Abstract
In this paper, the performance of proposed Direction Independent U-function based sensorless controlled brushless DC motor has been analyzed. The proposed U-function based virtual hall sensor has the good control characteristics as compared with the real solid state hall sensors. Analysis of the proposed method has been done with the view point of position estimation error, speed estimation error, dynamic tracking response with different observer gains and speed accuracy. The simulation studies have been carried out with different speed and load torque references. U-function based virtual hall sensing provides the commutation positions which is the exact replication of the actual solid state hall sensor and provides excellent control over all four quadrants with added sensitivity at starting and reversal. The proposed method has excellent sustainability in the vibration and load disturbance together with the step load changes. The method has added stability for both directions of rotation.

Keywords: Sensorless control; Brushless Motor; Commutation; Back EMF; U-function.

INTRODUCTION
Brushless DC motors has widely applications in computers, aerospace, medical, robotics, electric vehicles, and industries due to the reason that these motors posses high power/weight ratio, high torque to current ratio and high efficiency [1, 2]. Variable voltage variable frequency voltage source inverters or current source inverters are used to operate brushless DC motors. The self commutated BLDC motors are commutated either by the Hall effect or encoder based position sensors fixed in the rotor periphery or by sensorless schemes [3]. Frus and Kuo [4] first intilialized the milestone research in sensorless control. Due to the limitation of hall sensing or encoder based control in terms of increased machine size, reduced reliability and higher noise, sensorless methods are preferred [5]. Various detection based sensorless control schemes are detection of interval of freewheeling diode conduction [6]-[8], back EMF ZCP detection [9]-[22], third harmonic back EMF detection [11],[23], back EMF Integration till ZCP [24],[25]. There are many disadvantages in the detection of the back EMF. Indirect and direct methods of back EMF detection has circuit losses, delays due to filtering and deteriorated motor performance due to effect of detection circuit in the phase variables [20]. Various PWM strategies used in detection set up unwanted oscillations in torque and degrade the motor performance [14, 26]. Observer based control techniques [27]-[35] were introduced to overcome these problems. Unknown observer method has widely been implemented [33, 38] especially in the field of the fault detection. Unknown Input Observer (UIO) assumes the back EMF information to be unknown and the estimation of unknown value of back EMF is done with the appropriate Kalman gain matrix. After estimation of phase to phase back EMF, some special functions e.g. Commutation Function (CF) [33], Speed independent position function (SIPF) [36] were devised. These are achieved in the form of a train of high magnitude impulses. The magnitude and width varies as there is disturbance in motor performance and so the threshold cannot be appropriately set. These methods not work properly for high speeds. To overcome this problem simple function named ZCP for getting commutation pulses was introduced [37], in which the simulation study was carried out and detected phase back EMF was used for calculation of Zero Difference Points (ZDP). Determining the threshold in ZDP method is easy and the position detection is precise over wide speed range. Together it is fault tolerant and free of acquisition and measurement noises. Also the method provided the very first commutation signal at starting comparatively to ZCP based method based on initial rotor position. ZCP of estimated Line to line back EMF were used for getting phase back EMF ZDPs in [38]. All above method proved unsuitable for the four-quadrant application and give unstable results. So a new function named direction independent U-function has been introduced in the proposed work which is useful in the four quadrant application and low to high speed operation as well. The estimated speed is used as feedback to the U-function and speed proportional current control loop. Two methods have been used for speed estimation in proposed work. One method provides exact speed but in positive direction only, while the other method [39] provided rippet speed in both directions. So the properties of both estimated speeds are applied. Proposed method has excellent robustness towards estimation and acquisition noise. In [40] the U-function has been introduced first for sensorless control of BLDC motor. Main motivation for the direction independent U-function is that, if direction changes during reversal, the hall signal sequence only changes and one of hall sensor signal level doesn’t instantly change during that sector. So U-function also must be direction independent and stable. Furthermore, the control should be self sustained for all speeds. It should be able to control the speed and torque in all four quadrants, without additional logics. Function should have the transition edges exactly coinciding with solid state hall signal transition edges. The
MODELLING OF VSI FED BLDC MOTORS

The equivalent circuit of PWM Inverter fed Brushless DC motor drive system is shown in Fig. 1.

![Figure 1. VSI fed BLDC motor drive.](image)

The voltage equation for all the three phases with respect to neutral can be expressed as Eq. (1) assuming that the inductance and resistance parameters for all phases are equal and time and rotor position doesn’t affect the self and mutual inductances.

\[
\begin{align*}
\frac{d}{dt}\begin{bmatrix}
v_{an} \\
v_{bn} \\
v_{cn}
\end{bmatrix} &= \begin{bmatrix}
R & 0 & 0 \\
0 & R & 0 \\
0 & 0 & R
\end{bmatrix}\begin{bmatrix}
i_{bn} \\
i_{cn}
\end{bmatrix} + \begin{bmatrix}
L & 0 & 0 \\
0 & L & 0 \\
0 & 0 & L
\end{bmatrix}\frac{d}{dt}\begin{bmatrix}
i_{an} \\
i_{bn} \\
i_{cn}
\end{bmatrix} + \begin{bmatrix}
e_{an} \\
e_{bn} \\
e_{cn}
\end{bmatrix}
\end{align*}
\]

(1)

Where, \(v_{an}\), \(v_{bn}\) and \(v_{cn}\) are phase to neutral voltages and \(i_{an}\), \(i_{bn}\) and \(i_{cn}\) are phase to neutral currents. The per phase stator inductance \(L_s\) is calculated from the stator self inductance \(L\) and mutual inductance \(M\) between phases i.e. \(L_s = L - M\).

The ideal waveforms of trapezoidal phase back EMFs with relative position of rectangular current have been shown in Fig.2. The developed electromagnetic torque is determined by the values of phase currents and phase back EMFs as given by Eq.(2).

\[
T_e = \frac{e_{an}.i_{an} + e_{bn}.i_{bn} + e_{cn}.i_{cn}}{\omega_r}
\]

(2)

![Phase A](image)

![Phase B](image)

![Phase C](image)

**Figure 2.** Waveforms of phase back EMF, phase currents and Electromagnetic torque developed [37].

PROPOSED SENSORLESS BRUSHLESS DC MOTOR DRIVE

A. Back EMF Estimation

The phase back EMFs cannot be measured directly due to unavailability of neutral point of motor. So the phase to phase back EMF can be estimated from known phase currents and line voltages using the following equation,

\[
\frac{d}{dt}(i_{an}i_{bn}) = -\frac{R}{L}(i_{an}i_{bn}) + \frac{1}{L}(v_{an}v_{bn}) - \frac{1}{L}(e_{an}e_{bn})
\]

(3)

Similarly for other phase groups also, the equations can be written. The quantities \(v_{an} - v_{bn}\), \(i_{an}\), \(i_{bn}\) in Eq.(3) are known state variables as they can be measured. The phase to phase back EMF \(e_{an} - e_{bn}\) is unknown disturbance quantity.

The above equation can be written in state space matrix form as,

\[
x = Ax + Bu + Fw, \quad y = Cx + Du
\]

(4)
The disturbance quantity \( w \) is can be expressed as a polynomial of order \( \gamma \), which satisfies the condition that \( w \) is completely observable dynamic model.

\[
w = \sum_{i=0}^{\gamma} a_i t^i, \quad \gamma \geq 1
\]

Augmented state space matrix involving unknown disturbance in phase to phase back EMF can be determined by the representation as below:

\[
\dot{x}_a = A_a x_a + B_a u
\]

\[
y = C_a x_a
\]

(5) (6)

Where,

\[
x_a = \begin{bmatrix} i_{an} - i_{bn} \\ e_{an} - e_{bn} \end{bmatrix}, \quad u = \begin{bmatrix} v_{an} - v_{bn} \end{bmatrix}, \quad y = \begin{bmatrix} i_{an} - i_{bn} \end{bmatrix}
\]

\[
A_a = \begin{bmatrix} R & 1 \\ -L & -L \end{bmatrix}, \quad B_a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_a = [1 \ 0]
\]

As the system in equations (4) and (5) are observable system, the complete state space equation can be reduced in the form of observer given by (7) and shown by Fig.3.

\[
\dot{x}_a = A_a x_a + B_a u + K(y - \hat{y})
\]

(7)

Where, \( \hat{e}_{ab} = (i_{an} - i_{bn}) - (\hat{i}_{an} - \hat{i}_{bn}) \) is the estimation error in phase currents in phase-A and phase-B. Similarly other estimation errors can also be defined. Observer gain matrix is defined by,

\[
K = \begin{bmatrix} K1 \\ K2 \end{bmatrix}
\]

(8)

From the variation of estimated quantities

\[
\frac{d\hat{e}_i}{dt} = K2.e_i
\]

(9)

The variables \( e_i \) and \( \hat{e}_i \) are generalized errors in the actual and estimated values of line to line currents and line to line back EMF. The complete equation showing the difference between the actual and estimated value is given by.

\[
\frac{d}{dt} \begin{bmatrix} e_i \\ e_{\hat{i}} \end{bmatrix} = \begin{bmatrix} -R/L & -K1/L \\ -K2/L & 0 \end{bmatrix} \begin{bmatrix} e_i \\ e_{\hat{i}} \end{bmatrix}
\]

(10)

The objective is to find out the optimal value of the observer gains so that the observer error can be reduced approximately to zero. The observer gain calculation can be obtained by method discussed in later section.

B. Observer Gain Calculation

The state feedback gain matrix can be determined Ackermans [39] method in which the system poles are placed at preferred positions. Let the desired eigen values are \( \lambda_1 \) and \( \lambda_2 \). The characteristic polynomial is obtained as below,

\[
\alpha(s) = (s - \lambda_1)(s - \lambda_2) = s^2 - (\lambda_1 + \lambda_2)s + \lambda_1\lambda_2
\]

(11)

Where \(-\alpha_1 = \lambda_1 + \lambda_2, \quad \alpha_0 = \lambda_1\lambda_2\)

The characteristic polynomial can be expressed in terms of matrix \( A \) by the Caley-Hamilton Theorem as

\[
\alpha(A) = A^2 + \alpha_1 A + \alpha_0.1
\]

(12)

The observer gain matrix \( G \) with observability matrix \( Q \) can be determined by

\[
K = \begin{bmatrix} K1 \\ K2 \end{bmatrix} = \alpha(A).Q^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -R/L + \alpha_1 \\ -\alpha_0 L \end{bmatrix}
\]

(13)

\[
Q = \begin{bmatrix} C \end{bmatrix} A = \begin{bmatrix} 1 \\ -R/L \end{bmatrix}
\]

(14)

Different Eigen values can be chosen for the best performance considering the mutual balance between the fast convergence which occurs when eigen values are set near minus infinity, sluggishness in case of small eigen values and sensitivity towards the noise and other disturbances while large eigen values are selected.

C. Rotor Speed and Position Estimation

The actual speed of the motor depends upon the load torque applied and internal electromagnetic torque developed by
motor, which is obtained by solving the following differential equation for \( \omega_r \),

\[
T_c = T_1 + J \frac{d\omega_r}{dt} + B \omega_r
\]  

(15)

The coefficients \( J \) and \( B \) are Inertia and friction coefficient respectively.

The estimated speed can be calculated by the following equation using line to line back EMF information and back EMF constant.

\[
\hat{\omega}_r = \frac{2}{P} \max \left\{ \left| \hat{e}_{an} - \hat{e}_{bn} \right|, \left| \hat{e}_{bn} - \hat{e}_{cn} \right|, \left| \hat{e}_{cn} - \hat{e}_{an} \right| \right\}
\]  

(16)

The rotor angle can be calculated using

\[
\hat{\theta}_r = \int \hat{\omega}_r dt + \theta_0
\]  

(17)

Where, \( \theta_0 \) is initial angle of rotor with the phase-A axis and \( K_e \) is per phase BEMF constant. The proper tracking of actual and estimated value is needed so as to protect motor from going into the saturation especially when motor reverses and direction becomes negative. So, for the direction estimation, another speed estimation method was followed in [39], which can be useful for direction detection, as discussed below. Referring to the Fig.4, the line to line back EMF can be transformed to dq form as below [39].

\[
\hat{e}_{ds} = \frac{1}{3} \left( \hat{e}_{ab} - \hat{e}_{ca} \right)
\]  

(18)

\[
\hat{e}_{qs} = -\frac{1}{\sqrt{3}} \hat{e}_{bc}
\]  

(19)

\[
\hat{\theta}_e = \tan^{-1} \left( \frac{\hat{e}_{qs}}{\hat{e}_{ds}} \right)
\]  

(20)

Where, \( \hat{e}_{ds} \) and \( \hat{e}_{qs} \) are the direct and quadrature back EMFs.

Estimated rotor angle and rippled angular speed from the above method,

\[
\hat{\theta}_r = \frac{2}{P} \hat{\omega}_r, \quad \hat{\omega}_r = \frac{d\hat{\omega}_r}{dt}
\]  

(21)

The estimated speed obtained from the Equation (16) is always achieved positive magnitude, while the magnitude is close to actual speed. So direction measurement is necessary for getting the direction with near exact value. Direction of the rotation can be obtained by the ratio of estimated rippled speed with the magnitude of rippled speed. The ratio of the rippled speed vs magnitude of rippled speed may be equal to 1 or -1. This value is multiplied with the Exact estimated speed magnitude from Eq.(16), to get near exact speed with direction information. So both the methods are used together at same time to get the rippleless exact speed with direction information is obtained. Therefore the rippleless exact speed is given by,

\[
\hat{\omega}_2 = \frac{2}{P} \max \left\{ \left| \hat{e}_{an} - \hat{e}_{bn} \right|, \left| \hat{e}_{bn} - \hat{e}_{cn} \right|, \left| \hat{e}_{cn} - \hat{e}_{an} \right| \right\}
\]  

(22)

The different types of speeds relative to the reference speed obtained by performing the simulation with proposed method are shown in Fig.5, which are calculated by Eq. (15), Eq.(16), Eq. (21) and Eq. (22). The reference speeds of 3000, -2000 and 500 rpm were applied at the instants 0, 0.2 and 0.6 seconds respectively with the load of 0.5 Nm.

Figure 4. Vector representation of back EMFs
(b) Figure 5 (a) Actual speed, exact estimated speed without direction, rippled speed with direction information and exact estimated speed with direction information relative to the reference speed are shown. (b) zoomed view from 0.162 to 0.26 sec. The motor has been operated with the load of 0.5 Nm with various reference speeds of 3000, -2000 and 500 at instants 0, 0.2 and 0.6 seconds respectively.

D. Direction Independent U-Function

The proposed U-function has the property that it crosses the zero axis only at the actual solid state hall transition edges unlike Commutation Function and Speed Independent Position Function. U-functions need not the setting of threshold values as were needed in other functions for sensorless operation. The position of phase mmf axis may be obtained wherever the derivative of U-function is zero. The U-function is defined by the following equation based on the line to line back EMFs.

\[
U_{ab} = \frac{\sqrt{E_{ab}^2 + E_{bc}^2 + E_{ca}^2}}{3E_{ab}} \tag{23}
\]

\[
U_{bc} = \frac{\sqrt{E_{bc}^2 + E_{ca}^2 + E_{ab}^2}}{3E_{bc}} \tag{24}
\]

\[
U_{ca} = \frac{\sqrt{E_{ca}^2 + E_{ab}^2 + E_{bc}^2}}{3E_{ca}} \tag{25}
\]

Whenever reversal of the direction starts, the back emf polarity and sequences become opposite which can generate ambivalence in decision for right commutation. For this problem, the direction normalization is done. The direction independent U-functions are given by,

\[
U_{ab} = \frac{\hat{\omega}_r}{[\hat{\omega}_r]} \frac{\sqrt{E_{ab}^2 + E_{bc}^2 + E_{ca}^2}}{3E_{ab}} \tag{26}
\]

\[
U_{bc} = \frac{\hat{\omega}_r}{[\hat{\omega}_r]} \frac{\sqrt{E_{bc}^2 + E_{ca}^2 + E_{ab}^2}}{3E_{bc}} \tag{27}
\]

Where, the angular speed \( \hat{\omega}_r \) may be positive, negative or zero. U-functions cross the zero axis only at real Hall sensor transition edges. The U-function based virtual hall sensor behaves as it is fixed as real solid state hall sensors. The virtual hall sensor signal outputs are obtained by comparing U-functions to zero, which may be High or Low (logic 1 or 0) which is shown in Fig.6 and illustrated in Table.1.

Table 1. Actual versus u-function based virtual hall sensors

<table>
<thead>
<tr>
<th>Condition of U-function</th>
<th>U-function based Virtual Hall Sensor States</th>
<th>Corresponding Real Hall Sensor States</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{ca} \leq 0 )</td>
<td>( H'_a = 1 )</td>
<td>( H_a = 1 )</td>
</tr>
<tr>
<td>( U_{ca} &gt; 0 )</td>
<td>( H'_a = 0 )</td>
<td>( H_a = 0 )</td>
</tr>
<tr>
<td>( U_{ab} \leq 0 )</td>
<td>( H'_b = 1 )</td>
<td>( H_b = 1 )</td>
</tr>
<tr>
<td>( U_{ab} &gt; 0 )</td>
<td>( H'_b = 0 )</td>
<td>( H_b = 0 )</td>
</tr>
<tr>
<td>( U_{bc} \leq 0 )</td>
<td>( H'_c = 1 )</td>
<td>( H_c = 1 )</td>
</tr>
<tr>
<td>( U_{bc} &gt; 0 )</td>
<td>( H'_c = 0 )</td>
<td>( H_c = 0 )</td>
</tr>
</tbody>
</table>

\[
U_{ca} = \frac{\hat{\omega}_r}{[\hat{\omega}_r]} \frac{\sqrt{E_{ab}^2 + E_{bc}^2 + E_{ca}^2}}{3E_{ca}} \tag{28}
\]

Figure 6. Virtual hall sensors obtained from U-functions compared to zero.

E. Overall Block Diagram

Complete block diagram of the proposed U-function based sensorless controlled VSI fed brushless DC motor system has been shown in Fig.7.
The three phase currents and line voltages are taken as feedback for rectangular reference current generator and Unknown input observer. The Back EMFs obtained from the observer are used for U-function calculation and speed estimation. Based on the virtual hall signals from calculated U-functions, the switching sequence signals are decoded. The current from reference current generators and the actual phase current are compared and then hysteresis current controllers provides the PWM signals to the switches of VSI based on the switching signals obtained from sequence signals. The effect of increasing feed-forward torque error proportional current control gain \( K_{pT} \) leads to fast speed response with higher starting torque and current but leads to uncontrolled torque with ripple. Higher value of speed error proportional gain \( K_{pN} \) leads to fast response, less rise time but increased torque ripple. So the PID parameters of torque as well as speed error proportional current controllers need to be set properly to keep speed rise time and torque ripple in reasonable limit. Also the steady state errors are affected by these parameters. To keep the starting torque and current within permissible limits, a current limiter is used.

**SIMULATION RESULTS**

The simulation diagram as shown by Fig.8 has been designed on PC with MATLAB R2011b. The simulation results have been obtained and analyzed for various load and speed references using the motor and controller parameters as shown in Table 2. Different case studies have been done in succeeding sections. During the starting or zero (or near zero) speed condition at any time, only the reference speed is fed to the current controller so as to lead the system in reference direction. When a very small speed is achieved and motor starts running in either direction, the rippled speed feedback having direction information (from Eq.21) is given after a delay of 0.1 milliseconds. The difference of reference and rippled speed (with direction information as per Eq.21) is fed as error for current controller. Again after a small delay, the exact speed with direction information (Eq. 22) is fed to the current controller.

**Table 2. Simulation parameters for Motor and Controller**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per Phase Inductance</td>
<td>8.5 mH</td>
</tr>
<tr>
<td>Per phase Resistance</td>
<td>2.875 ohm</td>
</tr>
<tr>
<td>Voltage Constant</td>
<td>49 V peak L_L / Krpm</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>0.46792 Nm/A-peak</td>
</tr>
<tr>
<td>Flux linkage by rotor magnets</td>
<td>0.23396 V.sec</td>
</tr>
<tr>
<td>Rotor Inertia</td>
<td>0.0008 kg.m²</td>
</tr>
<tr>
<td>Viscous damping</td>
<td>0.001 N.m.sec</td>
</tr>
<tr>
<td>Pole Pairs</td>
<td>1</td>
</tr>
<tr>
<td>Nominal DC link voltage</td>
<td>300 volt DC</td>
</tr>
<tr>
<td>Kalman gain ( K_1 )</td>
<td>3500</td>
</tr>
<tr>
<td>Kalman Gain ( K_2 )</td>
<td>-100000</td>
</tr>
<tr>
<td>Forward Gain ( K )</td>
<td>1</td>
</tr>
<tr>
<td>Threshold for phase back EMF ZDP</td>
<td>( V_{dc}/1500 )</td>
</tr>
<tr>
<td>Hysteresis band</td>
<td>0.000005</td>
</tr>
<tr>
<td>PWM generator ( K_p, K_i )</td>
<td>0.000005, 0.00000005</td>
</tr>
<tr>
<td>Torque proportional control ( K_pT, K_iT )</td>
<td>1.075, 0.0000005</td>
</tr>
<tr>
<td>Speed Proportional Control ( K_pN,K_iN )</td>
<td>0.006, 0.0000006</td>
</tr>
<tr>
<td>Saturation</td>
<td>0.000006</td>
</tr>
<tr>
<td>Current limiter</td>
<td>20, -20</td>
</tr>
</tbody>
</table>
A. Case 1. Dynamic response

1) Reference and Actual Values of Speed, Torque and Current: The speed, torque and current response of the system with the reference speed and load variation as per the Table 3 is shown in Fig. 9. The load torque applied to the motor are 3 Nm, 2 Nm and 4 Nm respectively at 0, 0.15 and 0.4 seconds respectively. The speed references at 0, 0.15, and 0.3 seconds are -2000, 1000 and 500 rpm respectively. The simulation time is 1 sec.

Table 3. Loading Torque and reference speed

<table>
<thead>
<tr>
<th>Time (Sec.)</th>
<th>0</th>
<th>0.15</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load torque (Nm)</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Reference Speed(rpm)</td>
<td>-2000</td>
<td>1000</td>
<td>500</td>
<td>500</td>
</tr>
</tbody>
</table>

Figure 8. Simulation block diagram in MATLAB/SIMULINK Platform.

Figure 9. Reference speed and actual speed achieved, current and torque.
2) Three Phase Currents and Back EMF Waveform: The three phase currents and the line to line estimated back EMF waveforms are shown in Fig. 10, for the variations of load and reference speed as per Table. 3.

![Figure 10. Three phase currents and line to line BEMFs.](image)

3) U-functions based Virtual Hall signals & Actual hall signals: When U-functions are compared with zero, they give the virtual hall signal values (Logical 1 or 0). Because the value of U-function has very high magnitude at phase back EMF ZDPs or hall signal transition edges, so a limiter (max 10) is used to show the U-functions properly as in Fig. 11(a).

![Figure 11. U-functions, virtual hall signals and corresponding actual hall signals.](image)

Zoomed view (time axis from 0.13 to 0.2 seconds) in Fig. 11(b) shows the additional sensitivity and stability of sensorless virtual hall sensor scheme using U-function during speed reversal and load disturbance at 0.173 second. It can be seen that virtual hall sensor signals using U-function gives extra short duration pulse signal during the reversal, while the sequence is same as the real hall signals at all other instants. Also during starting, the virtual hall signals are seen performing in more sensitive manner.

B. Case 2 Dynamic response

1) Reference and Actual Values of Speed, Torque and Current: The response of the system with the reference speed and load variation as per the Table. 4 is shown in Fig. 12. With high load torque and reference speed, the power requirement is high. So current is not controlled and the motor has high torque ripple as can be seen in the Fig. 12.

![Figure 12. Speed reference, actual speed, load versus actual torque and currents.](image)

<table>
<thead>
<tr>
<th>Time (Sec.)</th>
<th>0</th>
<th>0.15</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Load torque (Nm)</strong></td>
<td>5</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td><strong>Reference Speed (rpm)</strong></td>
<td>5000</td>
<td>5000</td>
<td>2000</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

![Table 4. Load and reference speed variation](image)
2) Three Phase Currents and Back EMF Waveform:
The three phase currents and the line to line estimated back EMF waveforms are shown in Fig.13, for the variations of load and reference speed as per Table.4.

Figure 13. Back EMF and three phase currents for loading and speed reference as per Table.4.

3) U-functions based Virtual Hall signals & Actual hall signals: for the loading pattern as per Table.4, the U-function based virtual hall signals and actual hall signals are shown in Fig.14 (a). Zoomed view (time axis from 0.18 to 0.35 seconds) in Fig. 14(b) shows the additional sensitivity and stability of sensorless virtual hall sensor scheme using U-function during speed reversal at 0.248 second and 0.319 second. It can be seen that virtual hall sensor signals using U-function gives extra short duration pulse signal during the reversal from negative to positive to negative at 0.248 second, whereas the sequence is same as the real hall signals at all other instants. During the reversal when speed is going positive from negative at 0.32 second, the virtual hall sensor \( H_v^c \) from U-function for phase C (i.e. ) goes LOW for short duration. Also during starting, the virtual hall signals are seen performing in more sensitive manner.

Figure 14(a) U-functions based virtual hall sensor signals and actual hall sensor signals for loading pattern as per Table.2

Figure 14(b) Zoomed view of U-functions, virtual hall signals, actual hall signals at 0.248 and 0.319 seconds.

C. Dynamic response to load disturbances and fluctuations imposed at different step loading conditions

The robustness of the proposed U-function based sensorless scheme has been checked for the load disturbance imposed at the step load changes of 3, 2 and 4 Nm at 0-0.15, 0.15-0.4 and 0.4-0.5 seconds respectively for a fixed forward motoring speed reference of 2000 rpm. Fig.15 shows that the speed of the motor is not differing much with reference speed and remains unaffected by noise and disturbance in load.

Figure 15. Performance verification with load disturbance and fluctuations applied at different step loadings.

D. Position Estimation Error for Different Observer Gains K1 and K2

For the improvement in the position tracking performance with U-function based sensorless method, first the system performance has been investigated with fixed value of K1 and varying the value of K2 at fixed speed reference of 2000 rpm.
and Load Torque of 2Nm. It can be observed from Fig.16, that increasing the observer gain $K_2$, turns into the reduced the slope of position estimation error. After this, the Kalman gain $K_2$ is kept fixed and $K_1$ is varied and then both are varied.

**Figure 16.** Effect of varying the observer state feedback gain $K_2$ on Position error

The position error (radians) is constant $0.25 \times 10^{-3}$ for $K_1=10000$ and $K_2 = -3.5 \times 10^7$ for the reference speed of 2000 rpm as shown in Fig.20.

**Figure 17.** Position estimation error for different observer gains.

**E. Actual speed feedback vs. Estimated Speed feedback**

For the reference speed of the 5000 rpm and 1 Nm load, the speed performance of U- function based sensorless technique has been investigated with the feedback of actual speed and estimated speed as shown in Fig.18. The percentage difference in later condition is only 0.2 percent.

**F. Reference Tracking of Estimated and Actual Speeds**

The reference tracking performance of the proposed U-function based sensorless technique has been evaluated as shown in Fig.22 (a) and (b). In case (a), the speed references of -3000, 3000 and -5000 rpm is applied with 2 Nm load input, while in case(b) the references of 3000, -3000 and 5000 rpm is given at instants 0, 0.2 and 0.4 seconds respectively.

**Figure 18.** Performance of U-function method when actual speed estimated speed is used as feedback for speed control.

**Figure 19.** The performance of proposed U-function based sensorless method for various speed references at load of 2Nm showing the reference tracking behavior of actual speed and estimated speed.
G. Percentage Steady state error between actual and estimated speed with direction:

The percentage error using proposed method obtained based on the reference speed, has been shown in Fig.20.

![Percentage error in estimated speed](image)

Figure 20 (a) Percentage error in the estimation of speed with speed reference of 2000 rpm. (b) Percentage error with input reference speed of 5000 rpm.

The reference speed has been given 3000 rpm. It has been observed that for higher observer gain obtained with corresponding Eigen values, the position estimation and speed estimation error reduce to very low value and remain constant.

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