Performance Analysis of Sparse Unmixing for Hyperspectral Data

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Abstract
In remote sensing, Hyperspectral Cameras (HSCs) are generally mounted on satellite or on airborne vehicles. Due to the high altitude of HSCs, the spatial resolution of hyperspectral images (HSI) is very poor, in order of $4m \times 4m$ to $20m \times 20m$. HSC measures electromagnetic energy scattered in their instantaneous field of view (IFOV) in hundreds of thousands of spectral bands with very high spectral resolution. HSC generally captured 224 images in the spectrum 0.4µm to 2.5µm with the spectral resolution of 10nm. Due to the poor spatial resolution more than one material present in a one pixel. The process of accurate estimation of a number of material present in the scene, known as endmembers, their spectral signature, and their corresponding fractional abundance is known as Hyperspectral Unmixing (HU). Hyperspectral unmixing enables different applications like agriculture assessment, anomaly detection, manmade materials identification, environmental monitoring, target identification, and change recognition.

Spectral Unmixing by Variable Splitting and Augmented Lagrangian (SUnSAL) and Collaborative SUnSAL (CLSUnSAL) approach are sparsity-based unmixing algorithms. In this paper performance analysis of SUnSAL and CSUnSAL is done with the help of publically available spectral libraries and synthetically generated hyperspectral data cubes. Simulation results shows that performance of CLSUnSAL is better SUnSAL.

Keywords: Hyperspectral unmixing, ADMM, SUnSAL, CLSUnSAL, spectral signature, spectral libraries, sparse unmixing, $l_1 - l_2$ sparsity

I. INTRODUCTION
Remote Sensing (RS) is the field of science for obtaining information about objects or area, from satellite or aircraft. Hyperspectral remote sensing is a relatively new technique that is currently being investigated by scientist and researchers. It is also known as imaging spectroscopy. Hyperspectral Cameras (HSC) are generally mounted on satellite or on airborne vehicles and contributes important role to earth observation and remote sensing. HSC measures Electromagnetic Radiation scattered in their Instantaneous Field Of View (IFOV) in hundreds or thousands of spectral bands with a very high spectral resolution of 10nm. The hyperspectral image has three dimensional, two spatial dimensions and one spectral dimension. It contains more than 200 bands and it covers most electromagnetic spectrum range from 0.4 to 2.5µm. The hyperspectral image in all number of a band is adjacent to each other. So we consider this particular band is contiguous. It can be a stake of 200 images and each mixed pixel reflection of a geographical area at a specific wavelength.

Due to the high altitude of Hyperspectral Cameras (HSCs), the spatial resolution of Hyperspectral Images (HSI) is very poor, in order of $4m \times 4m$ to $20m \times 20m$. Due to poor spatial resolution, more than one material present in a single pixel of an image so it is not possible to finding the number of material present in one pixel but the hyperspectral image makes it possible. The information which is not extracted by the human eye can be extracted by spectral imaging. Spectral signature observed by the camera of the mixed pixel is considered as a linear combination of several spectral signatures of materials contain in that pixel. High spectral resolution enables the accurate estimation of a number of materials present in the scene, known as endmembers, their spectral signature and fractional proportion within the pixel, known as abundance map. This process is known as Hyperspectral Unmixing (HU) [1]. Due to large data size, environmental noise, poor spatial resolution, endmember variability, not the availability of pure endmembers hyperspectral unmixing is an ill-posed problem and challenging task. HU empower variety applications like agricultural assessment, environmental monitoring, ground cover classification, mineral exploitation, change detection, target detection, and surveillance.

There are several approaches to tackle the problem of Hyperspectral unmixing; Geometrical, Statistical and Sparse regression-based approaches. Geometrical and statistical are Blind Source Separation (BSS) kind of approaches [1]. As per standard literature it has been well found that Pixel Purity Index (PPI) [11], Nonnegative Matrix Factorization (NMF) [12] and Vertex Component Analysis (VCA) algorithm [13] with some constraints related to HSI are the best solutions for a linear spectral unmixing problem. In a sparse regression approach, the problem of HU is simplified to finding the optimal subset of signatures from (potentially very large) a spectral library that can best model each mixed pixel in the scene [2]. The sparsity-based technique assumes the availability of standard and publically available spectral libraries. It is a semi-blind approach. The approach of sparse regression is Spectral Unmixing via variable Splitting and Augmented Lagrangian (SUnSAL) and Collaborative Spectral Unmixing via variable Splitting and Augmented Lagrangian (CLSUnSAL) [5]. A spectral library contains the spectral signatures of thousands of materials available on the earth surface. For example, The Airborne Visible Infra-Red Imaging Spectrometer (AVIRIS) Spectral library contains 2400 spectral signatures [15] and the United States Geological Survey (USGS) [14], which contains over 1300 spectral
signatures are standard spectral libraries for researchers. In this paper, we discuss the linear mixing model, sparse unmixing, Signal to Reconstruction Error (SRE) is observed in SUNSAL and CLSUcSAL sparsity-based algorithm analyses. The simulation result is performed on the synthetic dataset.

II. LINEAR MIXING MODEL

A material can reflect, absorb and emit electromagnetic radiation according to their molecular size and structure [1]. The spectral signature of material is used to uniquely identify material present in the scene. The Linear Mixing Model (LMM) assumes that the spectral response of a pixel in any given spectral band is a linear combination of all of the endmembers present in the pixel at the respective spectral band [1]. Consider a single pixel of an HSI consisting of three different materials. Spectral signatures of materials present in the pixel are \( m_1, m_2 \) and \( m_3 \). The fractional proportion of the materials are \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) respectively. In HSI if pixel contains only one material then it is known as a pure pixel and if it contains more than one materials than known as mixed pixel. As per LMM, the observed spectral signature of the linear combination of spectral signatures of constituent materials. i.e. \( \alpha_1m_1, \alpha_2m_2 \) and \( \alpha_3m_3 \).

For each pixel of the three-dimensional hyperspectral data cube, the mathematically LMM can be expressed as

\[
y_i = \sum_{j=1}^{q} m_{ij} \alpha_j + n_i \tag{1}
\]

Where, the subscript \( i \) represents the number of spectral band and subscript \( j \) represent number of endmember from endmember matrix \( M \). The length of observed vector \( y \) is \( L \), i.e. \( i = 1, 2, \ldots, L \) and endmember number in endmember matrix are \( q \), i.e. \( j = 1, 2, \ldots, q \) represents the reflectance at spectral band \( j \) of \( j \)th endmember, \( \alpha_j \) represent the fractional abundance of the \( j \)th endmember in a pixel, and \( n_i \) denoted by the error term for the \( j \)th spectral band. If suppose the hyperspectral sensor used in data acquisition has spectral bands \( L \), equation (1) can be rewritten in compact matrix form such as [2]

\[
Y = Ma + n \tag{2}
\]

Where \( Y \in R^{L \times 1} \) observed the spectrum of the single pixel \( M \in R^{L \times q} \) the q pure spectrum signatures(endmembers) \( \alpha \in R^{q \times 1} \) the fractional abundances of the endmembers.

The value of fractional abundance is always nonnegative, lie in the range of 0 to 1 and the sum of its values for a single pixel is always one. These are abundance sum to one constraint(ASC) and abundance non-negative constant (ANC) [2], which are written in compact form as

\[
ANC: \alpha_j \geq 0 \tag{3}
\]

\[
ASC: \sum_{j=1}^{q} \alpha_j = 1 \tag{4}
\]

In the linear mixing model given hyperspectral image \( Y \) and our objective is to estimate the number of material, their spectral signature and their fractional abundance for each pixel is \( q, M \) and \( \alpha \) of the hyperspectral image.

III. SPARSITY BASED ALGORITHMS

Recently the availability of spectral library enables the representation of mixed spectral signature as a linear combination of several spectral signatures available in the library. This approach is also known as semi-blind because we assume that the constituent spectral signatures are from the library. In Sparse Unmixing the endmember matrix is the subset of spectral signature found in a spectral library that can be better for each mixed pixel in the scene [2]. The linear sparse regression is an exceptionally dynamic field of research that is connected to compressive sensing.

**Fig. 1.** Linear mixing and unmixing present at the pixel

As we discuss above in sparse unmixing endmember are not finding from the hyperspectral image. In the sparse unmixing large number of the spectral signature are denoted by \( A \in R^{L \times m} \). Where \( L \) is the number of bands and \( m \) is the number of endmember in a spectral library.
Let us assume that \( m \) endmembers available in spectral library A. In the compressive sensing, from \( m \) samples the original signal \( x \) can be reconstructed without any information loss. Number of the non-zero element is \( k \), \( k \) is less than material present in the scene, where \( k << m \). The fractional abundance vector for one pixel is \( y_{ij} \) can be expressed as \( k \) sparse because most of the elements of the vector are zero, so vector \( x \) must be \( k \) sparse.

A. Sparse Unmixing via variable Splitting and Augmented Lagrangian (SUuSAL)

In SUuSAL algorithm, given hyperspectral data cube, we used pixel sparsity for finding non zero elements. In this, we use the \( l_1 \) norm. As shown in the figure, the spectral signature of a single pixel, denoted by \( y \), spectral signature \( A \), and the fractional abundance map denoted by \( x \) is the solution of below optimization problem.

\[
\min_x ||AX - Y||_2^2 + \lambda||X||_1 + \gamma R(x) 
\]

where \( \gamma R(x) \) = indicator function

Applying the concept of variable splitting as,

\[
\min_{U, V_1, V_2, V_3} \frac{1}{2} ||V_1 - Y||_2^2 + \lambda||V_2||_1 + \gamma R(V_3)
\]

Subject to \( V_1 = AU \) \( V_2 = U \) \( V_3 = U \)

In compact form,

\[
\min_{U, V} g(V)
\]

Subject to \( GU + BV = 0 \)

Where,

\[
g(V) = \frac{1}{2} ||V_1 - Y||_2^2 + \lambda||V_2||_1 + \gamma R(V_3)
\]

In which \( V = (V_1, V_2, V_3) \)

Given that \( V_1 = AU \) \( V_2 = U \) \( V_3 = U \)

Using this we can write

\[
V_1 = AU \Rightarrow AU - V_1 = 0 \\
V_2 = U \Rightarrow U - V_2 = 0 \\
V_3 = U \Rightarrow U - V_3 = 0
\]

Using above equation, we can write in matrix form like,

\[
G = \begin{bmatrix} A & I \\ I & -I \end{bmatrix} \quad B = \begin{bmatrix} -I & 0 & 0 \\ 0 & -I & 0 \\ 0 & 0 & -I \end{bmatrix}
\]

Consider augmented Lagrange multiplier for above equation and it can be written as,

\[
\mathcal{L}(U, V, D) = g(U, V) + \mu \frac{1}{2} ||GU + BV - D||_2^2
\]

Subject to \( GU + BV = 0 \)

Expansion of above expression is given as,

\[
\mathcal{L}(U, V_1, V_2, V_3, D_1, D_2, D_3) = \frac{1}{2} ||V_1 - Y||_2^2 + \lambda ||V_2||_1 + \gamma R(V_3) + ||AU - V_1 - D_1||_2^2 + \mu ||U - V_2 - D_2||_2^2 + \frac{\mu}{2} ||U - V_3 - D_3||_2^2
\]

Algorithm of Alternating Direction Method of Multipliers (ADMM) can be used for solve above problem [5].

1. Initialization: set \( k = 0 \), choose \( \mu > 0 \), \( U^{(0)}, V^{(0)}, D^{(0)} \)
2. Repeat:
   3. \( U^{(k+1)} \leftarrow \text{arg min}_U \mathcal{L}(U^{(k)}, V^{(k)}, D^{(k)}) \)
   4. \( V^{(k+1)} \leftarrow \text{arg min}_V \mathcal{L}(U^{(k+1)}, V^{(k)}, D^{(k)}) \)
   5. \( D^{(k+1)} \leftarrow (D^{(k)} - GU^{(k+1)} - BV^{(k+1)}) \)
6. Algorithm is stop when all criterion satisfied.

By using Lagrange multiplier, we can obtain the optimal value of \( k \) which can be done by this iterative process till stopping criterion is satisfied. The Constraint Sparse Regression problem is solved by using SUuSAL algorithm.

B. Collaborative Sparse Unmixing via variable Splitting and Augmented Lagrangian (CSSuSAL)

In CSSuSAL algorithm, given hyperspectral data cube instead of pixel sparsity we used line sparsity for finding non zero elements in line. In this algorithm, we use \( l_{2,1} \) norm.
The optimization problem is,
\[
\min_x \|AX - Y\|_2^2 + \lambda \sum_{k=1}^m \|X^k\|_2^2
\]  
(13)
Subject to: \(X \geq 0\)

In the above equation where \(X^k\) denotes the \(k\)-th line of \(X\) and \(X \geq 0\) is to be understood component-wise. The convex term \(\sum_{k=1}^m \|X^k\|_2\) is the so-called \(l_{2,1}\) mixed norm which encourages sparsity among the lines of \(X\), i.e., it an encourage solutions of above equation (13) with the less number of nonzero lines of \(X\) [3]. The objective function in CLSUnSAL is,
\[
\min_x \|AX - Y\|_2^2 + \lambda \|X\|_{2,1} + \lambda R_1(X)
\]  
(14)

Figure 4. shows the effect of the mixed \(l_{2,1}\) norm \(\sum_{k=1}^m \|X^k\|_2\)imposing sparsity among the endmembers simultaneously (collaboratively) for all pixels. The \(l_{2,1}\) regularizer already exploit sparsity in the solution, making an \(l_1\) term in (13) somehow redundant [3]. In certain situations, slightly improves the unmixing result by using \(l_1\) term.

IV. SIMULATION RESULTS

A. Hyperspectral Image Generation

Performance of SUnSAL algorithm and CLSUnSAL has been generated with the help of synthetically generated Data Cubes (DC). In this, we used \(l_1\) norm in SUnSAL and \(l_{2,1}\) norm in CLSUnSAL. Each spectral signature has 224 bands, which is denoted by \(L\). For simulation, a subset of original USGS spectral library is used, which consist 498 spectral signatures and denoted with \(A\). Fractional abundance map for All data cube are below shown. Signal to Reconstruction Error (SRE) is used as a performance evaluation parameter as,
\[
SRE = 20 \log_{10} \frac{\|x\|_2^2}{\|x - \hat{x}\|_2^2}
\]  
(15)

Here real fractional abundance is denoted by \(x\) and estimated fractional abundance is \(\hat{x}\). SRE give more information concerning the error power in relation to the signal power. SRE is just opposite of RMSE. Which indicate higher SRE better the unmixing performance.

We have randomly selected spectral signatures from \(A\) and generated two data cubes of image size 100×100 shown in TABLE I. The synthetically generated data cubes are corrupted with different noise levels like 30dB, 40dB, 50dB, and 60dB. simulation, we have measured the value of SRE for different values of \(\lambda\). In this Algorithms mentioned above were implemented in MATLAB.

### Table I: Detail of Implementation

<table>
<thead>
<tr>
<th>Data cube</th>
<th>Image Size</th>
<th>No. of Endmember</th>
<th>Index of endmember</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC#1</td>
<td>100×100</td>
<td>5</td>
<td>21,31,129,135,176</td>
</tr>
<tr>
<td>DC#2</td>
<td>100×100</td>
<td>9</td>
<td>21,31,129,135,176,377,394,409,487</td>
</tr>
</tbody>
</table>

Fig.5(a) True abundance map used for data cube # 1

Fig.5(b) Estimated abundance map using SUnSAL
**Fig. 5(c)** Estimated abundance map using CLSUnSAL

**Fig. 6(a)** True abundance map used for data cube # 2

**Fig. 6(b)** Estimated abundance map using SUnSAL

**Fig. 6(c)** Estimated abundance map using CLSUnSAL

**Fig. 7** SRE Vs. SNR

(a) \( \lambda = 0.0001 \)  
(b) \( \lambda = 0.0005 \)  
(c) \( \lambda = 0.001 \)  
(d) \( \lambda = 0.005 \) for DC#1
V. CONCLUSION

In the field of Remote Sensing, Hyperspectral Unmixing is the inviting topic of the researcher. Hyperspectral Unmixing is the challenging task and also the inverse ill-posed problem due to the environmental conditions, model inaccuracy, noise, and large data size, endmember variation. Unmixing is the process to determine the number of material present in the scene is called endmembers, their spectral signature, and their abundance map. In this article we have done Comparative analysis of different approaches for hyperspectral unmixing. From these different methods we have selected sparse regression based method. The different number of materials and image size is generated using USGS spectral library with an implemented algorithm SUnSAL and CLSUnSAL. This algorithm used for synthetic Hyperspectral data, by changing the subsequent parameters such as SNR, Image Size, Endmembers, Weighted parameter. Add different constraints in CLSUnSAL algorithm to improve performance as compare to SUnSAL algorithm and also we try to use different constraints for estimates the fractional abundance maps of materials present in the scene inefficient manner.

REFERENCES


