention. Aged people and those with underlying medical conditions like cardiovascular disease, diabetes, chronic respiratory disease, or cancer are more likely to develop serious illness. Anyone can get sick with COVID-19 and become seriously ill or die at any age.

To study the dynamics of the disease a compartmental model involving non-linear ordinary differential equations has been formulated for humans. Controls in terms as preventive measures of self-quarantine, isolation, vaccination are to be taken applied to humans. Numerical Simulation has been carried out to show the impact of control on different compartments.

Keywords: Mathematical model, System of non-linear ordinary differential equations, COVID 19, Stability, Control

1. INTRODUCTION
COVID-19 symptoms range from none to life-hanging. Severe illness is more likely in senior patients and those with certain underlying medical conditions. Transmission of COVID-19 occurs when people breathe in air defiled by driblets and small airborne particles. The threat of breathing these in is loftiest when people are nearby, but the contagion can transmit over longer distances, particularly indoors and in inadequately ventilated areas. Transmission can also occur, infrequently, via defiled surfaces or fluids. People remain contagious for up to 20 days and can spread the contagion even if they do not develop symptoms.

The flu is veritably common, especially in the season, and generally the symptoms are fever, headache, muscle ache, but also upper respiratory symptoms similar as sneezing and coughing. For COVID-19 it's the same symptoms, principally, but in addition, we have specific symptoms such as anosmia, which is a lack of smell and ageusia, which is a lack of taste. And numerous people, especially young people, have endured these additional and specific symptoms for COVID-19 [10]. Some of the most common symptoms of post COVID-19 condition or long COVID include briefness of breath, cognitive dysfunction, which people call brain fog, as well as fatigue [11].

Several vaccines have been approved and distributed in different countries, which have initiated mass vaccination juggernauts since December 2020. Other suggested preventive measures include social distancing, wearing face masks in public, ventilation, and air-filtering, covering one's mouth when sneezing or coughing, hand washing, disinfecting surfaces and quarantining people who have been exposed or are symptomatic. Treatments concentrate on addressing symptoms, but work is underway to develop specifics that inhibit the contagion. Authorities worldwide have responded by enforcing travel restrictions, lockdowns, business closures, workplace hazard controls, testing protocols, and systems for tracing contacts of the infected.


In this paper simulation of COVID 19 with control have been discussed by considering human population. Human population has been divided into susceptible individuals, exposed individuals, infected individuals, hospitalized individuals, death of individuals, recovered individuals, super-spreader individuals and infectious but asymptomatic individuals. Mathematical model for the transmission of COVID 19 has been described in Section 2. Section 3, 4 and 5 includes stability, control and simulation for the compartments. Conclusion is described in Section 6.

2. MATHEMATICAL MODEL
Here, we formulate a mathematical model for Human population. The notations with its description for each parameter are described in Table 1.
Table 1: Notations and its Parametric Values

<table>
<thead>
<tr>
<th>Notations</th>
<th>Description</th>
<th>Parametric Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_H$</td>
<td>New Recruitment Rate for Human</td>
<td>0.01</td>
</tr>
<tr>
<td>$\mu_H$</td>
<td>Humans Death Rate</td>
<td>0.07</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Transmission rate of humans from susceptible to exposed</td>
<td>0.01</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>Transmission rate of humans from exposed to infected</td>
<td>0.03</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>Transmission rate of humans from infected to hospitalized</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_4$</td>
<td>Transmission rate of humans from hospitalized to death</td>
<td>0.08</td>
</tr>
<tr>
<td>$\beta_5$</td>
<td>Transmission rate of humans from infected to death</td>
<td>0.08</td>
</tr>
<tr>
<td>$\beta_6$</td>
<td>Transmission rate of humans from super spreaders to death</td>
<td>0.08</td>
</tr>
<tr>
<td>$\beta_7$</td>
<td>Transmission rate of humans from super spreaders to recovered</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_8$</td>
<td>Transmission rate of humans from super spreaders to hospitalized</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_9$</td>
<td>Transmission rate of humans from infected to recovered</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_{10}$</td>
<td>Transmission rate of humans from exposed to super spreaders</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_{11}$</td>
<td>Transmission rate of humans from exposed to infectious but asymptomatic</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>Transmission rate of humans from infected to susceptible</td>
<td>0.01</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>Transmission rate of humans from super spreaders to susceptible</td>
<td>0.02</td>
</tr>
<tr>
<td>$\beta_{14}$</td>
<td>Transmission rate of humans from infectious but asymptomatic to recovered</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_{15}$</td>
<td>Transmission rate of humans from infectious but asymptomatic to super spreaders</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_{16}$</td>
<td>Transmission rate of humans from infectious but asymptomatic to infected</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_{17}$</td>
<td>Transmission rate of humans from hospitalized to recovered</td>
<td>0.05</td>
</tr>
<tr>
<td>$u_1$</td>
<td>Control rate in terms of self quarantined</td>
<td>[0,1]</td>
</tr>
<tr>
<td>$u_2$</td>
<td>Control rate in terms of isolation</td>
<td>[0,1]</td>
</tr>
<tr>
<td>$u_3$</td>
<td>Control rate in terms of vaccination</td>
<td>[0,1]</td>
</tr>
</tbody>
</table>

In the model, total population is divided in eight compartments: susceptible individuals ($S$), exposed individuals ($E$), symptomatic and infectious individuals ($I$), hospitalized individuals ($H$), dead individuals ($D$), recovered individuals ($R$), super-spreader individuals ($P$) and infectious but asymptomatic individuals ($A$).

The transmission diagram of COVID 19 from different compartments is shown in Figure 1.
Infected human \( (I) \) can infect susceptible human \( (S) \). Susceptible human infects human at the rate \( \beta_1 \). The rate of \( \beta_3 \) infected humans are seeking hospitalization so total individuals in the hospitalized compartment \( (H) \) are \( \beta_1 I \). Infectious but Asymptomatic \( (A) \) can infects human at the rate of \( \beta_{16} \). Super spreaders \( (P) \) can infect recovered human \( (R) \) at the rate of \( \beta_7 \). \( \mu_H \) denotes natural death rate of humans.

So, from the above figure 1, we have the following set of nonlinear ordinary differential equations describing the causes of COVID 19 from one compartment to other.

\[
\begin{align*}
\frac{dS}{dt} &= B_H - \beta_1 SE + \beta_{12} I + \beta_{13} P - \mu_H S \\
\frac{dE}{dt} &= \beta_1 SE - \beta_2 E - \beta_{11} E - \beta_{10} E - \mu_H E \\
\frac{dI}{dt} &= \beta_2 E - \beta_3 I - \beta_5 I - \beta_{12} I + \beta_{10} A - \mu_H I \\
\frac{dH}{dt} &= \beta_3 I - \beta_4 H - \beta_7 H + \beta_6 P - \mu_H H \\
\frac{dD}{dt} &= \beta_4 H + \beta_5 I + \beta_6 P - \mu_H D \\
\frac{dR}{dt} &= \beta_7 H + \beta_8 I + \beta_9 P + \beta_{14} A - \mu_H R \\
\frac{dP}{dt} &= \beta_{10} E + \beta_{15} A - \beta_8 P - \beta_7 P - \beta_6 P - \beta_{13} P - \mu_H P \\
\frac{dA}{dt} &= \beta_{11} E - \beta_{16} A - \beta_{14} A - \mu_H A
\end{align*}
\]
The feasible region for the above set of equations (1) is

\[
\begin{align*}
\Lambda &= \left\{ \frac{S + E + I + H + D + R + P + A}{\mu} \right\} \\
\end{align*}
\]

On equating set of equations (1) to zero, following equilibrium points are obtained:

1. **COVID 19 free equilibrium point** \(E_0\)

\[
E_0 = (S, E, I, H, D, R, P, A) = \left( \frac{B_H}{\mu_H}, 0, 0, 0, 0, 0, 0, 0 \right)
\]

2. **COVID existence equilibrium point** \(E_1\)

\[
E_1 = (S, E, I, H, D, R, P, A) = (S_0, 0, I_1, H_1, D_1, R_1, 0, 0)
\]

where,

\[
D = \frac{I \left( \beta \beta_3 + \beta_4 \left( \beta_4 + \beta_3 + \mu_H \right) \right)}{\mu_H \left( \beta + \beta_3 + \mu_H \right)} \quad \text{and} \quad H = \frac{\beta_3}{\beta_4 + \beta_3 + \mu_H} \quad \text{and} \quad R = \frac{I \left( \beta_4 \beta_5 + \beta_5 \left( \beta_4 + \beta_5 + \mu_H \right) \right)}{\mu_H \left( \beta_4 + \beta_5 + \mu_H \right)}
\]

Now, our actual interest lies in calculating the basic reproduction number which is calculated using the next generation matrix method which is defined as \(FV^{-1}\) where \(F\) and \(V\) both are Jacobian matrices of \(\mathcal{X}\) and \(v\) evaluated with respect to infected humans at the point \(E_0\).

Let \(X = (S, E, I, H, D, R, P, A)\)

\[
\frac{dX}{dt} = \mathcal{X}(X) - v(X)
\]

where \(\mathcal{X}(X)\) denotes the rate of newly recruited and \(v(X)\) denotes the rate of derived recruited which is given as

\[
\mathcal{X}(X) = \begin{bmatrix} 0 \\ \beta_1 SE \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\]

\[
v(X) = \begin{bmatrix} -B_H + \beta_1 SE - \beta_2 I - \beta_3 I + \mu_H S \\ \beta_2 E + \beta_4 E + \beta_{10} E + \mu_H E \\ -\beta_5 E_H + \beta_7 I + \beta_8 I + \beta_9 I - \beta_{16} A + \mu_H I \\ -\beta_3 I + \beta_4 H + \beta_5 H - \beta_6 P + \mu_H H \\ -\beta_4 H - \beta_5 I + \beta_6 P + \mu_H D \\ -\beta_7 I - \beta_8 I - \beta_9 P - \beta_{14} A + \mu_H R \\ -\beta_1 I - \beta_{15} A + \beta_8 P + \beta_7 P + \beta_6 P + \beta_{13} P + \mu_H P \\ -\beta_1 E + \beta_{15} A + \beta_{16} A + \beta_{14} A + \mu_H A \end{bmatrix}
\]

Now, the derivative of \(\mathcal{X}\) and \(v\) evaluated at a COVID 19 free equilibrium point \(E_0\) gives matrices \(F\) and \(V\) of order 8×8 which is defined as

\[
F = \left[ \frac{\partial \mathcal{X}(E_0)}{\partial X_j} \right] \quad V = \left[ \frac{\partial v(E_0)}{\partial X_j} \right] \quad \text{for} \ i, j = 1, 2, 3, 4, 5, 6, 7, 8
\]
\[ F = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\beta_1 B_H}{\mu_H} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

So, \[ V = \begin{bmatrix}
\mu_H & \frac{\beta_1 B_H}{\mu_H} & -\beta_{12} & 0 & 0 & 0 & -\beta_{13} & 0 \\
0 & \frac{\beta_2 + \beta_1}{\mu_H} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\beta_2 & \left( \beta_3 + \beta_6 \right) & +\beta_3 & +\mu_H & 0 & 0 \\
0 & 0 & 0 & -\beta_3 & \mu_H & 0 & -\beta_6 & 0 \\
0 & 0 & 0 & 0 & 0 & -\beta_7 & -\beta_{14} \\
0 & 0 & 0 & 0 & 0 & 0 & -\beta_{10} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_{11} \\
\end{bmatrix} \]

where \( V \) is non-singular matrix. Thus, the basic reproduction number \( R_0 \) which is the spectral radius of matrix \( FV^{-1} \) is given as

\[ R_0 = \frac{\beta_1 B_H}{\mu_H \left( \beta_1 + \beta_{10} + \beta_{11} + \mu_H \right)} \]

### 3. STABILITY ANALYSIS

In this section, the local stability at \( E_0 \) and \( E_1 \) using the linearization method and matrix analysis are to be studied.

Theorem 3.1 (stability at \( E_0 \)) The system is locally asymptotically stable at COVID 19 free equilibrium point \( E_0 \) if \( \beta_2 + \beta_{11} + \beta_{10} + \mu_H > \beta_1 \frac{B_H}{\mu_H} \).
Proof: Jacobian Matrix of the system evaluated at point $E_0$ is

$$J(E_0) = \begin{bmatrix}
-\mu_H & -\beta_1 \frac{B_H}{\mu_H} & \beta_{12} & 0 & 0 & 0 & \beta_{13} & 0 \\
0 & -A_i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_2 & -A_2 & 0 & 0 & 0 & 0 & \beta_{16} \\
0 & 0 & \beta_3 & -A_3 & 0 & 0 & \beta_8 & 0 \\
0 & 0 & \beta_5 & \beta_4 & -\mu_H & 0 & \beta_6 & 0 \\
0 & 0 & \beta_9 & \beta_{17} & 0 & -\mu_H & \beta_7 & \beta_{14} \\
0 & \beta_{10} & 0 & 0 & 0 & 0 & -A_4 & \beta_{15} \\
0 & \beta_{11} & 0 & 0 & 0 & 0 & 0 & -A_5 \\
\end{bmatrix}$$

where, $A_i = \beta_2 + \beta_{11} + \beta_{10} + \mu_H - \beta_1 \frac{B_H}{\mu_H}$, $A_2 = \beta_3 + \beta_5 + \beta_{12} + \mu_H$, $A_3 = \beta_4 + \beta_{17} + \mu_H$, $A_4 = \beta_4 + \beta_5 + \beta_{13} + \mu_H$, $A_5 = \beta_{15} + \beta_{16} + \beta_{14} + \mu_H$.

The eigen values of the characteristic equation are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$ which satisfies the equation

$$a_0\lambda^8 + a_1\lambda^7 + a_2\lambda^6 + a_3\lambda^5 + a_4\lambda^4 + a_5\lambda^3 + a_6\lambda^2 + a_7\lambda + a_8 = 0$$

The coefficients of characteristic polynomial are positive if $\beta_2 + \beta_{11} + \beta_{10} + \mu_H > \beta_1 \frac{B_H}{\mu_H}$.

Theorem 3.2 (stability at $E_1$) The system is locally asymptotically stable at COVID existence equilibrium point $E_1$ if

$\beta_2 + \beta_{11} + \beta_{10} + \mu_H > \beta_1 S_1$.

Proof: Jacobian Matrix of the system evaluated at point $E_1$ is

$$J(E_1) = \begin{bmatrix}
-\mu_H & -\beta S_1 & \beta_{12} & 0 & 0 & 0 & \beta_{13} & 0 \\
0 & -A_6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \beta_2 & -A_7 & 0 & 0 & 0 & 0 & \beta_{16} \\
0 & 0 & \beta_3 & -A_8 & 0 & 0 & \beta_8 & 0 \\
0 & 0 & \beta_5 & \beta_4 & -\mu_H & 0 & \beta_6 & 0 \\
0 & 0 & \beta_9 & \beta_{17} & 0 & -\mu_H & \beta_7 & \beta_{14} \\
0 & \beta_{10} & 0 & 0 & 0 & 0 & -A_4 & \beta_{15} \\
0 & \beta_{11} & 0 & 0 & 0 & 0 & 0 & -A_{10} \\
\end{bmatrix}$$

where, $A_6 = \beta_2 + \beta_{11} + \beta_{10} + \mu_H - \beta_1 S_1$, $A_7 = \beta_3 + \beta_5 + \beta_{12} + \mu_H$, $A_8 = \beta_4 + \beta_{17} + \mu_H$, $A_9 = \beta_4 + \beta_5 + \beta_{13} + \mu_H$, $A_{10} = \beta_{15} + \beta_{16} + \beta_{14} + \mu_H$.

The eigen values of the characteristic equation are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8$ which satisfies the equation

$$a_0\lambda^8 + a_1\lambda^7 + a_2\lambda^6 + a_3\lambda^5 + a_4\lambda^4 + a_5\lambda^3 + a_6\lambda^2 + a_7\lambda + a_8 = 0$$

where

The coefficients of characteristic polynomial are positive if $\beta_2 + \beta_{11} + \beta_{10} + \mu_H > \beta_1 S_1$. 

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4. OPTIMAL CONTROL MODEL

In this section, a control function has been implemented to decrease the spread of COVID 19 in human population. The objective function along with the optimal control variable is given by

\[
J(u, \Omega) = \int_0^T \left( A_2 S^2 + A_2 E^2 + A_3 I^2 + A_4 H^2 + A_5 D^2 + A_6 R^2 + A_7 P^2 + A_8 A^2 \\
+ w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2 \\
+ \lambda_S \left[ B_{11} - \beta_S E + \beta_{13} I + \beta_{14} P - \mu_S S \right] \\
+ \lambda_E \left[ \beta_S E - \beta_E E - \beta_{11} E - \beta_{10} E - \mu_E E - u_1 E \right] \\
+ \lambda_I \left[ \beta_E E - \beta_S I - \beta_{13} I - \beta_{14} I + \beta_{16} A - \mu_I I + u_1 E + u_2 A \right] \\
+ \lambda_H \left[ \beta_H H - \beta_I H - \beta_{17} H + \beta_{18} P - \mu_H H - u_3 H \right] \\
+ \lambda_J \left[ \beta_I H - \beta_J H + \beta_{18} P - \mu_J A - u_3 A \right] \\
+ \lambda_D \left[ \beta_J H + \beta_I A + \beta_{17} A - \mu_J A - u_3 A \right] \\
+ \lambda_P \left[ \beta_{19} E + \beta_{15} A - \beta_E P - \beta_I A + u_4 A + u_5 P \right] \\
+ \lambda_R \left[ \beta_{19} E + \beta_{15} A - \beta_E P - \beta_I A + u_4 A + u_5 P \right] \\
+ \lambda_A \left[ \beta_{19} E + \beta_{15} A - \beta_I A + u_4 A + u_5 P \right]
\]

where \( \Omega \) denotes the set of all compartmental variables, \( A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8 \) denotes non-negative weight constants for the compartments \( S, E, I, H, D, R, P, A \) respectively and \( w_1, w_2, w_3 \) are the weight constant for the control variable \( u_1, u_2, u_3 \) respectively.

The weights \( w_1, w_2, w_3 \) which are constant parameters for \( u_1, u_2, u_3 \) will standardized using the optimal control condition.

Now, we will calculate the value of control variables \( u_1, u_2, u_3 \) from \( t = 0 \) to \( t = T \) such that

\[
J(u_1(t), u_2(t), u_3(t)) = \min \{ J(u_1^*, \Omega) / u_1^*, u_2^*, u_3^* \in \phi \}
\]

where \( \phi \) = smooth function on the interval \([0,1]\).

Using, Fleming and Rishel results, the optimal control denoted by \( u_i^* \) is obtained by collecting all the integrands of the objective function using the lower bounds and upper bounds of the both the control variables respectively.

Using Pontrygin’s principle, we construct a Lagrangian function consisting of state equation and adjoint variables \( A_v = (\lambda_S, \lambda_E, \lambda_I, \lambda_H, \lambda_D, \lambda_P, \lambda_A) \) which is as follows:

\[
L(\Omega, A_v) = A_2 S^2 + A_2 E^2 + A_3 I^2 + A_4 H^2 + A_5 D^2 + A_6 R^2 + A_7 P^2 + A_8 A^2 \\
+ w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2 \\
+ \lambda_S \left[ B_{11} - \beta_S E + \beta_{13} I + \beta_{14} P - \mu_S S \right] \\
+ \lambda_E \left[ \beta_S E - \beta_E E - \beta_{11} E - \beta_{10} E - \mu_E E - u_1 E \right] \\
+ \lambda_I \left[ \beta_E E - \beta_S I - \beta_{13} I - \beta_{14} I + \beta_{16} A - \mu_I I + u_1 E + u_2 A \right] \\
+ \lambda_H \left[ \beta_H H - \beta_I H - \beta_{17} H + \beta_{18} P - \mu_H H - u_3 H \right] \\
+ \lambda_J \left[ \beta_I H - \beta_J H + \beta_{18} P - \mu_J A - u_3 A \right] \\
+ \lambda_D \left[ \beta_J H - \beta_I A + \beta_{17} A - \mu_J A - u_3 A \right] \\
+ \lambda_P \left[ \beta_{19} E + \beta_{15} A - \beta_E P - \beta_I A + u_4 A + u_5 P \right] \\
+ \lambda_R \left[ \beta_{19} E + \beta_{15} A - \beta_I A + u_4 A + u_5 P \right] \\
+ \lambda_A \left[ \beta_{19} E + \beta_{15} A - \beta_I A + u_4 A + u_5 P \right]
\]

Now, the partial derivative of the Lagrangian function with respect to each variable of the compartment gives us the adjoint equation such that

\[
\dot{\lambda}_S = -\frac{\partial L}{\partial S} = -2A_2 S + (\beta_S E)(\lambda_S - \lambda_E) + \mu_S \lambda_S \\
\dot{\lambda}_E = -\frac{\partial L}{\partial E} = -2A_2 E + (\beta_S S)(\lambda_S - \lambda_E) + (\beta_S + u_1)(\lambda_E - \lambda_I) + \beta_{14} \left( \lambda_E - \lambda_A \right) + \beta_{10} \left( \lambda_E - \lambda_P \right) + \mu_E \lambda_E \\
\dot{\lambda}_I = -\frac{\partial L}{\partial I} = -2A_2 I + \beta_{13} \left( \lambda_I - \lambda_S \right) + (\beta_I + u_2)(\lambda_S - \lambda_I) + \beta_{14} \left( \lambda_I - \lambda_D \right) + \beta_{10} \left( \lambda_I - \lambda_R \right) + \mu_I \lambda_I \\
\dot{\lambda}_H = -\frac{\partial L}{\partial H} = -2A_2 H + \beta_{17} \left( \lambda_H - \lambda_D \right) + (\beta_{17} + u_3)(\lambda_D - \lambda_R) + \mu_H \lambda_H \\
\dot{\lambda}_D = -\frac{\partial L}{\partial D} = -2A_2 D + \mu_H \lambda_D
\]
\[ \dot{\lambda}_R = -\frac{\partial L}{\partial R} = -2A_i R + \mu_H \lambda_R \]
\[ \dot{\lambda}_P = -\frac{\partial L}{\partial P} = -2A_i P + \beta_3 (\lambda_P - \lambda_S) + \beta_6 (\lambda_P - \lambda_D) + \beta_4 (\lambda_P - \lambda_R) + \mu_H \lambda_P \]
\[ \dot{\lambda}_A = -\frac{\partial L}{\partial A} = -2A_i A + \beta_5 (\lambda_A - \lambda_P) + (\beta_{10} + u_2) (\lambda_A - \lambda_T) + \beta_{13} (\lambda_A - \lambda_R) + \mu_H \lambda_A \]

The necessary conditions for Lagrangian function \( L \) to be optimal for control are
\[
\frac{\partial L}{\partial u_1} = 2w_1 u_1 + E(\lambda_E - \lambda_E) = 0 \quad (3) \\
\frac{\partial L}{\partial u_2} = 2w_2 u_2 + A(\lambda_A - \lambda_A) = 0 \quad (4) \\
\frac{\partial L}{\partial u_3} = 2w_3 u_3 + H(\lambda_R - \lambda_R) = 0 \quad (5)
\]

On solving equation (3), (4) and (5) we get,
\[ u_1 = \frac{E(\lambda_E - \lambda_E)}{2w_1}, \quad u_2 = \frac{A(\lambda_A - \lambda_A)}{2w_2}, \quad u_3 = \frac{H(\lambda_R - \lambda_R)}{2w_3} \]

Hence, the required optimal control condition is obtained as
\[
U^*_1 = \max \left\{ a_1, \min \left( \frac{b_1, E(\lambda_E - \lambda_E)}{2w_1} \right) \right\}, \quad U^*_2 = \max \left\{ a_2, \min \left( \frac{b_2, A(\lambda_A - \lambda_A)}{2w_2} \right) \right\}, \quad U^*_3 = \max \left\{ a_3, \min \left( \frac{b_3, H(\lambda_R - \lambda_R)}{2w_3} \right) \right\}
\]

where \( a_1, a_2, a_3 \) = lower bounds and \( b_1, b_2, b_3 \) = upper bounds of the control variables \( u_1, u_2, u_3 \) respectively.

5. **NUMERICAL SIMULATION**

In this section, we will analyze and study the effect of control on each compartment numerically.

![Figure 2(a): Impact of without control on human population](image-url)
Figure 2(b): Impact of control on human population

Figure 2(a) and 2(b) suggests the impact of without and with control on human population. It can be observed that when control is applied number of infected individuals decreases from approximately 6 to 4 in comparison to without control. Hospitalized individuals are gradually decreasing from approximately 3 to 2 and recovered individuals are increased from approximately 4 to 5 which means they have recovered after control applied. It can be seen from the figure that number of deaths are also decreased in comparison to without control.

Figure 3: Control in terms of preventive measures on $E$ compartment

From figure 3, it can be seen that when control applied, then exposed human decreases.
From figure 4, it can be seen that when an individual gets medical treatment along with preventive measures, then infected human decreases.

Figure 5 depicts that when control is applied, hospitalized individuals decreases from 8 to 1.5 approximately.
From figure 6, it can be seen that recovered individuals increases when control applied.

From figure 7, it can be seen that when control applied, then infected but asymptomatic individuals decreases.
From figure 8, we can see that super spreader individuals decreases with controls.

Figure 9: Control Variables versus Time (in days)

Figure 9 show that each control has a vital role in protecting individuals from COVID 19. It is seen that 38% self-quarantine and isolation, 60% of vaccination are required in the initial days to stop one from becoming the victim of COVID 19. The high fluctuation in $u_3$ control variable at an initial stage suggests that vaccination should be applied to control further transmission of COVID-19.
From figure 10, we can see that initially approximately 99 individuals reduces to 86% approximately in two weeks duration but not vanishes so preventive measures are advocated to control the spread of COVID 19.

6. CONCLUSION

In this paper, a mathematical model has been constructed to study the spread of corona virus. It has been established that the number of COVID 19 affected individual can be reduced by using control on them. Two equilibrium points have been found: COVID 19 free equilibrium point and COVID 19 existence equilibrium points. At these points, system proves to be locally asymptotically stable. Results for different compartments have been calculated numerically which interprets that to reduce overall severity of the COVID 19, increase self quarantine, isolation, vaccination and preventive measures at the end of humans are required.

REFERENCES

[12] https://www.who.int/health-topics/coronavirus#tab=tab_1