Calculating Optimum Gear Ratios of Two Step Bevel Helical Reducer

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Abstract

The article focuses on determining optimum gear ratios of a two step bevel helical reducer to obtain the minimum reducer length. An investigation was carried out into the effects of the input factors consisting of the total reducer ratio, the face width coefficient of the bevel and the helical gear sets, the allowable contact stress and the output torque. Furthermore, a simulation experiment was established with the aim of weighing their impacts on the optimum gear ratios. The obtained results contained the information on the influences of the input parameter and proposed equations which proved to be a helpful way of determining the optimum gear ratios.

1. INTRODUCTION

In designing optimum reducers, calculating optimum gear ratios plays a significant role. This is because the dimension, the mass, and therefore the cost of a reducer is dependent on the gear ratios. This can be clarified by Figure 1 which represents the relation between the gear ratio of the second step and the gear mass of a two step helical reducer. From the figure, with optimum gear ratio \( u_2 = 2 \), the gear mass (about 178 kg) is much less than that when \( u_2 = 5 \) (about 242 kg).

Up to now, there have been numerous studies for determining the gear ratios of helical reducers. For this reducer type, the gear ratios have been determined for two steps [1-4], three steps [4-8], and four steps [4, 8-10]. In addition, their calculation has been carried out by using several methods including the graph method [1, 2 and 4], the practical method [2] and the model method [5-10].

For bevel helical reducer, for determining the gear ratio of the bevel gear set \( u_1 \), we can use the graph in Figure 2 [1]. Besides, in [2] a practical method to find the optimum gear ratios of a two step bevel helical reducer was proposed. Based on the data from reducer factories, the authors noted that the reducer weight was minimum if the ratio between the diameter of the bevel gear wheel and the center distance of the helical gear \( d_{w2} / d_{w1} \) ranged from 1.12 to 1.4 [2]. Therefore, the optimal gear ratios of the reducer were proposed in the tabulated form.

The model method is widely used to determine the optimum gear ratios. In this method, the equations to determine the optimum gears are used for different objectives. The objectives can be the minimum reducer height [11], the minimum length of the reducer [12] or the minimum reducer cross-section area [13].

![Figure 1. Partial ratios versus gear mass of a two-step helical reducer](image1)

Calculation with \( u_h = 10; T_1 = 60000 \text{Nmm}; \beta = 12^0; k_H = 1.3 \)

![Figure 2. The gear ratio of the bevel gear set versus the total reducer ratio](image2)

This paper is concerned with the determination of the optimum gear ratios of a two step bevel helical reducer to
attain the minimum reducer length. Besides, the influence of
the input parameters on the optimum gear ratios was
inspected.

2. OPTIMIZATION PROBLEM

The reducer length is gained by (see Figure 3):

\[ L = d_{21} / 2 + a_w + d_{w2} / 2 \]  \hspace{1cm} (1)

In which, \( d_{21} \) is the outer pitch diameter of the wheel of bevel
gear set; \( a_w \) and \( d_{w2} \) are the center distance and the pitch
diameters of the helical gear set.

Therefore, the optimization problem can be defined as:

\[ \text{minimize } L \]  \hspace{1cm} (2)

With the following constraints:

\[ 1 \leq u_t \leq 6 \]
\[ 1 \leq u_s \leq 9 \]  \hspace{1cm} (3)

Where, \( u_t \) is the gear ratio of the bevel gear set; \( u_s \) is the
the gear ratio of the helical gear set;

From the above equations, it can be drawn that to solve the
optimization problem (2), it is essential to determine \( d_{21}, a_w \)
and \( d_{w2} \).

2.1. Determining the outer pitch diameter of the wheel

For a straight bevel gear set, the outer pitch diameter of the
wheel can be determined by:

\[ d_{21} = 2 \cdot R_e \cdot \left(1 + u_t^2\right)^{-1/2} \]  \hspace{1cm} (4)

Wherein, \( R_e \) is the external cone distance of the bevel gear set;
\( R_e \) is calculated by [14]:

\[ R_e = \frac{k_r \cdot \left(u_t^2 + 1\right)^{1/2} \cdot \left(1 - k_{w}\right) \cdot u_t \cdot \left[\sigma_{u}^2\right]^{1/3}}{T_{11} \cdot k_{H\beta} / \left(1 - k_{w}\right) \cdot u_t \cdot \left[\sigma_{u}^2\right]^{1/3}} \]  \hspace{1cm} (5)

In which, \( k_r \) is the coefficient contingent upon the gear type
and gear material; \( k_r = 50 \) (MPa\(^{1/2}\)) [14]; \( k_{w} = 0.25 \ldots 0.3 \) is the
face width coefficient; \( \left[\sigma_{u}^2\right] \) is the allowable contact
stress (MPa); \( K_{H\beta} \) is the pitting resistance contact load ratio;

From the tabulated data in [14], \( K_{H\beta} \) can be determined by
the following regression equations (with the coefficient of
determination \( R^2 = 1 \)):

\[ K_{H\beta} = 0.25 \cdot k^2 + 0.2 \cdot k + 1.02 \]  \hspace{1cm} (6)

Where, \( k = k_{w} \cdot u_t / (2 - k_{w}) \).

\[ T_{11} = T_{out} / \left(u_g \cdot \eta_{bg} \cdot \eta_{bg} \cdot \eta_{bg}^3\right) \]  \hspace{1cm} (7)

In which, \( u_g \) is the reducer total ratio; \( T_{out} \) is the output torque
(Nmm); \( \eta_{bg} \) is the transmission efficiency of the bevel gear set
(\( \eta_{bg} = 0.95 \ldots 0.995 \) [14]); \( \eta_{bg} \) is the transmission efficiency of
the helical gear set (\( \eta_{bg} = 0.96 \ldots 0.98 \) [14]); \( \eta_{bg} \) is the
transmission efficiency of a pair of rolling bearing
(\( \eta_{bg} = 0.99 \ldots 0.995 \) [14]). Choosing \( \eta_{bg} = 0.96 \cdot \eta_{bg} = 0.97 \cdot
\eta_{bg} = 0.992 \) and substituting them into (7) gives:

\[ T_{11} = 1.101 \cdot T_{out} / u_g \]  \hspace{1cm} (8)

Substituting \( k_r = 50 \) and (8) into (5) gets:

\[ R_e = 51.6296 \cdot \left(u_t^2 + 1\right)^{1/2} \cdot \left(T_{out} \cdot k_{H\beta} / \left(1 - k_{w}\right) \cdot u_t \cdot \left[\sigma_{u}^2\right]^{1/3}\right) \]  \hspace{1cm} (9)

2.2. Determining the center distance of the second step

For the helical gear set, the center distance \( a_{w2} \) is calculated by [14]:

\[ a_{w2} = k_m \cdot \left(u_s + 1\right) \cdot \left(T_{12} \cdot k_{H\beta} / \left(1 - k_{w}\right) \cdot u_s \cdot [\sigma_e]^2\right)^{1/3} \]  \hspace{1cm} (10)

Where, \( k_m \) is the material coefficient; \( k_m = 43 \) because the
gear material is steel [14]; \( K_{H\beta} \) is the contact load ratio for
pitting resistance; we can choose $k_{H\beta} = 1.1$ because $k_{H\beta} = 1.02 \pm 1.28$ [14]; $[\sigma_H]$ is the allowable contact stress (MPa); In practice, $[\sigma_H] = 350 \ldots 420$ (MPa); $\psi_{ba2}$ is the wheel face width coefficient of the helical gear set; $\psi_{ba2} = 0.35 \ldots 0.4$ [14];

In addition, for this reducer we have:

$$T_{out} = T_{12} \cdot \eta_g \cdot \eta_h^2 \cdot u_2$$  \hspace{1cm} (11)

In which, $T_{12}$ is the torque on the pinion shaft (Nmm).

Choosing $\eta_g = 0.97$ and $\eta_h = 0.992$ as in Section 2.1 and substituting them into (11) gives:

$$T_{12} = 1.0476 \cdot T_{out} / u_2$$  \hspace{1cm} (12)

Substituting (12) and $k_{H\beta} = 1.1$ into (10) we have:

$$a_{w2} = 45.0814 \cdot (u_2 + 1) \cdot \left( \frac{T_{out}}{\left( \left[ \sigma_H \right]^2 \cdot u_2^2 \cdot \psi_{ba2} \right)^{1/3}} \right)$$  \hspace{1cm} (13)

The pitch diameter of the helical gear set then is identified by [14]:

$$d_{w2} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1)$$  \hspace{1cm} (14)

### 2.3. Experimental work

A simulation experiment with 2-level full factorial experimental design was performed with the assistance of a computer program to explore the influence of the input factors on the optimum gear ratios. Additionally, the computer program was designed based on equations (2) and (3). It is also reported that 5 factors were taken into the exploration (see Table 1). Consequently, 32 numbers of runs were carried out in the test. The experimental results are illustrated in Table 2.

### Table 1. Input parameters.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Code</th>
<th>Unit</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total reducer ratio</td>
<td>$u_g$</td>
<td>-</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Coefficient of the face width of bevel gear set</td>
<td>$k_{be}$</td>
<td>0.25</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Coefficient of wheel face width of helical gear set</td>
<td>$x_{ba2}$</td>
<td>-</td>
<td>0.35</td>
<td>0.4</td>
</tr>
<tr>
<td>Allowable contact stress</td>
<td>AS</td>
<td>MPa</td>
<td>350</td>
<td>420</td>
</tr>
<tr>
<td>Output torque</td>
<td>$T_{out}$</td>
<td>Nmm</td>
<td>$10^5$</td>
<td>$10^7$</td>
</tr>
</tbody>
</table>

### Table 2. Experimental plans and output response.

<table>
<thead>
<tr>
<th>StdOrder</th>
<th>RunOrder</th>
<th>CenterPt</th>
<th>Blocks</th>
<th>$u_g$</th>
<th>$k_{be}$</th>
<th>$x_{ba2}$</th>
<th>AS (MPa)</th>
<th>$T_{out}$ (Nm)</th>
<th>$u_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0.25</td>
<td>0.4</td>
<td>350</td>
<td>10000</td>
<td>1.75</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td>0.25</td>
<td>0.35</td>
<td>350</td>
<td>100</td>
<td>4.68</td>
</tr>
<tr>
<td>30</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td>0.25</td>
<td>0.4</td>
<td>420</td>
<td>10000</td>
<td>4.54</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td>0.25</td>
<td>0.4</td>
<td>420</td>
<td>100</td>
<td>4.54</td>
</tr>
<tr>
<td>19</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0.3</td>
<td>0.35</td>
<td>350</td>
<td>10000</td>
<td>1.83</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0.25</td>
<td>0.35</td>
<td>350</td>
<td>10000</td>
<td>1.80</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0.3</td>
<td>0.4</td>
<td>350</td>
<td>100</td>
<td>1.77</td>
</tr>
<tr>
<td>18</td>
<td>31</td>
<td>1</td>
<td>1</td>
<td>30</td>
<td>0.25</td>
<td>0.35</td>
<td>350</td>
<td>10000</td>
<td>4.68</td>
</tr>
<tr>
<td>25</td>
<td>32</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>0.25</td>
<td>0.35</td>
<td>420</td>
<td>10000</td>
<td>1.80</td>
</tr>
</tbody>
</table>
The main effects plot for $u_1$ is exhibited in Figure 4. It is noticeable that the optimum gear ratio is largest influenced by the total reducer ratio $U_g$, whereas it is almost not dependent on the allowable contact stress $AS$ and the output torque $T_{out}$. However, the wheel face width coefficients of bevel gear set and helical gear set $K_{be}$ and $X_{ba2}$ do affect the optimum gear ratio to a low extent.

The Pareto chart of the standardized effects is demonstrated in Figure 5. It is interpreted that those factors consisting of total reducer ratio, the wheel face width coefficients of the bevel gear set and the helical gear set are statistically significant at the 0.05 level with the response model. The indicator’s signal is on the bars denoting the factors passing the reference line.
The Normal Plot of the standardized effects on \( u_1 \) is presented in Figure 6. It can be seen from the figure that the total reducer ratio \( u_1 \) (factor A) is the most significant parameter for \( u_1 \). In addition, it has a positive standardized effect on \( u_1 \). Besides, \( k_{be} \) (factor B) and \( \psi_{ba2} \) (factor C) have a negative standardized effect on \( u_1 \).

Figure 7 illustrates the estimated effects and coefficients for \( u_1 \). It is elucidated from the figure that some factors such as the total reducer ratio, the wheel face width coefficients of the bevel gear and the helical gear sets and their interactions have P-values lower than 0.05, which signifies that they influence the response significantly. Subsequently, \( u_1 \) can be reached by:

\[
\begin{align*}
    u_1 &= 1.44053 + 0.157084 \cdot u_g + 0.3392 \cdot k_{be} - 1.2068 \cdot \psi_{ba2} - \\
    &- 0.075844 \cdot u_g \cdot k_{be} - 0.06598 \cdot u_g \cdot \psi_{ba2} + 1.442 \cdot k_{be} \cdot \psi_{ba2} \\
    &= 1.44053 + 0.157084 \cdot u_g + 0.3392 \cdot k_{be} - 1.2068 \cdot \psi_{ba2} - \\
    &- 0.075844 \cdot u_g \cdot k_{be} - 0.06598 \cdot u_g \cdot \psi_{ba2} + 1.442 \cdot k_{be} \cdot \psi_{ba2}
\end{align*}
\]

(15)

Figure 7 displays the adj-R\(^2\) and pred-R\(^2\) with the high values. Therefore, the equation (15) appropriately fits the data. This equation is used for determination of the optimum gear ratio of the bevel gear set \( u_1 \). After that, the optimum gear ratio of the helical gear set is calculated by \( u_2 = u_g / u_1 \).

3. CONCLUSIONS

The determination of the optimum gear ratios of a two step bevel helical reducer for the minimum reducer length was performed. Several input parameters such as the total ratio of the reducer, the wheel face width coefficients of the bevel gear and helical gear sets, the allowable contact stress and the output torque, and their impacts on the optimum gear ratios were taken into consideration. More importantly, to achieve the minimum reducer length, some equations for calculating the optimum gear ratios were recommended.

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REFERENCES


[10] Le Xuan Hung, Vu Ngoc Pi and Nguyen Van Du 2009 The international symposium on Mechanical Engineering ISME Ho Chi Minh city, Vietnam, 21-23, September


[13] Vu Ngoc Pi, Nguyen Thi Hong Cam and Nguyen Khac Tuan 2016 Environmental Science and Engineering A 5 566-69