Analysis of Elastic Scattering of $\alpha$ Particles on $^{70,72,74,76}$ Ge targets at $E_{Lab}=25\text{MeV}$

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Abstract
The elastic scatterings of $\alpha$ particles on $^{70,72,74,76}$ Ge targets at $E_{Lab}=25\text{MeV}$ have been analyzed by using combination between McIntyre model and Regge pole model which is based on concept of strong absorption parameterization of the scattering matrix elements, by trying to fit the experiment data of elastic scattering. The calculated elastic differential cross sections show agreement with the experimental data, seven parameters extract from models employed as fixed entries in the fitting process for forward angle and backward angle of angular distribution.

When increase in the size of target ion the interaction radius and total reaction cross-section are found increasing, parameters are obtained for colliding nuclei together for all reaction cross sections.

INTRODUCTION
The elastic scattering data is used to explain of heavy-ion one of the important researches in nuclear physics. The theoretical models have been used to analyze the experimental data. A phenomenological model based on the diffraction phenomenon its can analyze experimental data of elastic scattering of heavy nuclear ions in terms of the asymptotic properties of the scattered wave function.

In many nuclear scattering the incident particle is strongly absorbed by entering the target nucleus, Using strong absorption conditions. The experimental data can be explained without any knowledge about the details of the absorption process. This allowed for developing proper for both the amplitude and phase of scattering wave function.[1-3]

This work propose to analyze the experimental data of angular distribution for elastic scattering of $\alpha$-particles from $^{70,72,74,76}$Ge, at laboratory energy equal 25MeV using the McIntyre plus Regge models. The analysis will be start the numerical method based on parameterizations of scattering matrix elements and used by the strong absorption model (SAM). This analysis will explain the diffraactive condition, and then the parameters and quantities are extracted Such as the grazing angular momentum $l_g$, diffusivity, etc (1-3)

THEORY
The amplitude $f(\theta)$ as a function as partial wave expansion for elastic scattering expressed by:

$$ f(\theta) = \frac{1}{i\kappa} \sum_{\ell=0}^{\infty} (2\ell + 1) S_{\ell} (\cos \theta) P_{\ell}(\cos \theta). \quad (1) $$

$S_{\ell}$ is the complex amplitude of the $\ell$th scattered partial wave, $P_{\ell}(\cos \theta)$ is the Legendre polynomial of order $\ell$ and $k$ is the wave number. The scattering matrix amplitude is express by:

$$ S_{\ell} = \eta_{\ell} e^{2i\delta_{\ell}}. \quad (2) $$

The Coulomb phase shifts are given by the exact solution of the Rutherford scattering problem:

$$ \sigma_{\ell} = \arg \Gamma(\ell + 1 + in). \quad (3) $$

the semi-classical strong absorption model has the sharp cutoff conditions (1,3):

$$ \begin{align*}
\eta_{\ell} &= 0 \quad S_{\ell} = 0 \quad \text{if} \quad \ell \leq \ell_g \\
\eta_{\ell} &= 1 \quad S_{\ell} = e^{2i\delta_{\ell}} \quad \text{if} \quad \ell > \ell_g.
\end{align*} \quad (4) $$

Here $\eta_{\ell}$ is called the reflection coefficient of the outgoing $\ell$th partial wave determined by the boundary conditions at the nuclear surface. This means that waves up to the grazing angular momentum $\ell_g$ are completely absorbed.

The sum of radii R of the projectile $R_p$, and target nuclei $R_T$

This strong interaction radius has been defined:

$$ R = r_0 \left( A_p^{1/3} + A_T^{1/3} \right) \quad (5) $$

Regge-pole factor expressed as:

$$ S(\ell) = [1 + e^{-i\alpha} e^{i(\ell - - i\Delta)}]^{-1} \left[ 1 + \frac{\ell - \ell_g - i\delta(\ell)}{\ell - \ell_g - i\Gamma(\ell)/2} \right]. \quad (6) $$

Or

$$ S(\ell) = [1 + e^{-i\alpha} e^{i(\ell - - i\Delta)}]^{-1} \left[ 1 + \frac{iD(\ell)}{\ell - \ell_g - i\Gamma(\ell)/2} \right]. $$

The $z(\ell)$ and $p(\ell)$ functions, in equation, represent the Regge zero and Regge pole at a complex $\ell$. (1)
where amplitude of the pole is

\[ D(\ell) = D_0 [1 - \text{Re} S_\ell (BG)] \] (7)

and the width of the pole is

\[ \Gamma(\ell) = \Gamma_0 [1 - \text{Re} S_\ell (BG)] \] (8)

The phase of the pole is represented by \( \phi_{\ell,0} \). [1-4]

RESULT AND DISCUSSION

The available experimental data of scattering \(^4\text{He}\) by different target nuclei \(^{70}\text{Ge}, ^{72}\text{Ge}, ^{74}\text{Ge} \) and \(^{76}\text{Ge}\) are analyzed using the combined model of McIntyre and Regge by fitting the calculated results to the experimental data. We use FORTRAN code to obtain the best fit of experiment data of angular distribution and the final choice of extracted parameters depend on the value of \( \chi^2 \) for each model is. The \( \chi^2 \) is defined by the expression:

\[
\chi^2 = \frac{1}{N} \sum_i \left( \frac{\sigma_{\text{theory}}^i - \sigma_{\text{exp}}^i}{\Delta \sigma_{\text{exp}}^i} \right)^2
\]

Here, \( \sigma_{\text{theory}}^i \), \( \sigma_{\text{exp}}^i \), and \( \Delta \sigma_{\text{exp}}^i \) are theoretical, experimental cross section and the corresponding error in cross section, respectively, \( N \) is the number of experimental data points. However, the average value of 10% of the experimental measurement is taken for each experimental error of the energy of the elastic scattering under study (1,4).

One of the aims of this work is to investigate the change in total cross section of scattered ions when size of target ion change, for incident projectile. This study is also objected to better understanding of the diffractive features of elastic scattering of heavy ions and extract important parameters (like radius of interaction region of scattered ions \( R \) and diffusivity \( d \)). (3)

Elastic scattering of \(^4\text{He}+^{70,72,74,76}\text{Ge} \) target nuclei.

McIntyre plus Regge pole are used to analyze the experimental data of angular distribution for elastic scattering of \( \alpha \) particle by different target nuclei \(^{70}\text{Ge}, ^{72}\text{Ge}, ^{74}\text{Ge} \) and \(^{76}\text{Ge} \) at Energy 25 MeV

Figures 1 present the theoretical results of angular distribution for the elastic scatterings \(^4\text{He}+^{70,72,74,76}\text{Ge} \) at laboratory energy 25 MeV, The experimental data (symbols) of angular distribution of elastic scattering of \( \alpha \) particles by the target nucleus \(^{70,72,74,76}\text{Ge} \) at laboratory energies 25 MeV, compared to the theoretical results obtained using the combined model of McIntyre plus Regge (solid line) are shown.

![Figure 1](image-url)

**Figure 1:** The experimental data (symbols) of angular distribution of elastic scattering of \( \alpha \) particles by the target nucleus \(^{70,72,74,76}\text{Ge} \) at laboratory energies 25 MeV, are shown and compared to the theoretical results obtained using the combined model of McIntyre plus Regge (solid line).
The list of fitting parameters which also includes the values of $\chi^2$ and other physical quantities for the elastic scatterings $4\text{He+76Ge, 4He+74Ge, 4He+72Ge and 4He + 70Ge}$ at fixed energy is shown in Tables 1.

**Table 1.** List of parameters for elastic scattering of $\alpha$ particles by the target nucleus $^{70,72,74,76}\text{Ge}$ at laboratory energies 25MeV, which are extracted from the analyses using McIntyre plus Regge pole model. The total reaction cross-sections $\sigma_{r}(\text{FM})$ and $\sigma_{r}(\text{GFM})$ are obtained from FM and GFM, respectively.

<table>
<thead>
<tr>
<th>Elastic scattering</th>
<th>$4\text{He + 76Ge}$</th>
<th>$4\text{He + 74Ge}$</th>
<th>$4\text{He + 72Ge}$</th>
<th>$4\text{He + 70Ge}$</th>
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<tr>
<td>$E_{lab.}$ (MeV)</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
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<tr>
<td>$r_0$(fm)</td>
<td>1.55</td>
<td>1.30</td>
<td>1.25</td>
<td>1.125</td>
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<td>$\rho$(Rad)</td>
<td>0.38</td>
<td>0.29</td>
<td>0.22</td>
<td>0.27</td>
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<tr>
<td>$d$ (fm)</td>
<td>0.34</td>
<td>0.325</td>
<td>0.23</td>
<td>0.155</td>
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<tr>
<td>$\ell_s$</td>
<td>0.50</td>
<td>5.50</td>
<td>6</td>
<td>3.5</td>
</tr>
<tr>
<td>$\phi_{f_s}$ (Deg.)</td>
<td>1</td>
<td>10</td>
<td>2.5</td>
<td>6</td>
</tr>
<tr>
<td>$D_s$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>5</td>
<td>10</td>
<td>4.2</td>
<td>6</td>
</tr>
<tr>
<td>$\ell_g$</td>
<td>14</td>
<td>10</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>7.30x10^{-1}</td>
<td>7.19x10^{-1}</td>
<td>5.13x10^{-1}</td>
<td>3.50x10^{-1}</td>
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<td>$R$(fm)</td>
<td>9.025</td>
<td>7.252</td>
<td>7.18</td>
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<td>$d/R$</td>
<td>3.76x10^{-2}</td>
<td>4.32x10^{-2}</td>
<td>3.20 x10^{-2}</td>
<td>2.41 x10^{-2}</td>
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<td>$\rho$</td>
<td>8.147</td>
<td>8.22</td>
<td>8.23</td>
<td>8.22</td>
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<tr>
<td>$n$</td>
<td>4.03</td>
<td>4.03</td>
<td>4.03</td>
<td>4.03</td>
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<tr>
<td>$h$</td>
<td>10.21</td>
<td>12.25</td>
<td>12.82</td>
<td>14</td>
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<tr>
<td>$\theta_{f_s}$(Rad)</td>
<td>5.6x10^{-1}</td>
<td>7.66x10^{-1}</td>
<td>7.66x10^{-1}</td>
<td>9.33 X10^{-1}</td>
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<td>$\theta_{rot}$(Rad)</td>
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<td>-2.0X10^{-1}</td>
<td>-1.09X10^{-1}</td>
<td>-1.03X10^{-1}</td>
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<td>$\sigma_r$ (mb) [GFM]</td>
<td>160</td>
<td>83.8</td>
<td>69.7</td>
<td>45.5</td>
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<tr>
<td>$\sigma_r$ (mb) [FM]</td>
<td>164</td>
<td>88.7</td>
<td>73.3</td>
<td>59.6</td>
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<tr>
<td>$\chi^2$</td>
<td>0.014</td>
<td>0.012</td>
<td>0.033</td>
<td>0.025</td>
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</table>

**CONCLUSIONS**

We have analyzed the elastic scattering $\alpha$ particle by different target nuclei $^{70,72,74,76}\text{Ge}$ at Energy 25MeV within the framework of the parameterized model. There are two quantities determined type of diffraction pattern in the angular distribution; the diffraction parameter $kr$ (which is equivalent to $\Lambda$ or $\ell_g$ the angular momentum) and (the strength of the Coulomb interactions which represents by Sommerfield parameter n ). The Fresnel features in the angular distribution, which is determined by the parameter $p$, that the parameter $p$ increases if the size of target nuclei increase, similar values of the parameters (n, h) or ($\ell_g$, p) for oscillatory structure in elastic scatterings. we used the parameters ($r_0$, $d$, $\mu$, $\ell_s$, $\phi_{f_s}$, $D_s$ and $\Gamma_s$) in the combined model, the radius interaction region $R$ is increasing when the atomic mass of target nucleus is increased the better formula of ($R=r_s A_{1/3}$ + $r_0$ ). The diffusivity parameter $d$ has an effect on the slope of the oscillation and its increase with increase the target size . The increase in nuclear phase shift parameter $\mu$ cause to change the diffraction structure for the angular distributions calculated , the parameters ($\ell_s$, $\phi_{f_s}$, $D_s$ and $\Gamma_s$) used in backward angel, $\ell_s$ the orbital angular momentum which estimate the location for the pole, the $D_s$ Amplitude and the $\Gamma_s$ width exhibit the similar behavior the width of the pole , $\phi_{f_s}$ which represent the phase angel determined the size of oscillation, the parameters ($\ell_s$, $\phi_{f_s}$, $D_s$ and $\Gamma_s$) are found vulnerable in the fitting process but accountable for reproducing the oscillatory structure of the experimental data. The quality of fittings using the adopted models is as good as that using the standard model.
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REFERENCE