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Plane Micropolar Fluid Through a Porous Medium: Exact Solution by Hodograph Transformation

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Abstract

The motion of a steady, homogenous, incompressible, plane micropolar fluid through a porous medium has been considered. The governing continuity, momentum and angular momentum equations are converted into a system of linear partial differential equations by means of hodograph transformation. Further the flow equations have been obtained in terms of Legendre transform function of the stream function. Results are summarized in the form of a theorem. Lastly some examples have been taken as application to illustrate the developed theory and exact solutions and geometry of the flow are obtained in each case.

Keywords: Micropolar fluid, exact solution, hodograph transformation, Legendre transform function, porous medium

1. INTRODUCTION

Eringen[1] [2]first gave and further developed the theory and the constitutive equations of flow of micropolar fluids. The characteristic of these fluids is that they show certain microscopic effects due to local structure and micro motion of the fluid elements [3]. A micro-rotation vector describes, in this class of fluid, the rigid particles that are contained in a small volume element which rotate about the center of the volume element. This local rotation of the particles is in addition to the usual rigid body motion of the entire volume element [4]. Liquid crystals, suspension solutions, some fluids, flow of colloidal fluids, blood, fluid with bar like elements or dumbbell molecules may be represented by the mathematical model underlying micropolar fluids [5].This model includes the effects of local rotary and couple stresses.

In the studies of micropolar fluids the non-linearities increase considerably and present a great deal of difficulty in finding the exact solutions. For obtaining exact solutions transformations are used for reformulation of equations in solvable form. Some researchers have used hodograph transformation in order to linearise the system of governing equations and got some exact solutions. This method has been widely used and W.F. Ames [6] has given an excellent survey to this method together with applications to various other fields.

Hodograph transformation method is used in various fields of research such as gas dynamics [7,8], linear viscous fluids [9], non-Newtonian fluids [10,11], and MHD Newtonian and non-Newtonian fluid flows [12,13]. Chandna, et al. [9,14,15,16] applied hodograph transformation method to study various kinds of flows. Many other authors [17,18,19,20,21,22,23,24] used this technique to study various fluid flows.

Fluid flow through porous media has become very important because of ground water flows and variety of tertiary oil recovery processes. Also there are many practical applications of fluid flow through porous medium, including filtration flow in packed column, permeation of water or oil within matrix of porous rock etc. Many researchers [25,26,27,28,29] have studied fluid flows through porous medium in different flow problems.

In this paper we have considered the flow of a steady, homogenous, incompressible, plane micropolar fluid through a porous medium. We have applied the hodograph transformation method to convert the governing non-linear system of partial differential equations into linear form. Further we have obtained the flow equations in terms of Legendre transform function of the stream function. We have considered two examples as application to illustrate the developed theory and found out exact solutions of the flow.

2. BASIC EQUATIONS

The basic equations governing the motion of steady plane flow of homogenous, incompressible, micropolar fluid through porous media are

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho(\mathbf{V}\cdot\nabla)\mathbf{V} = -\nabla p + \kappa(\nabla\times\boldsymbol{\nu}) - (\mu+\kappa)\nabla\times(\nabla\times\mathbf{V}) - \frac{\eta}{k}\mathbf{V}, \tag{2}$$

$$\rho j(\mathbf{V}\cdot\nabla)\boldsymbol{\nu} = (\alpha+\beta+\gamma)\nabla(\nabla\cdot\boldsymbol{\nu}) - \gamma\nabla\times(\nabla\times\boldsymbol{\nu}) + \kappa(\nabla\cdot\mathbf{V}) - 2\kappa\boldsymbol{\nu}, \tag{3}$$

where $\mathbf{V}=$ velocity field vector, $p=$ pressure function, $\rho=$ the constant fluid field density, $\boldsymbol{\nu}=$ micro-rotation vector, $\eta=$ coefficient of viscosity, $k=$ permeability of the medium, $j=$ micro inertia or gyration and $\mu,\kappa,\alpha,\beta,\gamma$ are the material constants.

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If the velocity and micro-rotation components are $\left(u(x,y), v(x,y)\right)$ and $\left(0,0,\Omega(x,y)\right)$ respectively then the governing equations (1)-(3) takes the form as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\ , \tag{4}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\kappa\frac{\partial \Omega}{\partial y}-(\mu+\kappa)\frac{\partial \omega}{\partial y}-\frac{\eta}{k}u\ , \tag{5}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\kappa\frac{\partial \Omega}{\partial x}+(\mu+\kappa)\frac{\partial \omega}{\partial x}-\frac{\eta}{k}v\ , \tag{6}$$

$$\rho j\left(u\frac{\partial \Omega}{\partial x}+v\frac{\partial \Omega}{\partial y}\right)=-2\kappa\Omega+\kappa\omega+\gamma\left(\frac{\partial^2 \Omega}{\partial x^2}+\frac{\partial^2 \Omega}{\partial y^2}\right), \tag{7}$$

where $\omega(x, y)=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$, being the two-dimensional vorticity function.

Equations (4)-(7) form four partial differential equations in four unknown functions $u(x,y)$, $v(x,y)$, $\Omega(x,y)$ and $p(x,y)$.

Let us define a generalized energy function h(x, y) as:

$$h(x,y)=\frac{1}{2}\rho\left(u^2+v^2\right)+p, \tag{8}$$

also $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$.

Using the above, the equations (5), (6) and (7) become

$$\frac{\partial h}{\partial x}=\rho v\omega+\kappa\frac{\partial \Omega}{\partial y}-(\mu+\kappa)\frac{\partial \omega}{\partial y}-\frac{\eta}{k}u\ , \tag{9}$$

$$\frac{\partial h}{\partial y}=-\rho u\omega-\kappa\frac{\partial \Omega}{\partial x}+(\mu+\kappa)\frac{\partial \omega}{\partial x}-\frac{\eta}{k}v\ , \tag{10}$$

$$\rho j\left(u\frac{\partial \Omega}{\partial x}+v\frac{\partial \Omega}{\partial y}\right)=-2\kappa\Omega+\kappa\omega+\gamma\nabla^2\Omega, \tag{11}$$

The above system of three partial differential equations (9)-(11) along with equation (4) and equation for vorticity function ω contains five unknown functions u, v, ω , Ω and h as functions of (x, y) govern steady plane flows of an incompressible micropolar fluid through porous media. Once a solution of these equations are found, the pressure function is determined from the expression for h(x, y) given in (8).

3. EQUATIONS IN HODOGRAPH PLANE

Letting the function $u=u(x, y)$ and $v=v(x, y)$ to be such that, in the region of flow, the Jacobian

$$J(x,y)=\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}-\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\neq 0,\ |J|<\infty\ . \tag{12}$$

We may consider x and y as functions of u and v. By means of $x=x(u, v)$, $y=y(u, v)$, we derive the following relations:

$$\frac{\partial u}{\partial x}=J\frac{\partial y}{\partial v}\ ,\ \frac{\partial u}{\partial y}=-J\frac{\partial x}{\partial v}\ ,$$

$$\frac{\partial v}{\partial x}=-J\frac{\partial y}{\partial u}\ ,\ \frac{\partial v}{\partial y}=J\frac{\partial x}{\partial u}\ . \tag{13}$$

We also obtain the relations

$$\frac{\partial g}{\partial x}=\frac{\partial(g,y)}{\partial(x,y)}=\bar{J}\frac{\partial(g,y)}{\partial(u,v)}\ ,$$
$$\frac{\partial g}{\partial y}=-\frac{\partial(g,x)}{\partial(x,y)}=\bar{J}\frac{\partial(x,g)}{\partial(u,v)}\ . \tag{14}$$

Where g = g(x, y) = g(x(u, v), y(u, v)) = g(u, v) is any continuously differentiable function and

$$J=J(x,y)=\frac{\partial(u,v)}{\partial(x,y)}=\left[\frac{\partial(x,y)}{\partial(u,v)}\right]^{-1}=\bar{J}(u,v)\ . \tag{15}$$

Employing these transformation relations for the first order partial derivatives and the transformation equations for the functions $\boldsymbol{\omega}, \boldsymbol{\Omega}$ and $\mathbf{h}$, defined by

$$\omega(x,y)=\omega\left(x(u,v),y(u,v)\right)=\boldsymbol{\omega}(u,v),$$

$$\Omega(x,y)=\Omega\left(x(u,v),y(u,v)\right)=\boldsymbol{\Omega}(u,v)$$

$$h(x,y)=h\left(x(u,v),y(u,v)\right)=\mathbf{h}(u,v),$$

equation (4) and the system of equations (9)-(11) is replaced by the following system in the hodograph plane (u, v) :

$$\frac{\partial x}{\partial u}+\frac{\partial y}{\partial v}=0\ , \tag{16}$$

$$\bar{J}\frac{\partial(\mathbf{h},y)}{\partial(u,v)}=\rho v\boldsymbol{\omega}+\kappa\bar{J}Q_1-(\mu+\kappa)\bar{J}w_1-\frac{\eta}{k}u\ , \tag{17}$$

$$\bar{J}\frac{\partial(x,\mathbf{h})}{\partial(u,v)}=-\rho u\boldsymbol{\omega}-\kappa\bar{J}Q_2+(\mu+\kappa)\bar{J}w_2-\frac{\eta}{k}v, \tag{18}$$

$$\rho j\bar{J}\left(vQ_1+uQ_2\right)=$$
$$-2\kappa\boldsymbol{\Omega}+\kappa\boldsymbol{\omega}+\gamma\bar{J}\left\{\frac{\partial\left(x,\bar{J}Q_1\right)}{\partial(u,v)}+\frac{\partial\left(\bar{J}Q_2,y\right)}{\partial(u,v)}\right\}, \tag{19}$$

$$\bar{J}\left(\frac{\partial x}{\partial v}-\frac{\partial y}{\partial u}\right)=\boldsymbol{\omega}, \tag{20}$$

where,

$$Q_1 = Q_1(u,v) = \frac{\partial(x,\boldsymbol{\Omega})}{\partial(u,v)}, \quad Q_2 = Q_2(u,v) = \frac{\partial(\boldsymbol{\Omega},y)}{\partial(u,v)}, \tag{21}$$

$$w_1 = w_1(u,v) = \frac{\partial(x,\boldsymbol{\omega})}{\partial(u,v)}, \quad w_2 = w_2(u,v) = \frac{\partial(\boldsymbol{\omega},y)}{\partial(u,v)}. \tag{22}$$

System of equations (16) to (20) is a system of five equations for the five unknown functions x (u, v), y (u, v), $\boldsymbol{\omega}$ (u, v), $\boldsymbol{\Omega}$ (u, v) and **h** (u, v).

The equation of continuity implies the existence of a stream-function ψ (x, y) such that

$$d\psi = -v dx + u dy \text{ or } \frac{\partial\psi}{\partial x} = -v, \ \frac{\partial\psi}{\partial y} = u. \tag{23}$$

Likewise equation (16) implies the existence of a function L(u, v) , called the Legendre transform function of the stream-function ψ (x, y) , so that

$$dL = -y du + x dv \text{ or } \frac{\partial L}{\partial u} = -y, \ \frac{\partial L}{\partial v} = x, \tag{24}$$

and the two functions $\psi(x, y)$ and L(u, v) are related by

$$L(u,v) = vx - uy + \psi(x,y). \tag{25}$$

Introducing L(u, v) in the system (16)-(19), with $\bar{J}$, Q_1, Q_2, w_1, w_2, given by (15), (21) and (22) respectively, it follows that (16) is identically satisfied and the system may be replaced by

$$\bar{J}\frac{\partial\left(\frac{\partial L}{\partial u},\mathbf{h}\right)}{\partial(u,v)} = \rho v\boldsymbol{\omega} + \kappa\bar{J}Q_1 - (\mu+\kappa)\bar{J}w_1 - \frac{\eta}{k}u, \tag{26}$$

$$\bar{J}\frac{\partial\left(\frac{\partial L}{\partial v},\mathbf{h}\right)}{\partial(u,v)} = -\rho u\boldsymbol{\omega} - \kappa\bar{J}Q_2 + (\mu+\kappa)\bar{J}w_2 - \frac{\eta}{k}v, \tag{27}$$

$$\rho j\bar{J}(vQ_1 + uQ_2) = -2\kappa\boldsymbol{\Omega} + \kappa\boldsymbol{\omega} + \gamma\bar{J}\left\{\frac{\partial\left(\frac{\partial L}{\partial v},\bar{J}Q_1\right)}{\partial(u,v)} + \frac{\partial\left(\frac{\partial L}{\partial u},\bar{J}Q_2\right)}{\partial(u,v)}\right\}, \tag{28}$$

$$\bar{J}\left[\frac{\partial^2 L}{\partial v^2} + \frac{\partial^2 L}{\partial u^2}\right] = \boldsymbol{\omega}, \tag{29}$$

where

$$\bar{J} = \left[\frac{\partial^2 L}{\partial v^2}\frac{\partial^2 L}{\partial u^2} - \left(\frac{\partial^2 L}{\partial u\partial v}\right)^2\right]^{-1}, \tag{30}$$

$$Q_1 = \frac{\partial\left(\frac{\partial L}{\partial v},\boldsymbol{\Omega}\right)}{\partial(u,v)}, \ Q_2 = \frac{\partial\left(\frac{\partial L}{\partial u},\boldsymbol{\Omega}\right)}{\partial(u,v)}, \tag{31}$$

$$w_1 = \frac{\partial\left(\frac{\partial L}{\partial v},\boldsymbol{\omega}\right)}{\partial(u,v)}, \ w_2 = \frac{\partial\left(\frac{\partial L}{\partial u},\boldsymbol{\omega}\right)}{\partial(u,v)}, \tag{32}$$

By using the integrability condition

$$\left[\bar{J}\frac{\partial^2 L}{\partial u\partial v}\frac{\partial}{\partial v} - \bar{J}\frac{\partial^2 L}{\partial v^2}\frac{\partial}{\partial u}\right]\left(\bar{J}\frac{\partial\left(\frac{\partial L}{\partial u},\mathbf{h}\right)}{\partial(u,v)}\right) = \left[\bar{J}\frac{\partial^2 L}{\partial u^2}\frac{\partial}{\partial v} - \bar{J}\frac{\partial^2 L}{\partial u\partial v}\frac{\partial}{\partial u}\right]\left(\bar{J}\frac{\partial\left(\frac{\partial L}{\partial v},\mathbf{h}\right)}{\partial(u,v)}\right),$$

i.e $\partial^2 h/\partial x\partial y = \partial^2 h/\partial y\partial x$ in the (x,y) plane, we eliminate $\mathbf{h}(u, v)$ from (26) and (27) and obtain

$$-\kappa\left[\frac{\partial(\partial L/\partial v,\bar{J}Q_1)}{\partial(u,v)} + \frac{\partial(\partial L/\partial u,\bar{J}Q_2)}{\partial(u,v)}\right] + (\mu+\kappa)\left[\frac{\partial(\partial L/\partial v,\bar{J}w_1)}{\partial(u,v)} + \frac{\partial(\partial L/\partial u,\bar{J}w_2)}{\partial(u,v)}\right] + \frac{\eta}{k}\left[\frac{\partial(\partial L/\partial v,u)}{\partial(u,v)} - \frac{\partial(\partial L/\partial u,v)}{\partial(u,v)}\right] - \rho(uw_2 + vw_1) = 0, \tag{33}$$

where $\bar{J}$, Q_1, Q_2, w_1, w_2 are given in (30) , (31) and (32) . Summing up we have the following theorem:

THEOREMI : If L(u, v) is the Legendre transform function of a stream-function of steady, plane, incompressible, flow of a micropolar fluid through a porous media then L(u, v) must satisfy equation (28) and (33) where $\boldsymbol{\omega}$ (u, v), $\bar{J}$ (u, v), Q_1(u, v), Q_2(u, v), w_1(u, v), w_2 (u, v) are given by (29), (30), (31) and (32).

On finding L (u, v) and Ω (u, v), the velocity components are obtained from (24). Then p(x, y) and Ω (x, y) can be determined in the physical plane using the velocity components u(x, y) and v(x, y).

4. APPLICATION I

Let

$$L(u,v) = A\left(u^2 - v^2\right), \tag{34}$$

be the Legendre-transform function, where A is arbitrary non zero constant and. Using (34) in equations (29)-(32) we obtain

$$\bar{J} = -\frac{1}{4A^2}, \ \boldsymbol{\omega} = 0, w_1(u,v) = w_2(u,v) = 0$$
$$Q_1 = 2A\frac{\partial\boldsymbol{\Omega}}{\partial u}, \quad Q_2 = 2A\frac{\partial\boldsymbol{\Omega}}{\partial v}. \tag{35}$$

Using (34) and (35), in equations (33) and (28) give the following system of equations for $\mathbf{\Omega}$ (u,v)

$$\frac{\partial^2\mathbf{\Omega}}{\partial u^2}+\frac{\partial^2\mathbf{\Omega}}{\partial v^2}=0,\ \rho j\left(u\frac{\partial\mathbf{\Omega}}{\partial v}+v\frac{\partial\mathbf{\Omega}}{\partial u}\right)=2\kappa\mathbf{\Omega}. \quad (36)$$

Solving (36), we get

$$\mathbf{\Omega}(u,v)=C_1\left(u+\frac{4A\kappa}{\rho j}v\right), \quad (37)$$

$$A=\frac{\rho j}{4\kappa}, \quad (38)$$

$$\frac{2\kappa}{\rho j}=1, \quad (39)$$

where C_1 is an arbitrary non-zero constant.

Using (34) in (24) and solving the resulting equations we get

$$u(x,y)=\frac{-y}{2A},\ v(x,y)=\frac{-x}{2A}. \quad (40)$$

Using (40) in (37) and (9), (10) we obtain Ω (x, y) and p(x, y) given by

$$\Omega(x,\ y)=C_1\left(\frac{-y}{2A}-\frac{2\kappa x}{\rho j}\right), \quad (41)$$

$$p(x,y)=\kappa C_1\left(-\frac{x}{2A}+\frac{2\kappa y}{\rho j}\right)-\frac{\rho}{8A^2}\left(x^2+y^2\right)+\frac{\eta}{kA}xy+p_\circ, \quad (42)$$

where $p_\circ$ is an arbitrary constant.

And the streamlines are given by

$x^2-y^2=$ Constant.

This shows that the streamlines of the flow equations are concentric hyperbolas.

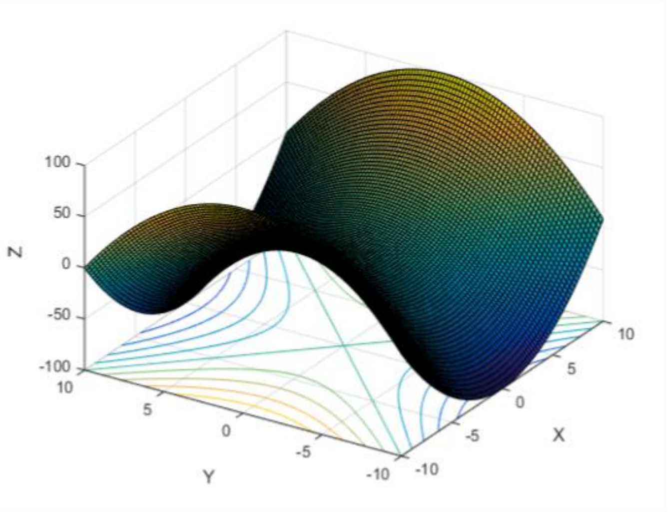

Figure 1. Streamsurface and streamlines for $x^2-y^2=$ constant

THEOREM II : If $L(u,v)=A\left(u^2-v^2\right)$ is the Legendre transform function of a stream function for steady, plane, incompressible, flow of a micropolar fluid through porous media, then the flow in the physical plane is a flow with concentric hyperbolic streamlines with flow variables given by (40), (41) and (42).

5. APPLICATION II

Let $L(u,v)=uv$, (43)

be the Legendre- transform function.

Using (43) in (29)-(32), we find that

$$\bar{J}=-1,\ \boldsymbol{\omega}=0,\ w_1=w_2=0,$$

$$Q_1=\frac{\partial\mathbf{\Omega}}{\partial v},\quad Q_2=-\frac{\partial\mathbf{\Omega}}{\partial u}. \quad (44)$$

Using (43) and (44) in equations (33) and (28) give the following system of equations for $\mathbf{\Omega}$ (u,v)

$$\frac{\partial^2\mathbf{\Omega}}{\partial u^2}+\frac{\partial^2\mathbf{\Omega}}{\partial v^2}=0,$$

$$\rho j\left(v\frac{\partial\mathbf{\Omega}}{\partial v}-u\frac{\partial\mathbf{\Omega}}{\partial u}\right)=2\kappa\mathbf{\Omega}. \quad (45)$$

Solving these equations , we get

$$\mathbf{\Omega}(u,v)=C_2 v, \quad (46)$$

$$\frac{2\kappa}{\rho j}=1, \quad (47)$$

where C_2 is an arbitrary constant.

On proceeding as in Application I, we get

$$u(x,y)=x,\qquad v(x,y)=-y, \quad (48)$$

$$\Omega(x,\ y)=-C_2 y, \quad (49)$$

and

$$p(x,y)=-C_2\kappa x-\frac{\eta}{2k}\left(x^2-y^2\right)-\frac{1}{2}\rho\left(x^2+y^2\right)+p_1, \quad (50)$$

where p_1 is an arbitrary constant.

And the streamlines are given by

$yx=$ Constant.

This shows that the streamlines of the flow equations are rectangular hyperbolae.

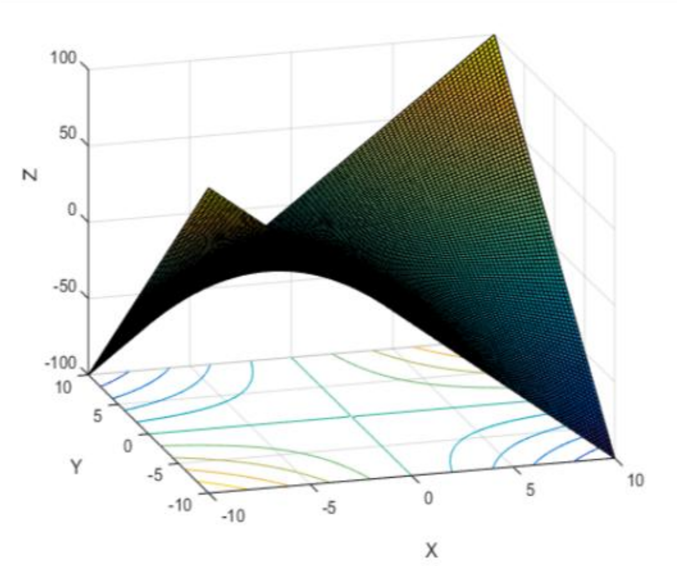

Figure 2. Streamsurface and streamlines for $yx = \text{constant}$

THEOREM III : If $L(u,v) = uv$ be the Legendre transform function of a stream function for steady, plane, incompressible, flow of micropolar fluid through porous media, then the flow in the physical plane is a flow with rectangular hyperbolae streamlines with flow variables given by (48) , (49) and (50).

In the absence of porous media i.e. the term $\frac{\eta}{k} \to 0$, and using appropriate constants we recover the results of Indrasena Adluri [5].

6. CONCLUSION

In this paper, the analytical solution of nonlinear equations governing the flow of micropolar fluid through porous media is obtained using hodograph transformation technique. Illustrations have been made taking different forms of Legendre transform function. The expressions for velocity, micro-rotation, streamline and pressure distribution are constructed in each case. Streamsurface and streamline patterns are also plotted. The present analysis is more general and the results of Indrasena Adluri [5] can be recovered in the limiting case.

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