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Sound Propagation Analysis in Porous Fibrous Acoustic Absorbers Using Johnson-Champoux-Allard Lafarge (JCAL) Model & Delaney Bazley Model

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Abstract

The characterization of the porous passive acoustic absorber for its acoustic and non-acoustic macroscopic properties is important in utilizing these materials for its passive noise reduction applications. The effective density and dynamic bulk modulus of the porous absorbers are determined using the characteristic impedance and propagation number predicted using the Delaney Bazley Modelling of the fibrous passive absorber. The tortuosity, thermal characteristic length and viscous characteristic length are in turn extracted from the real and imaginary parts of this effective density and dynamic bulk modulus. The expressions for effective density and dynamic bulk modulus according to Johnson-Chompoux Allard-Lafarge Model (JCAL) are used for this purpose. A typical porous fibrous layered passive absorber is selected as a specimen for the calculation of the above quantities.

Keywords: Viscous Characteristic length, Thermal Characteristic Length, Tortuosity, Characteristic Impedance, Porosity, Air flow resistivity.

INTRODUCTION

The important parameters defining the acoustic characteristics of porous passive sound absorbers are characteristic impedance, propagation constant, reflection coefficient, absorption coefficient, transmission coefficient, transmission loss etc. These properties can be measured using two-microphone, three-microphone or four-microphone impedance tube set up[1-2].

These properties are estimated using the transfer function and the elements of the transfer matrix which relates the input and output acoustic field with respect to the specimen. These physical quantities are related to the intrinsic and extrinsic properties of the specimen under consideration. The frequency response of the acoustic characteristics depends on the porosity, viscous characteristic length, thermal characteristic length, air flow resistivity, tortuosity and thickness of the specimen. The effective density and dynamic bulk modulus which takes into account the visco-inertial and thermal losses inside the porous material are functions of the above mentioned parameters[3-5]. The impedance tube measurement can be used to extract the effective density and dynamic bulk modulus. The parameters defining the effective density and dynamic bulk modulus can be determined using direct measurement and based on the calculations of indirect method and inversion method relying on the impedance tube measurements [6-7].

Acoustic and Non-Acoustic Properties in terms of the the Model

When the acoustic field travels through a porous material medium it suffers energy loss because of the different loss mechanisms happening in it during the propagation. The viscous and thermal loss associated with the acoustic field in a porous material is related to its effective density and dynamic bulk modulus which are functions of the intrinsic macroscopic properties of the bulk material. Different models have been proposed to explain the relationship of these macroscopic properties of the medium on the above quantities. The expressions showing the dependence of the macroscopic properties like the porosity, airflow resistivity, tortuosity, thermal and viscous characteristic lengths on the effective density and dynamic bulk modulus of the porous material are given by Johnson- Chompoux–Allard-Lafarge (JCAL) model [3-5]. The expression for effective density $\rho_{eff}(\omega)$ and dynamic bulk modulus $K_{eff}(\omega)$ are given by equations (a) and (b).

$$\rho_{eff}(\omega) = \frac{\alpha\rho_0}{\varepsilon}\left[1 + \frac{\sigma\varepsilon}{j\omega\rho_0\alpha} + \left[1 + j\,\frac{4\alpha\eta\rho_{0\omega}}{\sigma^2\,\Lambda^2\,\varepsilon^2}\right]^{1/2}\right] \quad \text{--- (a)}$$

$$K_{eff}(\omega) = \frac{\gamma P_0/_{\varepsilon}}{\gamma-(\gamma-1)\left[1-j\frac{8k_c}{\Lambda'^2\,c_p\rho_0\,\omega}\left[1+j\frac{\Lambda'^2\,c_p\rho_0\,\omega}{16k_c}\right]^{1/2}\right]^{-1}} \quad \text{--- (b)}$$

Where α is the tortuosity of the porous medium, ρ_0 is the density of air, ε is the porosity, σ is the air flow resistivity, ω is the frequency of sound, Υ is the ratio of specific heats, k_c is the wave propagation number inside the medium, c_p is the specific heat at constant pressure of the ambient medium, P_0 is the atmospheric pressure, Λ is the viscous characteristic length and Λ' is the thermal characteristic length of the porous medium.

The effective density and dynamic bulk modulus are related to the frequency of sound ω, and the characteristic impedance Z_c and propagation number k_c of the porous medium through the relations given by equation (1) and equation (2).

$$\rho_{eff} = \frac{Z_c k_c}{\omega} \text{-----(1)}$$

$$K_{eff} = \frac{\omega Z_c}{k_c} \text{-----(2)}$$

The characteristic impedance Z_c and wave propagation number k_c measured using the impedance tube apparatus can be utilized for the determination of the effective density ρ_{eff} and dynamic bulk modulus K_{eff} of the porous passive acoustic material.

In this particular modelling studies the modelling of fibrous passive acoustic absorbers are done using the emperical model proposed by Delaney and Bazley [8]. The characteristic acoustic impedance and wave propagation number acccording to this emperical model in the case of fibrous sound absorbing material media is given by the equations (3) and (4).

$$Z_c = \left\{ p_0\, c(1 + 0.0571 \left(\frac{p_0 f}{\sigma}\right)^{-0.754} - j0.087 \left(\frac{p_0 f}{\sigma}\right)^{-0.732} \right\} \text{---(3)}$$

$$K_c = \frac{\omega}{c} \left\{ 1 + 0.0978 \left(\frac{p_0 f}{\sigma}\right)^{-0.7} - j0.189 \left(\frac{p_0 f}{\sigma}\right)^{-0.595} \right\} \text{---(4)}$$

Where p_0 is the density of the ambient fluid medium air, f is the frequency of sound wave, σ is the air flow resisitivity of the material. The limiting condition for the validity of the above relation is $0.001 \leq \left(\frac{p_0 f}{\sigma}\right) \geq 1$. There are various emperical formulas proposed for determining the air flow resistivity of the fibrous media depending on the value of the fiber diameter. The relation for calculating the air flow resistivity is given by the following expressions [9-10].

$$\sigma = \frac{10.56\eta(1-\varepsilon)^{1.531}}{a^2\varepsilon^3} \text{---(5)}$$

$$\sigma = \frac{6.8\eta(1-\varepsilon)^{1.296}}{a^2\varepsilon^3} \text{---(6)}$$

Where η is the viscosity of the ambient medium and ε is the porosity of the medium and a is the radius of the fiber constituting the material medium. The expression(5) is valid for determining the air flow resisitivty for fiber radius varying in the range of 6 to 10μm and the expression(6) is valid for determining the same for the fiber radius in the range of 20-30μm. The porosity ε of the material medium is determined by the expression

$$\varepsilon \approx \frac{\rho}{\rho_f} \text{---(7)}$$

where ρ is the density of the bulk material and ρ_f is the fiber density.

The thermal characteristic length, viscous characteristic length, tortuosity and air flow resistivity are related to the effective density and dynamic bulk modulus of the material medium as is evident from Jhonson Chompoux Allard Lafarge model (JCAL) by equations (a) and (b). These parameters can be calculated from the real and imaginary parts of effective density and bulk modulus knowing the density, bulk modulus and viscosity of the ambient medium [3-5]. The viscous characteristic length, thermal characteristic length and totuosity are determined by the following expressions proposed by Jhonson et al viscous model and Lafarge et al thermal model.

$$\alpha = \frac{1}{\rho_0}\left[Re(\rho_{eff}) - \left\{ Im(\rho_{eff})^2 - \left(\frac{\sigma\varepsilon}{\omega}\right)^2 \right\}^{1/2} \right] \text{---(8)}$$

$$\Lambda = \alpha \left[\frac{2\rho_0\eta}{\omega\, Im(\rho_{eff})(\rho_0\,\alpha - Re(\rho_{eff}))} \right]^{1/2} \text{---(9)}$$

$$\Lambda' = \delta_t\sqrt{2} \left[-Im\left(\left(\frac{\varepsilon - \frac{K_{eff}}{K}}{\varepsilon - \gamma\frac{K_{eff}}{K}} \right)^2 \right) \right]^{-1/2} \text{---(10)}$$

$$\delta_t = \left[\frac{2\eta}{\rho_0\omega P_r} \right] \text{---(11)}$$

Where α is the tortuosity, ρ_0 is the density of the ambient medium, Re(ρ_{eff}) is the real part of the effective density, Im(ρ_{eff}) is the imaginary part of the effective density, σ is the air flow resistivity, ε is the porosity of the material, ω the frequency of sound, ʌ is the viscous characteristic length, η is the viscosity, ʌ ' is the thermal characteristic length, K_{eff} is the effective bulk modulus, K is the bulk modulus of air, γ is the ratio of specific heats of air, P_r is the prandtl number of air.

Acoustic characterization of the porous fibrous acoustic absorber for the and estimation of Tortuosity, Thermal Characteristic Length and Viscous Characteristic Length

In this particular modelling studies the complex effective density and bulk moduls given by expressions (1) and (2) are determined using the characteristic impedance Z_c, and the wave propagation number k_c determined with the help of Delaney Bazley modelling. For this purpose cotton fiber based porous layer passive acoustic absorber is used as the specimen sample. The values of Z_c and k_c are determined using the values of air flow resistivity and porosity. The porosity is determined using equation(7) by substituting the estimated value of the bulk density of the sample and the standeard value of density of the fiber. The radius of the fiber is found out experimentally using the CCD imaging of the same. Knowing the values of the fiber radius and porosity of the material the air flow resistivity of the material is determined using either the expression (5) or (6) according to the value of radius of the fiber. After determining the frequency dependent values of the characteristic impedance Z_c and the wave propagation number k_c for the porous sound absorbing medium using equations (3) and (4) they can be utilized for the determination of effective density and dynamic bulk modulus of the material medium using the expressions given by equations (1) and (2). Once the values of σ, ε, ρeff and Keff are determined, the thermal characteristic length, viscous characteristic length and tortuosity can be evaluated using the same with the help of equations (8)-(11). The frequency response of these quantities are determined in this manner using the Delaney Bazely Model in combination with the analytical solutions provided by the Jhonson-Chompoux –Allard-Lafarge model(JCAL) in the frequency range of 0-6000Hz. The average value of Tortuosity, Viscous Characteristic Length and Thermal Characteristic Length are taken in the frequency range corresponding to which these quantities remain almost constant without much variation.

RESULTS AND DISCUSSION

The frequency resposnse of the real and imaginary part of the effective density ρ_{eff} of the sample are shown in figure.1 and that of the dynamic bulk modulus is shown in figure.2. With increase in the value of frequency the real part of the effective density decreases initially and approches a constant value in the higher frequency range. The imaginary part of the effective density also approaches a constant value in the higher frequency range. In the higher frequency range the imaginary part of the effective density has a vanishing trend. The real part of the effective bulk modulus shows an increasing trend in the lower frequency region and appraches a constant value in the higher frequency region. Whereas the imaginary part of the effective bulk modulus decreases with frequecny and approaches to a constant value in the higher frequency region. The variation of both the real and imaginary parts of the same are shown in figure.2. The details of the calculated values of totuosity,viscous characteristic length, thermal characteristic length,porosity, airflow resisitivity of the sample are shown in the table.I. Thus all the parameters defining the effective density and dynamic bulk modulus are estimated using the the values of the characteristic impedance and propagation constant estimated using the Delaney Bazley model.

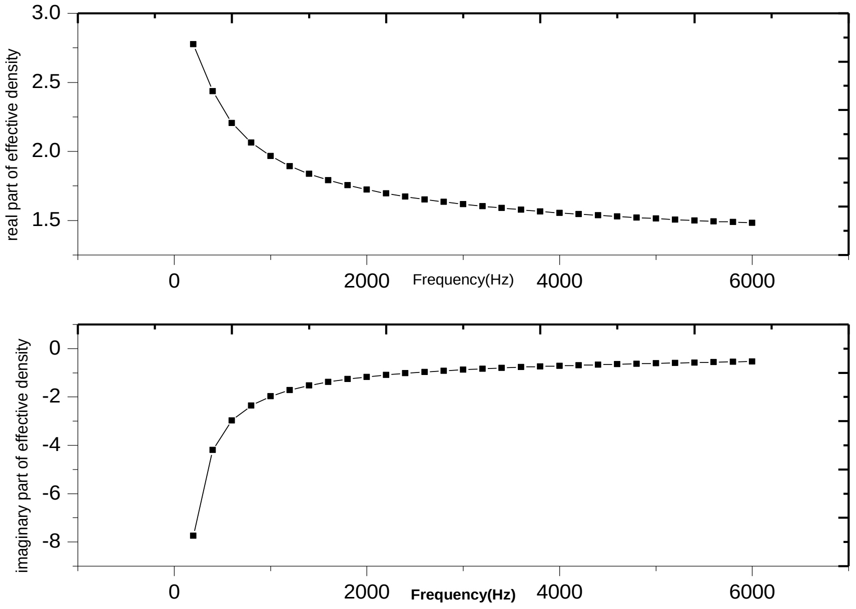

Figure.1. Variation of the real and imaginary part of the effective density with respect to frequency.

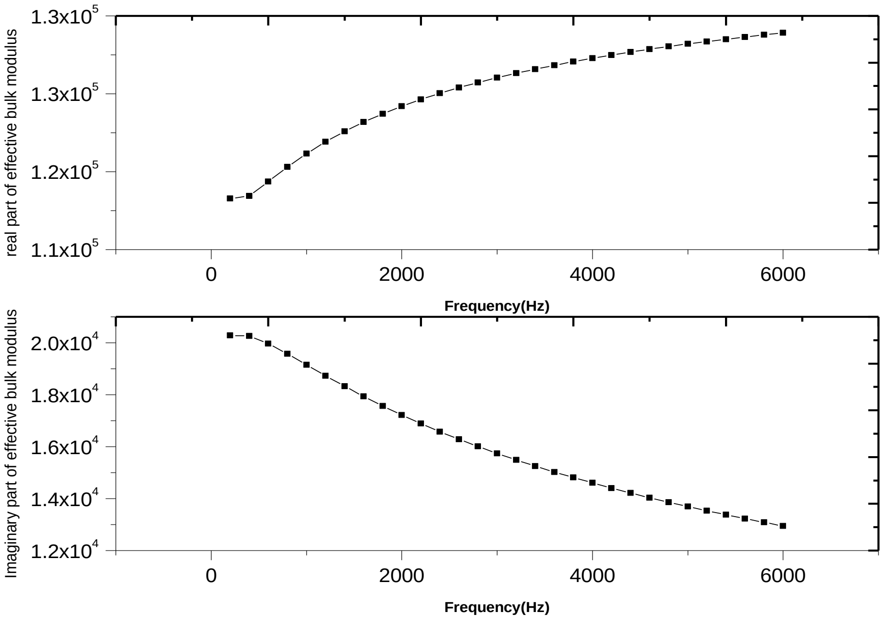

Figure.2. Variation of the real and imaginary part of the dynamic bulk modulus with respect to frequency.

Table.1. The estimated values of the properties defining the JCAL model.

Macroscopic Property	Estimated Value using the Model
Air Flow resistivity	$1.7339\text{x}10^{-4}\text{Nsm}^{-4}$
Porosity	0.9775
Tortuosity	1.0957
Thermal Characteritic Length	208.48µm
Viscous Characteristic Length	102.12µm

CONCLUSION

The frequency response of the characteristic impedance Z_c and the wave propagation number k_c of the selected specimen of cotton fiber based porous acoustic absorber is determined using Delaney Bazley model. This characteristic impedance and propagation number is used for estimating the frequency response of the effective density and dynamic bulk modulus. The real and imaginary parts of both of these two quantities approches a constant value in the high freqency range. The effective density and dynamic bulk modulus in turn deliver the non-acoustic parameters namely the tortuosity,viscous characteristic length and thermal charactersitic length. The parameters determined using the above models can be verified using the values of the characteristic impedance and propagation number estimated from the impedance tube measuremnt. Thus Delaney Bazley model blended with JCAL model can be utilized for the calculation of the acoustic and nonacoustic paramerts of the porous passive acoustic absorber.

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