A Special Study on Homo Cordial Labeling of Alternate Triangular Belt Graph

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Abstract
Let $G = (V, E)$ be a graph with $p$ vertices and $q$ edges. A Homo Cordial labelling of a graph $G$ with vertex set $V$ is a bijection from $V$ to $\{0, 1\}$ such that each edge $uv$ is assigned the label 1 if $f(u)=f(v)$ or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1. The graph that admits a Homo-Cordial labelling is called a Homo Cordial graph. In this paper we prove that Alternate triangular belt is Homo-Cordial labelling graph and further study on the generalisation of labelling an Alternate triangular belt graph.

Keywords: Homo Cordial graphs, Homo Cordial labelling

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1. INTRODUCTION
A graph $G$ is a finite nonempty set of objects called vertices and edges. All graphs considered here are finite, simple and undirected. Gallian[1] has given a dynamic survey of graph labelling. The origin of graph labelings can be attributed to Rosa. A Path related Homo Cordial graph was introduced by Dr.A.NellaiMurugan and A.Mathubala[2,3,4]. Motivated to be called a homo cordial labelling graph.

2. PRELIMINARIES
Definition 2.1: Let $L_n = P_n \times P_2 (n \geq 2)$ be the ladder graph with vertex set $U_i$ and $V_j$, $i=1,2...n$. The Alternate Triangular Belt is obtained from the ladder by adding the edges $u_{2i-1}v_{2i}$ for all $1 \leq i \leq n$ and $v_{2i}u_{2i+1}$ for all $1 \leq i \leq n$. This graph is denoted by $ATB(n)$.

3. MAIN RESULTS
Theorem 3.1: The Alternate Triangular Belt graph $ATB(n)$ is a homo cordial labelling graph.

Proof: Let $G = ATB(n)$ be the alternate triangular belt.
Let $L_n = P_n \times P_2 (n \geq 2)$ be the ladder graph with vertex set $U_i$ and $V_j$, $i=1,2...n$. The Alternate Triangular Belt is obtained from the ladder by adding the edges $u_{2i-1}v_{2i}$ for all $1 \leq i \leq n$ and $v_{2i}u_{2i+1}$ for all $1 \leq i \leq n$. This graph is denoted by $ATB(n)$. The vertex set is $V = \{u_1,u_2,...,u_n,v_1,v_2,...v_n\}$ and the edge set is $E = \{u_{2i-1}v_{2i} \mid 1 \leq i \leq n\} \cup \{u_{2i}v_{2i+1} \mid 1 \leq i \leq n\} \cup \{u_{2i-1}v_{2i+1} \mid 1 \leq i \leq n\}$

Now to label the vertices let us consider the bijective function $f : V \rightarrow \{0,1\}$ such that each edge $uv$ is assigned the label 1 if $f(u)=f(v)$ or 0 if $f(u) \neq f(v)$ with the condition that the number of vertices labelled with 0 and the number of vertices labelled with 1 differ by at most 1 and the number of edges labelled with 0 and the number of edges labelled with 1 differ by at most 1.

We define the labelling of vertices $u_1,u_2,..,u_n$ and for $v_1,v_2,...v_n$ as follows:

$f(u_i) = 1$ for $1 \leq i \leq n$
$f(v_i) = 0$ for $1 \leq i \leq n$

Then the induced edge labelling for the alternate triangular belt $ATB(n)$ are

$f^*(u_{2i-1}v_{2i}) = 1$ for $1 \leq i \leq n-1$
$f^*(v_iv_{i+1}) = 1$ for $1 \leq i \leq n-1$
$f^*(u_{2i}v_{2i+1}) = 0$ for $1 \leq i \leq n$
\[ f^*(u_{2i+1}v_{2i+2}) = 0 \text{ for } 0 \leq i \leq n-1 \]
\[ f^*(v_2u_{2i+1}) = 0 \text{ for } 1 \leq i \leq n-1 \]

Noticing the induced edge labelling we find that the number of vertices labelled with 0 is \( n \) and the number of vertices labelled with 1 is \( n \) and that the number of edges labelled with 0 is \( n+1 \) and the number of edges labelled with 1 is \( n \). Hence

\[ (0) - (1) \leq 1 \]
\[ (1) - (1) \leq 1 \]

Therefore the Alternate triangular belt graph is a homocordial labelling graph.

**Example 3.2**

Consider the Triangular belt graph \( ATB(6)(\downarrow\downarrow\ldots) \)

**Definition 3.3**: One part in Alternate Triangular Belt graph \( ATB(n)(\downarrow\downarrow\ldots) \):

For alternate triangular belt graph \( ATB(n)(\downarrow\downarrow\ldots) \) we define one part denoted by \( T(F) \) as shown below where each part consists of 3 0’s and 3 1’s which further signifies that number of vertices and edges labelled with 0 is denoted by \( T_0(\downarrow\downarrow\ldots) \) and number of vertices and edges labelled with 1 is denoted by \( T_1(\downarrow\downarrow\ldots) \)

One part in Alternate Triangular belt graph \( ATB(n)(\downarrow\downarrow\ldots) \) is added for every \( n \) is even

\[ T_0(\downarrow\downarrow\ldots) = T_0(\downarrow\downarrow\ldots) + (n-2)T_0(F) \quad \text{and} \quad T_1(\downarrow\downarrow\ldots) = T_1(\downarrow\downarrow\ldots) + (n-2)T_1(F), \quad n \geq 3 \]

where \( F \) is the one part of \( ATB(n)(\downarrow\downarrow\ldots) \) and so on to construct any order alternate triangular belt graph \( ATB(n)(\downarrow\downarrow\ldots) \). Which we state as a result in the following theorem.

**Theorem 3.4**:

If Alternate Triangular Belt graph \( ATB(n)(\downarrow\downarrow\ldots) \) is homocordial graph then

\[ (0) - (2) \leq 8 \]
\[ (1) - (2) \leq 7 \]

which is true

Now let us assume for \( n = k \)

\[ T_0(\downarrow\downarrow\ldots) = T_0(\downarrow\downarrow\ldots) + (k-2)T_0(F) \]
\[ T_1(\downarrow\downarrow\ldots) = T_1(\downarrow\downarrow\ldots) + (k-2)T_1(F) \]

Now let us prove for \( n = k+1 \)
Consider alternate triangular belt graph \( ATB(n) \) as being proved in Theorem 3.1 by labelling of vertices \( u_1, u_2, \ldots u_n \) and for \( v_1, v_2, \ldots v_n \) as follows

\[
f(u_i) = 1 \text{ for } 1 \leq i \leq n
\]
\[
f(v_i) = 0 \text{ for } 1 \leq i \leq n
\]

Then the induced edge labelling for the alternate triangular belt \( ATB(n) \) are

\[
f^*(u_{i+1} u_i) = 1 \text{ for } 1 \leq i \leq n-1
\]
\[
f^*(v_{i+1} v_i) = 1 \text{ for } 1 \leq i \leq n-1
\]
\[
f^*(u_i v_i) = 0 \text{ for } 1 \leq i \leq n
\]
\[
f^*(u_{2i+1} v_{2i+2}) = 0 \text{ for } 0 \leq i \leq n-1
\]
\[
f^*(v_{2i+1} u_{2i+2}) = 0 \text{ for } 0 \leq i \leq n-1
\]

Noticing the induced edge labelling we find that the number of vertices labelled with 0 is \( n \) and the number of vertices labelled with 1 is \( n \) and that the number of edges labelled with 0 is \( n+1 \) and the number of edges labelled with 1 is \( n \). Hence \( |v_j(0) - v_j(1)| \leq 1 \) and \( |e_j(0) - e_j(1)| \leq 1 \). Therefore the alternate triangular belt \( ATB(n) \) is a homo cordial labelling graph.

From the definition of one part \( ATB(n) \) we can claim that

\[
T_0(\text{ATB}(n)) = T_0(\text{ATB}(2)) + (n-2)T_0(F)
\]
\[
T_1(\text{ATB}(n)) = T_1(\text{ATB}(2)) + (n-2)T_1(F)
\]

Hence (a) implies (b)
Since each part of $ATB(n)(\downarrow\uparrow\downarrow\ldots)$ consists of 3 0’s and 3 1’s continuing in this pattern of calculating we can obtain the labelling procedure defined for the alternate triangular belt graph $ATB(n)(\downarrow\uparrow\downarrow\ldots)$ resulting in proving that $ATB(n)(\downarrow\uparrow\downarrow\ldots)$ is homo cordial labelling graph.

Hence the above statements are equivalent. Hence the proof.

CONCLUDING REMARKS

In this paper we have considered alternate triangular belt graph $ATB(n)(\downarrow\uparrow\downarrow\ldots)$ and proved that it is homo cordial labelling graph and have identified a generalisation method for alternate triangular belt graph to label the vertices and edges.

REFERENCES


