Abstract
A set $S$ of vertices of a graph $G$ is an edge geodetic set if every edge of $G$ lies on an $x$-$y$ geodesic for some elements $x$ and $y$ in $S$. The minimum cardinality of an edge geodetic set of $G$ is the edge geodetic number of $G$ denoted by $g_1(G)$. In this paper, we explore the concept of edge geodetic parameters in the context of various types of special graphs such as Cocktail party graph, Crown graph, Dutch windmill graph, Friendship graph, Shadow graph, Tadpole graph, Windmill graph, Jump graph.

Keywords: Edge geodetic set, Connected edge geodetic set, Restrained edge geodetic number, Split edge geodetic set.

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1. INTRODUCTION

Let $G = (V, E)$ be a connected graph with node set $V = V(G)$ and the edge set $E = E(G)$. A set $S$ of vertices of a graph $G$ is an edge geodetic set if every edge of $G$ lies on an $x$-$y$ geodesic for some elements $x$ and $y$ in $S$. The minimum cardinality of an edge geodetic set of $G$ is the edge geodetic number of $G$ denoted by $g_1(G)$. The edge geodetic number $g_1(G)$ was introduced and studied in [9]. The concept of connected edge geodetic number $g_1(G)$ was introduced in [8]. A. P. Santhakumaran et al. introduced the concept of restrained edge geodetic number $g_2(G)$ in [6]. The concept of split edge geodetic number $g_{1e}(G)$ was introduced in [2]. In [1] Shobha and Venkanagouda M Goudar introduced the concept of total edge geodetic number $g_{1c}(G)$. For any undefined terms or notations in this paper can be found in Harary [11].

2. MAIN RESULTS

Crown Graph

Definition 2.1. The Crown graph $H_{n,n}$ is the graph obtained from the complete bipartite graph $K_{n,n}$ by removing a perfect matching. It is defined in [5].

Example: For a Crown graph $H_{4,4}$ given in Figure 1, the darkened vertices is its $g_1$- set.

The set $S = \{u_1, v_1\}$ is $g_1$- set so that $g_1(H_{4,4}) = 2$.

![Figure 1: Crown graph](image)

Theorem 2.2. For a Crown graph $H_{n,n}$, $n \geq 3$, $g_1(H_{n,n})=2$.

Proof. Let $G = H_{n,n}$ with vertex set $\{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n\}$ and the edge set $\{(u_i, v_j) : 1 \leq i \neq j \leq n\}$ such that $|V(G)| = 2n$ and $|E(G)| = n$. Let $S = \{u_r, v_s\}$ such that $d(u_r, v_s) = 3$. Clearly, all the edges of $G$ lies in the geodesic joining $u_r$ and $v_s$ and hence $S$ is $g_1$- set. Thus, $g_1(H_{n,n}) = 2$.

Theorem 2.3 If $H_{n,n}$, $n \geq 3$ is any crown graph then $g_{1c}(H_{n,n}) = 4$.

Proof. Let $G = H_{n,n}$. By Theorem 2.2, $S = \{u_r, u_s\}$ is $g_1$- set of $G$ and $g_1(G) = 2$. But $G$ is disconnected. Let $S' = S \cup \{u_i, u_j\}$ where $u_i$ is adjacent to $v_s$ and $v_j$. Clearly, $S'$ is connected. Therefore, $S'$ is $g_{1c}$- set of $G$. Hence, $g_{1c}(H_{n,n}) = 4$.

Theorem 2.4 For any crown graph $H_{n,n}$, $n \geq 4$ $g_{1c}(H_{n,n}) = a_0(H_{n,n}) + 1$ where $a_0$ is the vertex covering number of $H_{n,n}$.

Proof. Consider $G = H_{n,n}$. Let $a_0$ be the vertex covering number of $G$. We have By Theorem 2.2 $S = \{u_r, v_s\}$ is the $g_1$- set of $G$ and $g_1(G) = 2$. Consider $S_1 = \{u_{ki}, v_{ki} : 2 \leq k, l \leq n - 2\}$. Let $S' = S \cup S_1$. Clearly, $S'$ has no isolated vertices.
vertices. Therefore, \( S' \) is \( g_{1t} - \) set. Hence \( g_{1t}(H_{n,n}) = |S'| = a_0(H_{n,n}) + 1 \).

**Corollary 2.5** For any crown graph \( H_{n,n}, n \geq 4 \)

\( g_{1t}(H_{n,n}) = g_{1t}(H_{n,n}) \).

**Definition 2.6** The Friendship graph is a planar undirected graph with \( 2n+1 \) vertices and \( 3n \) edges. The friendship graph \( F_n^3 \) can be constructed by joining \( n \) copies of the cycle \( C_3 \) with a common vertex. It is defined in [3].

**Example:** For a friendship graph \( F_3^4 \) given in Figure 4, the colorless vertices is its edge geodetic set.

\[ S = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\} \] is an edge geodetic set of \( F_3^4 \) so that \( g_1(F_3^4) = 8 \).

**Theorem 2.7** For any friendship graph \( F_n^3, n \geq 2 \)

\( g_1(F_n^3) = 2n \).

**Proof.** Let \( G = F_n^3 \) with \( V(G) = \{v_1, v_2, v_3, \ldots, v_{3n}\} \) such that \( |V(G)| = 2n + 1 \) and \( |E(G)| = 3n \) and \( \{v_1, v_2, v_3\} \) be the common vertex. Let \( S_1 = \{v_1, v_2, v_3\} \) be \( g_1 \)-set of \( G \) which covers all the edges of \( G \). Therefore, \( g_1(G) = 2n \).

**Corollary 2.8** For any friendship graph \( F_n^3, n \geq 2 \)

\( g_{1t}(F_n^3) = g_1(F_n^3) = g_1(F_n^3) \).

**Theorem 2.9** For a friendship graph \( F_n^3, g_{1c}(F_n^3) = 2n + 1 \).

**Proof.** By the Theorem 2.7, \( S_1 = \{v_1, v_2, v_3, \ldots, v_{3n}\} \) is \( g_1 \)-set and \( g_1(F_n^3) = 2n + 1 \). Let \( S_2 = S_1 \cup \{v_k\} \) forms \( g_{1c} \)-set of \( F_n^3 \). Therefore, \( g_{1c}(F_n^3) = 2n + 1 \).

**Corollary 2.10.** For any friendship graph \( F_n^3 \) there is no split edge geodetic number and restrained edge geodetic number.

**Windmill Graph**

**Definition 2.11.** The Windmill graph \( Wd(k, n) \) is an undirected graph obtained by taking \( k \) copies of the complete graph \( K_n \) with a vertex in common. It is defined in [3].

**Example:** For a Windmill graph \( Wd(3, 4) \) given in Figure 3. The colorless vertices is its \( g_1 \)-set.

\[ S = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\} \] be \( g_1 \)-set so that \( g_1(Wd(3, 4)) = 9 \).

**Theorem 2.12** For any Windmill graph \( Wd(k, n) \),

\( g_1(Wd(k, n)) = k(n - 1) \).

**Proof.** Let \( G = Wd(k, n) \) and \( V(G) = \{u_1, u_2, \ldots, u_{k(n-1)}, x\} \) and \( x \) is a common vertex. Let \( S = \{u_1, u_2, \ldots, u_{k(n-1)}\}_{k\ times}/x \). Clearly, \( S \) is \( g_1 \)-set of \( G \). Therefore, \( g_1(Wd(k, n)) = k(n - 1) \).

**Corollary 2.13** For a Windmill graph , \( g_{1t}(Wd(k, n)) = g_1(Wd(k, n)) \).

**Theorem 2.14** For any Windmill graph \( Wd(k, n) \),

\( g_{1c}(Wd(k, n)) = k(n - 1) + 1 \).

**Proof.** Let \( G = Wd(k, n) \). By the Theorem 2.12, \( S = \{u_1, u_2, \ldots, u_{k(n-1)}\}_{k\ times}/x \) is \( g_1 \)-set of \( G \) and \( g_1(G) = k(n - 1) \). But \( < S > \) is disconnected. Let \( S_1 = S \cup \{x\} \). Clearly, \( < S_1 > \) is connected and hence \( S_1 \) is \( g_{1c} \)-set. Therefore, \( g_{1c}(Wd(k, n)) = k(n - 1) + 1 \).

**Dutch windmill Graph**

**Definition 2.15** The Dutch windmill graph \( D^k_n \) is the graph obtained by taking \( k \) copies of the cycle graph \( C_n \) with a vertex in common. It is defined in [3].

**Example:** For a Dutch windmill graph \( D^k_4 \), \( S = \{v_1, v_6, v_9\} \) is edge geodetic set of \( D^k_4 \) so that \( g_1(D^k_4) = 3 \).
Theorem 2.16. Let $D_n^k$ be the Dutch windmill graph then
\[ g_1(D_n^k) = \begin{cases} k & \text{if } n \equiv 0 \pmod{2}, \\ 2k & \text{if } n \equiv 1 \pmod{2}. \end{cases} \]

Proof. Let $G = D_n^k$ consisting of $k$ copies of the cycle graph $C_n$ with a vertex in common. Let $V(G) = \{U v_1^i, v_2^i, \ldots, v_{n-1}^i, v_n^i\}$ where $1 \leq i \leq n$ and $v_i$ is a common vertex. Then $G = k(n-1) + 1$ and $E(G) = nk$. We discuss the following cases:

Case 1. Let $n \equiv 0 \pmod{2}$. Let $S_1 = \{v_1^i, v_2^i, v_3^i, \ldots, v_n^i\}$ such that $d(v_1^i, v_2^i) = d(v_3^i, v_4^i) = \ldots = d(v_{n-1}^i, v_n^i) = \text{diam}(G)$ where $\{v_1^i, v_2^i, v_3^i, \ldots, v_n^i\}$ are the antipodal vertices of $G$. Clearly, every edge of $G$ lies in geodesic joining any two vertices of $S_1$. Therefore, $S_1$ is $g_1$-set of $G$. Hence $g_1(G) = |S_1| = k$.

Case 2. Let $n \equiv 1 \pmod{2}$. Let $S_2 = \{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, \ldots, v_n^i\}$ such that $d(v_1^i, v_2^i) = d(v_2^i, v_3^i) = d(v_4^i, v_5^i) = \ldots = d(v_{n-2}^i, v_{n-1}^i) = \text{diam}(G)$ be $g_1$-set of $G$. Hence $g_1(G) = |S_2| = 2k$.

Theorem 2.17. For any Dutch windmill graph $D_n^k$, $g_{st}(D_n^k) = 2k$.

Proof. Let $G = D_n^k$. We discuss the following cases:

Case 1. When $n$ is even. By the theorem 2.16, $S_1 = \{v_1^i, v_2^i, v_3^i, \ldots, v_n^i\}$ is $g_1$-set of $G$. But $S_1$ has isolated vertices. Let $S'' = S \cup \{v_{i+1}^0, v_{i+3}^0, v_{i+4}^0, \ldots, v_{i+k}^0\}$. Clearly, $S''$ has no isolated vertices and hence $g_{st}(G) = 2k$.

Case 2. When $n$ is odd. By the theorem 2.16, $S_2 = \{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, \ldots, v_n^i\}$ is $g_1$-set of $G$. But $S_2$ has no isolated vertices. Therefore, $S_2$ is $g_{st}$-set and hence $g_{st}(G) = 2k$.

Theorem 2.18. For any Dutch windmill graph $D_n^k$, $g_{st}(D_n^k) = \begin{cases} k+1 & \text{if } n \equiv 0 \pmod{2}, \\ 2k+1 & \text{if } n \equiv 1 \pmod{2}. \end{cases}$

Proof. Let $G = D_n^k$. Let $V(G) = \{v_i^j\}$ where $1 \leq i \leq n$ and $v_i$ is a common vertex. We have the following cases:

Case 1. For $n$ is even. By the theorem 2.16, $S_1 = \{v_1^i, v_3^i, \ldots, v_n^i\}$ is $g_1$-set of $G$. But $V(G) - S_1$ is connected. Let $S'' = S_1 \cup \{v_{n+1}^0\}$. Clearly, $S''$ has no isolated vertices and hence $g_{st}(G) = k+1$.

Case 2. For $n$ is odd. By the theorem 2.16, $S_2 = \{v_1^i, v_3^i, v_5^i, v_7^i, v_9^i, \ldots, v_n^i\}$ is $g_1$-set of $G$. But $V(G) - S_1$ is connected. Let $S'' = S_2 \cup \{v_{n+1}^0\}$ forms $g_{st}$-set of $G$. Therefore, $g_{st}(G) = 2k+1$.

Shadow Graph

Definition 2.19. The Shadow graph of $G$, denoted by $D_2(G)$ is the graph constructed from $G$ by taking two copies of $G$ namely $G$ and $G'$ and by joining each vertex $u$ in $G$ to the neighbours of the corresponding vertex $u'$ in $G'$. It is defined in [4].

Example: For the shadow graph $D_2(P_5)$ given in Figure 5, $S = \{ v_1, v_2, v_3, v_4, v_5 \}$ is an edge geodesic set so that $g_1(D_2(P_5)) = 4$.

![Figure 5: G](image)

Observation 2.20. For any path $P_n, n \geq 4$ the vertex covering number $\alpha_0$ ($P_n$) of shadow graph $D_2(P_n)$ is equal to $n$.

Theorem 2.21. For any path $P_n, n \geq 4, g_1(D_2(P_n)) = 4$.

Proof. Let $G = (D_2(P_n))$ and $V(G) = \{v_1, v_2, v_3, \ldots, v_n\}$ where $\{v_1, v_2, v_3, \ldots, v_n\}$ are the vertices of $P_n$ and $\{v_1', v_2', v_3', \ldots, v_n'\}$ are the shadow vertices of $P_n$ respectively. Then $|V(G)| = 2n$ and $|E(G)| = 4n-6$.

For $n=3$, it is easy to verify that, $g_1(G) = 3$.

For $n \geq 4$, let $S = \{ v_1, v_n, v_2', v_n' \}$. Since all the edges lie in geodesic joining any two vertices of $S$, clearly, $S$ is $g_1$-set. Therefore, $g_1(G) = |S| = 4$.

Corollary 2.22. Let $G, n \geq 4$, be any path then $e_{g_1}(D_2(P_n)) = g_1(D_2(P_n))$.

Theorem 2.23. For any path $P_n, n \geq 4, g_1c(D_2(P_n)) = \alpha_0(D_2(P_n)) + 2$ where $\alpha_0$ is the vertex covering number of $D_2(P_n)$.

Proof. Let $\alpha_0$ be the vertex covering number of $G = D_2(P_n)$. By the Theorem 2.21, $S = \{v_1, v_2, v_3, v_n\}$ be $g_1$-set of $G$ and $g_1c(G) = 4$. But $<S>$ is connected. Let $S' = S \cup \{v_{n+1}\}$. Clearly, $S'$ is connected. By Observation 2.21, it follows that $g_1c(D_2(P_n)) = |S'| = \alpha_0(D_2(P_n)) + 2$.

Theorem 2.24. For any path $P_n, n \geq 5, g_{st}(D_2(P_n)) = 5$.

Proof. Let $G = D_2(P_n)$. By the Theorem 2.21, $S = \{v_1, v_2, v_3, v_4, v_5\}$ be $g_1$-set of $G$ and $g_{st}(G) = 4$. But $<V-S>$ is connected. Let $S' = S \cup \{v_{n-2}\}$. Clearly, $<V-S'>$ is not connected. Hence, $S'$ forms $g_{st}$-set. Therefore, $g_{st}(D_2(P_n)) = 5$.

Theorem 2.25. For any star $K_{1,n}, n \geq 3, g_1(K_{1,n}) = \Delta(K_{1,n}) + 1$ where $\Delta$ is the maximum degree of $D_2(K_{1,n})$. 

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Proof. Let $G = D_2(K_{1,n})$ and two copies of star $G = G' \cup G''$. Let $V(G') = \{a_1, a_2, a_3, \ldots, a_n, x\}$ and $V(G'') = \{a'_1, a'_2, a'_3, \ldots, a'_n, x'\}$ where $x$ and $x'$ are the central vertices of $G'$ and $G''$. Let $S = V(G')$. Clearly, $S$ forms $g_1$-set. Therefore, $g_1(D_2(K_{1,n})) = |S| = \Delta(K_{1,n}) + 1$.

**Corollary 2.26** For any star $K_{1,n}$, $n \geq 3$, $\text{eg}(D_2(K_{1,n})) = g_1(D_2(K_{1,n}))$.

**Theorem 2.27.** For any cycle $C_n$, $n \geq 4$, $g_1[D_2(C_n)] = \begin{cases} 4 & \text{for } n \text{ is even,} \\ 6 & \text{for } n \text{ is odd.} \end{cases}$

Proof. Let $G = D_2(C_n)$ and $G = G' \cup G''$ where $G'$ and $G''$ are the two copies of $G$ such that $V(G') = \{v_1, v_2, \ldots, v_n\}$ and $V(G'') = \{v'_1, v'_2, v'_3, \ldots, v'_n\}$. We shall discuss the following cases:

**Case 1.** For $n$ is even. Let $S = \{v_r, v'_r, v_r, v'_r\}$ where $v_r$ is the antipodal vertex of $v_r$ which forms a $g_1$-set of $D_2(C_n)$. Hence, $g_1(D_2(C_n)) = 4$.

**Case 2.** For $n$ is odd. Let $S = \{v_p, v'_p, v_r, v_r, v_s, v'_s\}$ where $v_r$ is the antipodal vertex of $v_r$ and $v_s$ in $G'$ be a $g_1$-set. Hence, $g_1(D_2(C_n)) = 6$.

**Corollary 2.28.** For any cycle $C_n$, $n \geq 4$, $\text{eg}(D_2(C_n)) = g_1(D_2(C_n))$.

**Corollary 2.29.** For any cycle $C_n$, $n \geq 4$, $g_1[D_2(C_n)] = g_1(D_2(C_n))$.

**Cocktail Party Graph**

**Definition 2.27.** The Cocktail Party graph $\overline{C}_n$ is the graph consisting of two rows of paired vertices in which all vertices but the paired ones are connected with a graph edge. It is defined in [5].

**Example:** For a cocktail party graph $\overline{L}_4$ given in Figure 6, the colorless vertices is its edge geodetic set. Then $S = \{u_1, u_2, u_3, w_1, w_2, w_3\}$ is $g_1$-set so that $g_1(\overline{L}_n) = 6$.

![Figure 6: G](image)

**Theorem 2.28.** The edge geodetic number of a cocktail party graph $\overline{L}_n$ of order $2n$ is equal to $2n - 2$.

Proof. Let $G = \overline{L}_n$. Let $V_1 = \{u_1, u_2, u_3, \ldots, u_n\}$ and $V_2 = \{w_1, w_2, w_3, \ldots, w_n\}$ be the two vertex sets respectively such that every vertex in $V_1$ has a vertex pair in $V_2$ except the paired ones $(u_i, w_j)$ for $i = j$. Then $|V(G)| = 2n$. Let $S = \{u_1, u_2, u_3, \ldots, u_{n-1}, w_1, w_2, \ldots, w_{n-1}\}$ and every edge of $G$ lies in geodesic joining any pair of vertices of $S$. Therefore, $g_1(G) = |S| = 2n - 2$.

**Corollary 2.29** If $\overline{L}_n$ is any cocktail party graph of order $2n$, then $g_1(\overline{L}_n) = 2n - 2$.

**Corollary 2.30** For any cocktail party graph $\overline{L}_n$ of order $2n$, $g_1(\overline{L}_n) = g_1(\overline{C}_n)$.

**Corollary 2.31** For any cocktail party graph $\overline{L}_n$ of order $2n$, $g_1(\overline{L}_n) = g_1(\overline{L}_n)$.

**Tadpole Graph**

**Definition 2.32.** The $T(p, n)$-Tadpole graph, also called a dragon graph, is the graph obtained by joining a cycle graph $C_p$ to a path graph $P_n$ with a bridge. It is defined in [4].

**Example:** For a Tadpole graph $T(5, 2)$ given in Figure 7, the darkened vertices is its $g_1$-set.

Let $S = \{x_2, x_4, y_2\}$ is edge geodetic set of $T(5, 2)$ so that $T(5, 2) = 3$.

![](image)

**Figure 7: G**

**Theorem 2.33** For any tadpole graph $T(p, n), n \geq 3$, $g_1(T(p, n)) = \begin{cases} 2 & \text{if } n \equiv 0(\text{mod } 2), \\ 3 & \text{if } n \equiv 1(\text{mod } 2). \end{cases}$

Proof. Let $G = T(p, n)$. Consider $V(G) = \{x_1, x_2, x_3, \ldots, x_p, y_1, y_2, y_3, \ldots, y_n\}$ where $V(P) = \{x_1, x_2, x_3, \ldots, x_p\}$ and $V(C_p) = \{y_1, y_2, y_3, \ldots, y_n\}$. We discuss in following two cases:

**Case 1:** when $n$ is even. Let $S = \{x_i, y_n\}$ where $d(x_i, y_n) = \text{diam}(G)$ and $I[S] = V(G)$ Clearly, $S$ is $g_1$-set of $G$. Hence $g_1(G) = 2$.

**Case 2:** when $n$ is odd. Let $S = \{x_i, x_{i+1}, y_n\}$ where $d(x_i, y_n) = d(x_{i+1}, y_n) = \text{diam}(G)$ and $S$ forms $g_1$-set of $G$. Hence $g_1(G) = 3$.

**Corollary 2.34** For a tadpole graph $T(p, n)$, $\text{eg}(T(p, n)) = g_1(T(p, n))$.

**Theorem 2.35** For a tadpole graph $T(p, n)$, $g_{1e}(T(p, n)) = 4$.

Proof. Let $G = T(p, n)$. We discuss in following two cases:
Case 1. For $n$ is even. By the Theorem 2.33, $S = \{x_i, y_n\}$ is a cut vertex of $G$. Let $S_1 = S \cup \{x_{i+1}, y_{n-1}\}$. Clearly, $S_1$ is $g_1(S)$ of $G$. Therefore, $g_1(S) = 4$.

Case 2. For $n$ is odd. By the Theorem 2.33, $S = \{x_i, x_{i+1}, y_n\}$ is $g_1(S)$ of $G$. Let $S_2 = S \cup \{x_k\}$ where $\{x_k\}$ is any cut vertex of $G$. Clearly, $S_2$ is $g_1(S)$ of $G$. Therefore, $g_1(S) = 4$.

**Theorem 2.36** For a tadpole graph $T(p,n)$, $g_{1\alpha}(T(p,n)) = \begin{cases} 3 & \text{if } n \equiv 0 \pmod{2}, \\ 4 & \text{if } n \equiv 1 \pmod{2}. \end{cases}$

**Proof.** Let $G = T(p,n)$. We have the following cases:

Case 1. When $n$ is even. By the Theorem 2.33, $S = \{x_i, y_n\}$ is a $g_1$-set of $G$ and $g_1(S) = 2$. But $V - S$ is connected. Let $S_1 = S \cup \{x_k\}$ where $\{x_k\}$ is any cut vertex of $G$. Clearly, $S_1$ is $g_{1\alpha}(G) = 3$.

Case 2. When $n$ is odd. By the Theorem 2.33, $S = \{x_i, x_{i+1}, y_n\}$ is $g_1(S)$ of $G$ and $g_1(S) = 3$. Let $S_2 = S \cup \{x_k\}$ where $\{x_k\}$ is the cut vertex of $G$ and $V - S_2$ is disconnected. Hence, $g_{1\alpha}(G) = 4$.

**Jump Graph**

**Definition 2.37** The Jump graph $J(G)$ of a graph $G$ is defined as that graph whose vertices are the edges of $G$ and where two vertices of $J(G)$ are adjacent if and only if the corresponding edges of $G$ are independent. It is defined in [7].

**Example:** Consider the jump graph of path $P_8$ given in Figure 8, the set $S = \{u_1, u_2, u_3, u_4, u_5\}$ is an edge geodetic set of $J(P_8)$ so that $g_1(J(P_8)) = 5$.

**Figure 8: G**

**Theorem 2.38** For any path $P_n$, $n \geq 8$,

$$g_1(J(P_n)) = \begin{cases} 2 & \text{for } n = 5, \\ 3 & \text{for } n = 6,7, \\ n-3 & \text{for } n \geq 8. \end{cases}$$

**Proof.** For $n = 5, 6,7$, proof is obvious.

Further, let $V_1 = \{v_1', v_2', v_3', \ldots, v_n'\}$ be the set of vertices in $J(P_n)$ corresponding to the set of independent edges $e_1, e_2, e_3, \ldots, e_n$ of $P_n$. Suppose that $n \geq 8$. Let $S_1 = \{u_1', v_1', v_2', v_3', \ldots, v_{n-3}'\}$. Clearly, all the edges of $J(P_n)$ lie in geodesic joining any two vertices of $S_1$. Hence $S_1$ is $g_1$-set of $J(P_n)$. Therefore, $g_1(J(P_n)) = |S_1| = n - 3$.

**Corollary 2.39** For any path $P_n$, $n \geq 8$, $g_1(J(P_n)) = g_1(J(P_{n-3}))$.

**Corollary 2.40** For any path $P_n$, $n \geq 5$, $g_1(J(P_n)) = g_1(J(P_{n-3}))$.

**Theorem 2.41** For any path $P_n$, $n \geq 8$, $g_1(J(P_n)) = \begin{cases} 2 & \text{for } n = 6,7, \\ n-3 & \text{for } n \geq 8. \end{cases}$

**Proof.** Let $\omega_0$ be the vertex covering number of $P_n$. Proof is obvious for $n = 6,7$. Consider any subgraph $P_{n-2}$ of $P_n$. Let $E$ be the set of edges in $P_{n-2}$. By the Theorem 2.38, let $V' = \{v_1', v_2', v_3', \ldots, v_n'\}$ be the set of vertices in $J(P_{n-2})$ corresponding to the set of edges $E$ of $P_n$. Clearly, $S_1 = V'$ is a $g_1$-set of $J(P_n)$. But $V - S_1$ is connected. Let $S_2 = V \setminus (J(P_n) - S_1)$, such that $V - S_2$ is disconnected. Hence, $g_{1\alpha}(J(P_n)) = |S_1| + 2$.

**Theorem 2.42** For any cycle $C_n$, $n \geq 5$, then,

$$g_1(J(C_n)) = \deg((J(v_j))) + 1.$$  

**Proof.** For $n = 4$, $J(C_4)$ is disconnected and hence there is no $g_1$-set for $J(C_4)$. Let $\{x_1, x_2, x_3, \ldots, x_n\}$ be the set of vertices of $J(C_n)$ corresponding to the edges $E = \{e_1, e_2, e_3, \ldots, e_n\}$ of $C_n$ and $\deg((J(v_j))) = n - 3$. Let $S = \{x_1, x_2, x_3, \ldots, x_n\}$. Clearly, $S$ covers all the edges of $J(C_n)$. Hence $S$ is a $g_1$-set of $J(C_n)$. Therefore, $|S| = g_1(J(C_n)) = \deg((J(v_j))) + 1$.

**Corollary 2.43** For any cycle $C_n$, $n \geq 5$, $g_1(J(C_n)) = g_1(J(C_n))$.

**Corollary 2.44** For any cycle $C_n$, $n \geq 5$, $g_1(J(C_n)) = g_1(J(C_{n-2}))$.

**Corollary 2.45** For any cycle $C_n$, $n \geq 5$, $g_{1\alpha}(J(C_n)) = n - 2$.

**CONCLUSION**

In this paper, we obtained the results on edge geodetic parameters of several special graphs.

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