Abstract
Purpose of steering is to direct vehicle in desired direction. Composition of steering with suspension controls and stabilizes the vehicle. Conventional steering system has mechanical element called as pinion to connect with rack of the vehicle wheel. This pinion transfers disturbing forces to the steering directly which makes steering wheel uncontrollable to handle for the driver. This uncontrollability of steering wheel leads to an accident. The various methodologies for the controllability and stability of steering system are proposed in this paper.

Keywords: PID, SMC, MPC, etc.

I. INTRODUCTION
As we are moving toward autonomous world, every system is controlled by the controller used in it. Sensor output is utilized to operate whole system. But the external disturbances add noises which mal-functions the whole system. Whenever one of the elements of the system gets faulty, it gets stopped working at the same time, which also increases the chances of getting injured or to have huge accident. As steer by wire system is linear time variant system, its response will be always linear to the input provided by the driver avoiding external disturbances. To maintain the controllability in uncertainty, many controllers have been implemented by various methods. Controller estimates the future response and tolerates the fault for a period. It detects and estimates the real time future responses from the various linear time input signals. PID controllers used to stabilize the external disturbance. The defects of PID controller are removed by sliding mode control method. The advancement has been done over these methods. In this paper, All of the methods to improve controllability and stability of the steer by wire system is explored in detail for better accuracy and tracking performance.

II. RELATED WORK
Here, various methods to improve the stability and controllability of steer by wire system are explored.

A. Bond graph method
Bond graph method is a graphical way of representing dynamic mechanical system. It allows conversion of differential equation into state space equation. It has similarity with signal flow graph instead it represents the bi-directional flow of forces from one component to another. It can represent mechanical system as well as electrical system. The flow of energy is in term of power variables through bond. In mechanical system, the flow of energy is in the form of Mass, Inertia J, Damping B and Spring K. similarly, in electrical system, the energy flow is in current and voltage form. The bond graph can represent single-port elements e.g. sources and sinks, resistance, capacitance, etc. and two-port elements e.g. transformer, gyrator, etc. similarly it represents multi-port elements such as 0-junctions and 1-junctions. The bond graph method generates state space equation which allows complex multi-order system to be solved as first-order system. Differential equations define the behavior of the system as well as relationships of the system variables.

The bond graph method has been used to model the steer by wire system. The steer by wire is the wired technology in the field of steering system which has replaces the clutch mechanisms. The steer by wire is modeled using an equation’s of Newton’s law. It has been divided into two parts steering wheel subsystem and vehicle wheel subsystem. The front wheel steering has been modeled in terms of steering column damping Bsc, lumped torque stiffness ks1, steering lumped inertia Isw, motor electrical resistance Rm, motor electrical inductance Lm1, lumped inertia motor Im1 and steering motor emf Ks1.

Mathematical equations for steering wheel subsystem are given below:
Steering angle: \( \ddot{\theta}_{sw} = \frac{1}{L_{sw}} [T_{driver} - T_{frc} - B_{sw}\dot{\theta}_{sw} - K_{s1}\theta_{sw} + B_{sc}\dot{\theta}_{m1} + K_{s1}\theta_{m1}] \)

Current of steering Motor: \( i_{sw} = \frac{1}{L_{m1}} [- R_{m}\dot{\theta}_{m1} - K_{s1}\theta_{m1} + \theta_{sw}] \)

Steering Motor angular displacement: \( \dot{\theta}_{m1} = \frac{1}{L_{m1}} [- K_{s1}\theta_{m1} - B_{sc}\dot{\theta}_{m1} + K_{s1}\theta_{sw} + B_{sc}\dot{\theta}_{sw} + T_{m1}] \)

Where, \( T_{driver} = \) driver torque
\( T_{frc} = \) torque friction
\( \theta_{sw} = \) steering angle
\( T_{m1} = \) motor torque

Similarly, the state space equations for the steering wheel system are produced from the mechanical parameters. where, \( u (t) = \) Input to the steering wheel subsystem by the driver and steering wheel angle. Similarly, front wheel system has also been implemented, the related equations are given below:
Front motor current: \( i_{m2} = - \frac{R_{m}}{L_{m2}} (i_{m2}) - \frac{K_{s2}}{J_{m2}} (T_{m2}) + V_{c2} \)
The steer by wire system is modeled using the Newton’s law method.

The disadvantages of modeling steer by wire with bond graph are:

1. Bond graph models are lumped variable models which do not provide zero error. Then block diagram method uses to form control system of the steer by wire which has PID controller in its feed-forward path to stabilize disturbance occurred. The steering ratio is the ratio of steering wheel angle to the vehicle wheel angle which is selected by vehicle speed is known as Active steering. Small turn of steering wheel provides large forces for vehicle wheel movement. The steering ratio equation is given as:

\[ K = \frac{R}{v^2(\delta t) - 57.3(a + b)/R} \]

Where, \( \delta t \) = steering ratio
\( V \) = vehicle speed
\( \delta t \) = Front tire angle without steering ratio
\( K \) = adjustable gain

The proposed system using bond graph method has stabilized the disturbances up to certain limit. However, the system is not accurate. Future work includes addition of body slip angle and yaw rate to improve stability of the SBW system.

The disadvantages of modeling steer by wire with bond graph method are:

1. Bond graph models which interact through thermo-fluid and magneto fluid are highly complex. So that obtained equation are complex and requires intensive computation.
2. Bond graph models are lumped variable models which do not provide information related to shapes and geometries, though it can be described in terms of lumped variables. Disturbances vary with the geometry of the components.
3. The bond graph method cannot model non-energetic interaction between the subsystems.

B. Two mode control strategy for stability and controllability of steer by wire

The steer by wire system is modeled using the Newton’s law considering inertia J1 and angle \( \theta_1 \), the mathematical equation for steer by wire is

\[ T_h = T_m = J_1 \cdot \Omega_1 + B_1 \cdot \Omega_1 + F_1 \]

where, \( T_h \) = Steering torque
\( T_m \) = torque from road surface
\( F_1 \) = friction of the system

Similarly, mathematical equation of vehicle wheel subsystem:

\[ T_{np} \cdot \Psi = J_2 \cdot \Omega_2 + B_2 \cdot \Omega_2 + F_2 \]

where, \( i_{nm} \) and \( \Psi \) = ratio and efficiency of worm gear. \( Tg \) = aligning torque from the ground

Using this developed model, the two mode control strategy has been proposed to improve the stability and controllability of the system at initial phase of start of SBW system. When SBW system is off, steering gear is fully mobilized but the steering wheel can move certain angle and is locked by steering column. So when SBW system is power ON, this gap angles affect controllability of the whole system. It has two mode of operation i.e. angle synchronization mode and steering control mode. In angle synchronization mode, firstly it synchronizes the steering wheel angle with vehicle wheel angle. If the gap angle between steering wheel and vehicle wheel > 1, then system switches to angle synchronization mode and decrease angle difference. It has controllers for its function. The The Steering Torque Controller block can be expressed as:

\[ i_1 \cdot d_1 \cdot f(\nu, i_{2d}, \Omega_1) \]

where, road sense motor current i1d1 is a function of vehicle speed \( \nu \), reference steering motor current \( i_{2d} \) and steering wheel angle \( \Omega_1 \), PID controller helps in tracking angle difference. Two PID controllers are used to control the angle and angular speed of steering gear to track the reference angle. The simulation model consists of SBW model, driver model and Vehicle/road model as Car-SIM model. The tracking error of the track is given as:

\[ \Delta L = \sqrt{(x(t) - x(t)) + (y(t) - y(t))} \]

where, \( x(t) \) and \( y(t) \) = targeted track, \( x(t) \) and \( y(t) \) = vehicle track. The simulation shows that it has less than 0.7 which needs to overcome to increase accuracy of the system.

C. Sliding mode control method for controllability and stability of steer by wire system.

The sliding mode control is also known as variable structure system control it makes use of state space and produces control settings based on the inputs and outputs of the system. It can be used to control high-order plant operations. The sliding mode design for steer by wire is given below with external disturbances and faults:

\[ \begin{align*}
\dot{x}(t) &= A_m x(t) + B_m v(t) + w(t) \\
\dot{y}(t) &= C_m x(t)
\end{align*} \]

where, \( B_m \in R \times m \) is the control input matrix for \( v(t) \).
\( B_m2 \in R \times m \) is the input matrix for \( T_f(t) \).
\( W(t) = B_m2 T_f(t) + w_1(t) \in R \times n \).
\( Bm2 T_f(t) = system \ disturbances \)
\( w_1(t) = system \ matched \ uncertainty \)
\( v(t) = \text{virtual control input} \)

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\( F(k) = \) weighing vector that represents effectiveness of the actuator

If \( 0 < F(t) < 1 \), then actuator if faulty. \( F(t) = 1 \) shows actuator is good and \( F(t) = 0 \), actuator is not good. Adaptive Kalman filter is used to estimate \( F(t) \). \( \hat{F}(t) = F(t) - \hat{\delta}(t) \)

where, \( \hat{F}(t) = \) estimates of weighing vector and \( \hat{\delta}(t) = \) inaccuracies of the actuators and \( \hat{\delta}(t) \in [0, 1] \).

\( w(t) = Bmw(t), \quad 0 \leq w(t) \leq \hat{\delta}(t) < \infty, \quad 0 \leq \hat{w}' m(t) \leq \epsilon, \quad T \)

The system is considered controllable described as:

\[ \dot{x}(t) = Amx(t) + Bm(F(t) - \hat{\delta}(t))u(t) + w(t) \]

\[ = Amx(t) + Bm1F(t)u(t) + Bm2Tf(t) + Bm1\hat{\delta}(t)u(t) \]

\[ = Amx(t) + Bm1F(t)u(t) + \zeta(t), \]

By discretizing the system equation at sample time \( T \), the equation can be rewrite as: \( x(k+1) = Ax(k) + B^TF(k)u(k) + d(k) \)

\[ y(k) = Cx(k) \]

the sliding mode switching function are:

\[ e(k+1) = y(k+1) - yd(k+1) = Cx(k) + CB^TF(k)u(k) + Cd(k) - yd(k + 1) \]

\[ s(k+1) = Ge (k+1) + Klp(K+1) = (G + Kl)(CAx(k) + CB^TF(k)u(k) + Cd(k) - yd(k+1)) + Kl \]

\[ \hat{\delta}(t) = 0 \]

\[ U = A - B^TF(k)\Omega(CB^TF(k)-1)\Omega(\Delta G) \]

\[ U = I - B^F(k)\Omega(CB^TF(k)-1)\Omega(\Delta C), \]

\( U \) and \( \hat{U} \) should be inside the unit circle of \( z \)-plane for its stability. The sliding mode control basically used for stabilizing yaw rate motion of the vehicle. The disadvantages of sliding mode control are as follows:

The mutual interaction between the parameters are hard to predict, so that the satisfactorily system may not offer the best performance.[2]

**D. Model prediction control method**

To improve controllability and stability of the steer by wire system in a failure, the Type equation here, the equation here, control method has been introduced. It utilizes for the path tracking controller. It computes the future variables. The model predictive control uses state space variables, the current values of state space variables is used to compute future values of the variables. The discrete time steer by wire equation is:

\[ x(k+1) = Ax(k) + Bmuk + Gmkd \]

\[ y(k) = Cmx(k) \]

\[ \Delta x(k+1) = x(k+1) - x(k) \]

\[ \Delta x(k) = x(k) - x(k-1) \]

\[ \Delta uk = uk - uk-1 \]

\[ \Delta dk = dk - dk-1 \]

The difference of state space equation is \( \Delta x(k+1) = A\Delta x(k) + Bm\Delta uk + G\Delta dk \)

\[ y(k+1) - y(k) = Cm(x(k+1) - x(k)) \]

\[ = Cm\Delta x(k+1) \]

\[ = Cm\Delta x(k) + CmBm\Delta uk + CmGm\Delta dk \]

\[ x(k) = [\Delta x(k) y(k)]^T \]

A new state space model is generated used in MPC design.

The disadvantage of MPC is its computation demand and requirement of the value of state variables.[4]

**E. Sliding mode prediction control method**

Sliding mode control requires the discontinuous function and high control gain for the robustness and finite-time convergence. It also need knowledge of the bounds of the uncertainties and these bounds are over-estimated and requires extra gain. The main drawback of sliding mode control is chattering phenomenon. So that, adaptive sliding mode control came into existence. Every system has external disturbances which affects the functionality of the system. In this the disturbance from the ground surface and self-aligning torque is considered. The sliding mode control method handles the uncertainty and adaptive control method is also effective for disturbance related to time varying parameters. Integration of methods provides stability and tracking accuracy. The model of steer by wire is given as:

\[ J_d\ddot{x} + B_d\dot{x} = K_u u - \tau_{sel} \]

\[ F_c = \varepsilon_f \text{sign}(\dot{x}) \]

Where, \( J_d \) = moment of inertia

\( B_d \) = damping

\( x \) = steering angle of the front wheel

\( u \) = steering motor control input

\( \tau_{sel} \) = self-aligning torque

\( F_c \) = Coulomb friction with constant \( \varepsilon_f \)

\( K \) = scaling gain

Here, \( u(k) = \omega q(k) + up(k) \)

\[ s(k+1) = s(k) + \Omega CB^F(k)up(k) + \Omega Cx(k) \]

where,

\[ \varepsilon(k) = e(k) = d(k) - d(k-1) = \int_0^T eAcT - \int_0^T \varepsilon_e AcT - \varepsilon_f AcT - tkT^\tau \]

\[ \varepsilon(k) = e(k) = d(k) - d(k-1) \]

\[ s(k+1) \] describes the dynamics of the sliding mode and sliding mode dynamics prediction. The predicted sliding mode dynamics are given below:

\[ s(k+1) = s(k) + \Omega CB^F(k)up(k) + \Omega Cx(k) \]

\[ \Omega \]

\[ \Omega \]

\[ \Omega \]

\[ \Omega \]

\[ \Omega \]
where, \( m \) = control horizon and \( p \) = prediction function

\[
S(k) = A s(k) + \phi U p(k) + \Gamma \eta(k - 1)
\]

\[
J = ST (k) S(k) + \lambda \Delta UT p (k) \Delta U p(k)
\]

\[
\Delta u \text{min} \leq \Delta u p(k) \leq \Delta u \text{max}
\]

\[
u \text{min} \leq u p(k) \leq u \text{max}
\]

where \( \lambda \) = the future behavior weighting coefficient.

\( \Delta U p(k) \) = control variable increment. \( \Delta u \text{min}, \Delta u \text{max}, u \text{min}, \) and \( u \text{max} \) = limitations for the rate of control effect and control effect.

Disadvantages of sliding mode prediction control method is it has difficulty in solving real time matrix calculations.

F. Delta operator based model predictive control method

In this the faults are estimated in linear matrix inequalities and the same is used for fault detection. A delta operator based model prediction control has been generated to compensate the effect of fault of actuators and to operate the vehicle wheel normally. The delta operator has following advantages over merely model prediction control:

1. When sampling time becomes zero, the delta operator variables act as continuous time parameters with it respective values. Hence, it gives better discrete time analysis.

2. It also has computational related advantages.

3. The superior word length coefficients give less round off noises.

In the delta- operator based SBW system is proposed:

\[
x(t) = A mx(t) + B mu(t)
\]

\[
y(t) = C mx(t)
\]

\[
x(t) \in \mathbb{R}^n \text{ is the state vector of SBW given as: } x(t) = [ia2(t) \quad T m2(t) \quad y r a c k (t) \quad t f (t) \quad \delta w m(t) \quad v r t(t)]^T
\]

where, \( ia2(t) \) = the front motor current,

\( T m2(t) \) = the torque of the front motor,

\( y r a c k(t) \) = the rack force,

\( \delta w m(t) \) = front angular displacement,

\( v r t(t) \) = tie rod velocity,

and \( t f (t) \) = the torque acting on the tire.

The front wheel subsystem output is the front wheel angle \( y(t) = \delta f(t) \), the input to the front wheel subsystem:

\[
u(t) = [U w m(t) \quad t a r t(t)]^T.
\]

the delta operator is given as:

\[
\dot{x}(t) = x(t + T) - x(t) \quad \text{for } T != 0
\]

\[
=dx/ dt \ (tk) \quad \text{for } T = 0
\]

Where \( T \) = the sample period.

The discrete time model is rewritten as:

\[
\dot{x}(tk) = A \delta x(tk) + B \delta u(tk) + E \delta f (tk)
\]

\[
y(tk) = C \delta x(tk)
\]

\[
E \delta = \int_0^T e A m r \ dt \ E m / T = \text{the matrix for fault function, and } f (tk) \in E g = \text{fault signal to be detected and belongs to } L[2](0, \infty).
\]

Design of fault observer is done as:

\[
\delta \ * x(tk) = A \delta \ * x(tk) + B \delta u(tk) + E \delta f (tk) + L \delta (y(tk) - \gamma y(tk)) y(tk) = C \delta \ * x(tk)
\]

\[
\dot{\delta f}(tk) = \theta \ f(tk) - \_\delta(y(tk) - y(tk))
\]

where, \( \gamma x(tk) \) = the state vector observer,

\( \gamma y(tk) \) = the output estimation vector,

and \( f(tk) \) = estimate of \( f(tk) \).

\( L \delta \) = the observer gain. \( T \delta = \) is the weighting matrix, and \( \theta \) is a constant.

Similarly, the estimation of error is described as:

\[
ex(tk) = x(tk) - \_\gamma x(tk),
\]

\[
ey(tk) = y(tk) - \gamma y(tk), \quad \text{and }
\]

\[
e f(tk) = f(tk) - f(tk).
\]

\[
\dot{\delta e x} (tk) = (A \delta - L \delta \ C \delta) e x (tk) + E \delta e f (tk)
\]

\[
ey(tk) = C \delta e x (tk)
\]

\[
\delta e f (tk) = [\delta f(tk) - \theta f(tk)] + \theta e f(tk) + \_\delta C \delta e x (tk).
\]

The delta base fault tolerant MPC which uses fault information and manipulates the inputs to minimize error. The MPC model where real faults are replaced by estimated faults. The design of predictive control is given as:

\[
y = w + Gu + Q'Y \]

\[
w = W x(tk)
\]

then, Generate matrices required for the predictive control cost function \( J \), obtain the optimal control effect based on faulty estimation.

Disadvantages of Delta operator based model predictive control method is numerical differencing. Hence, the need of scaling of larger numerical values in the formulation reduces the number of bits for representing fractions and hence affects resolution.[5]

G. Sliding mode based learning control:

The defects in sliding mode control method such as bounds of information of disturbances needed and chattering phenomenon. Method used for tracking reference angle uses a \( \mu \)-synthesis robust control as a main-loop controller and utilizes the SMLC as a servo-loop controller. The steer by wire is modeled as second order system, where the dynamics of motor is given as:

\[
J_\mu \ddot{w} + B_\mu \dot{w} + T_{\omega 2m} = T_m
\]

Where, \( J_m \) = the moments of inertia of motor rotor,
\( \delta_m \) = the rotational angle of motor rotor,
\( B_m \) = the vicious friction of motor,
\( T_{w2m} \) = the torque from the wheel assembly to the motor,
And \( T_m \) is the torque generated by the steering motor,
\[ T_m + T_{cw} = \Delta T_{pert} \]
The differential equation for steering wheel: \( J_m \ddot{\delta}_m + B_m \dot{\delta}_m + T_m + T_c = T_{m2w} \)
The deduced equation of steer by wire:
\[ J_{equiv} \ddot{\delta}_w + B_{equiv} \dot{\delta}_w + T_f = T_{equiv} \]
\[ T_c = (l_c + l_p) F_{yi} = (l_c + l_p) C \alpha_i \]
Where, \( l_c \) is the wheel mechanical trail, \( l_p \) is the wheel pneumatic trail, \( F_{yi} \) is the lateral force acting on the steering, \( C \) is the tire cornering stiffness and \( \alpha_i \) is the body slip angle calculated depending on yaw rate. The symbol \( i \) denotes the front \((f)\) and the rear \((r)\) wheel selection. The lateral velocity varies with \( \beta \), \( \beta \) is directly proportional to the lateral velocity, its relation for both the rear and front wheel is termed as:
\[
\alpha_f = \delta_i - \arctan \left( \frac{v+w l_f}{\mu} \right) = \ddot{\delta}_f \beta - \frac{w l_f}{\mu} \\
\alpha_r = \delta_i - \arctan \left( \frac{v+w l_r}{\mu} \right) = \ddot{\delta}_r \beta - \frac{w l_r}{\mu}
\]
where \( l_f \) and \( l_r \) is the distance from center of gravity (CG) of front and rear wheel, \( w \) is the yaw velocity of the vehicle. In motors, the torque is always proportional to the current which is given as:
\[ T_m = K i T \]
\( K i \) = torque of the motor, \( i \) = motor current.
Motor torque equation can be rewritten as:
\[ T_m = T_{ctr} + \Delta T_{pert} = (K_T + \Delta K_T)(l + \Delta l) \]
To verify the robustness of the control scheme used. Error between the steering angle and tracking steering angle is given as:
\[ e(t) = \delta_{eq}(t) - \delta_{ref}(t) \]
The sliding variable for this method is \( s(t) = e(t) + \lambda e(t) \)
Where, \( \lambda \) is the sliding variable whose value should be always greater than zero.
The derivative of sliding variable: \( s(t) = a(t) + b T_{ctr} - \delta_{ref} \lambda [\delta_{eq}(t) - \delta_{ref}(t)] = f(t) + b T_{ctr} \)
Where, \( f(t) = a(t) - \delta_{ref} + \lambda [\delta_{eq}(t) - \delta_{ref}(t)] \]
The Lyapunov function is given as:
\[ V(t) = \frac{1}{2} s^2(t) \]
\[ V(t - \tau) = V(t) - V(t - \tau)/\tau \]
The difference of Lyapunov function and its estimate shows the error amount. The \( \mu \)-synthesis method is used with some controller order reduction control method. SMLC has faster convergence period up to 0.2s.[1]

### III. CONCLUSION

Various controller designing methods has been explored, among all of them the sliding mode learning control provides excellent tracking performance, robustness and energy efficiency. It has good hold over tracking performance on single lane and double road lane as well.

### REFERENCES


