Dynamic Analysis of a Single MEMS Vibratory Gyroscope with Decoupling Connection between Driving Frame and Sensing Proof Mass

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Abstract
This paper reports a methodology to solve the dynamic analysis issue for micro-mechanical vibratory gyroscope with two mass elements which are connected by the U-shaped flexible beams. The equivalent stiffness coefficient of these beams is calculated by the analytical and simulation method with negligible error 3%. The differential motion equations of the gyroscope are set up and solved to determine the amplitude and phase of drive and sense modes formed analytical solution. The dimension of flexible beams is surveyed to achieve the suitable value of the desired mismatched resonant frequency as 100 Hz. The solution is compared with the results achieved by using ANSYS Workbench software with the different value 0.33%. The obtained results can be applied to mechanical structure design of MEMS vibratory gyroscope for optimizing the geometrical parameters and performance.

Keywords: mismatched frequency, decoupling connection, MEMS Vibratory Gyroscope, U-shaped beam.

INTRODUCTION
Micro-Electro-Mechanical Systems (MEMS) are generally considered as devices and systems integrated with mechanical elements, sensors, actuators, and electronic circuits on a common silicon substrate through micro-fabrication technology. In recent years, MEMS technology has been being developed owing to its advantages: small size, low power consumption, batch fabrication, low cost and ease of integration.

MEMS Vibratory Gyroscope (MVG) is a device used to detect and measure external angular velocity or rotational angle of an object relatively rotated in an inertial frame of reference [1], [2]. The operation of this sensor is based on the Coriolis principle to transform energy from primary mode to secondary mode [1÷5]. The MVGs have been extensively applied in the aerospace industry, automotive industry and consumer electronics market thanks to their dramatically reduced cost, size and weight [1÷8].

The MVGs are classified according to some factors such as the measured subject, the type of vibration, the number of mass element, the used effect for primary (drive) and secondary (sense) mode. There are two types of MVG which measures rotational angle for type I and angular rate for type II [4]. According to the effect used to create drive mode, the MVGs can be divided into some types: thermal drive [5], piezoelectric drive [6], electromagnetic drive [7] and electrostatic drive [8÷11]. Despite small productivity, the electrostatic effect is used more widely because of simple fabrication and control. The drive and sense vibration can be torsional vibration [9] or linear vibration [8] according to the design in MVGs. Depending on the number of the mass element the MVGs can be classified into the single-mass or multi-mass gyroscope [12].

Figure 1 shows a schematic design of a proposed MVG with a single proof-mass and a decoupling frame. It consists of a proof-mass \( m \) placed inside the outer frame \( m_i \) by the spring-damping suspension system. The frame is suspended by the flexible beams which are defined as a second spring-damping system. The flexible beams allow both the frame and the proof-mass to oscillate along the driving (\( x \)) direction. In addition, the proof-mass can move along the sensing (\( y \)) direction. In the...
driving direction, the frame and the proof-mass are driven into the resonant state by an external force with the suitable frequency. In this case, both the driving and sensing directions are in-plane while the out-of-plane angular rate is detected and measured. When the gyroscope is rotated with an angular rate \( \Omega \), a Coriolis force at the frequency of drive mode appears in the sense direction. This force induces the sense mode to the proof-mass in the sense direction. This sense mode is detected and used to measure the angular rate. Thus, this vibratory gyroscope is presumed to the 2-degree-of-freedom system with two linear oscillations in two orthogonal directions.

In this type of MVG, the important problem is to determine the mismatched resonant frequency of the drive and sense modes. It is a major parameter for increasing the sensitivity of gyroscope due to increasing the mechanical quality factor of sense mode [13÷15]. In designing process of MVGs, it is necessary to solve the problem by the analytical and simulation method to reduce expenditure before fabrication. In addition, the tuning fork gyroscopes which consist of two identical tines are being studied in present to increase the performance of the devices. In their configuration, each tine is defined as a single gyroscope with two masses or more connected by decoupling beams. They are linked directly or indirectly to each other by a mechanism [16], [17]. So, the single gyroscope with two masses with decoupling connected beam need to be studied seriously. Each above gyroscope is defined as a resonator with the close frequency between drive and sense modes. This difference frequency is therefore called the mismatched frequency. It is one of the most important parameters to assess the performance of the MVGs.

This paper presents a methodology to determine the resonant frequency including the driving and sensing frequency of a single MVG. Hence, the suitable mismatched frequency between drive and sense mode is determine. This methodology allows to carry out the optimal dimension parameters of the desired single MVG to obtain the expected mismatched frequency. The solution is achieved by using both numerical analytical and simulated method. This proposed methodology is then used to design a silicon single MVG with a two-degree-of-freedom gyroscope. In this gyroscope structure, the sensing mass and out driving frame are connected together by using U-shaped flexible beams.

**CONFIGURATION OF PROPOSED MVG**

The configuration of proposed MVG is created in ANSYS Workbench and shown in Figure 2a. It has one proof-mass (1) which is suspended on an outer frame (2) thanks to four elastic beams (3). The frame is suspended on a substrate (not shown in Figure 2a) by four other elastic beams (4). The elastic beams (3, 4) have a U-shaped form with a fixed end and the guided other end. The three-dimensional (3D) solid model (Figure 2a) does not include the etch holes on the proof-mass and comb finger in drive and sense directions. When the driving signals are present, the outer frame and the proof-mass oscillate along the \( x \)-axis (drive axis); additionally the proof-mass oscillates along the \( y \)-axis (sense axis) when the gyroscope is effected by \( \Omega \) angular rate (in the \( z \)-axis). This MVG is defined as single mass with decoupling frame system.

![a) The 3D solid model b) The mesh model](image)

**Figure 2:** 3D schematic design of a single MVG

The specific material parameters of Silicon are shown in Table 1. The dimensions of MVG are listed in Table 2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>2230</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>169x10⁹</td>
<td>Pa</td>
</tr>
<tr>
<td>Bulk modulus</td>
<td>1.2879x10⁹</td>
<td>Pa</td>
</tr>
</tbody>
</table>

**Table 2:** The dimensions of the proposed MVG

<table>
<thead>
<tr>
<th>Design parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions of MVG (x ( \times ) y)</td>
<td>2240 x 1930 ( \mu ) m</td>
</tr>
<tr>
<td>Thickness of structure</td>
<td>60 ( \mu ) m</td>
</tr>
<tr>
<td>Surface area of the proof-mass</td>
<td>15468x10² ( \mu ) m²</td>
</tr>
<tr>
<td>Surface area of the outer frame</td>
<td>7678x10² ( \mu ) m²</td>
</tr>
<tr>
<td>Length of the drive springs</td>
<td>500 ( \mu ) m</td>
</tr>
<tr>
<td>Width of the drive springs</td>
<td>20 ( \mu ) m</td>
</tr>
<tr>
<td>Length of the sense springs</td>
<td>460 ( \mu ) m</td>
</tr>
<tr>
<td>Width of the sense springs</td>
<td>16 ( \mu ) m</td>
</tr>
<tr>
<td>Mass of the outer frame</td>
<td>1.0734x10⁻⁷ kg</td>
</tr>
<tr>
<td>Sensing mass</td>
<td>2.1624x10⁻⁷ kg</td>
</tr>
<tr>
<td>Sensing spring mass</td>
<td>4.8231x10⁻⁹ kg</td>
</tr>
<tr>
<td>Driving spring mass</td>
<td>5.578x10⁻⁹ kg</td>
</tr>
<tr>
<td>Gyroscope total mass</td>
<td>3.6518x10⁻³ kg</td>
</tr>
</tbody>
</table>
The equivalent stiffness coefficients of the driving and sensing spring beams are determined by James J. Allen [18] as follow:

$$K = \frac{Eh}{L} \left( L' b + 2Lb^3 \right) \left( \frac{L' b^3 + 2Lb^3}{2Lb^3 + Lb^5} \right)$$

(1)

In the above expression, $K_x$ is equivalent stiffness coefficient of a folded beam in $x$-direction; $E$ is Young’s modulus of Silicon; $L$, $L'$, $b$, $b'$ is the length, width of a folded beam and $h$ is the thickness of structure (Figure 3).

Besides, the stiffness coefficients could be calculated by using simulation software ANSYS Workbench. The value of equivalent stiffness coefficient of one folded beam in driving direction is determined and shown in Figure 4.

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**ANALYTICAL SOLUTION**

Considering in the non-inertial reference frame $Oxyz$ with origin at the gravity center of the proof-mass where $Ox$, $Oy$ and $Oz$ axes are representative of driving, sensing and angular velocity direction, the differential motion equations of system are obtained by using Lagrange equation of the second kind [19]. The position of gravity center of decoupling frame is given by a vector $\{x, 0, 0\}$, while the vector $\vec{r} = \{x, y, 0\}$ defines the position of the gravity center of the proof-mass. In common case, the angular velocity vector $\vec{\Omega}$ is defined by its projections on the reference frame as a vector $\{\Omega_x, \Omega_y, \Omega_z\}$. The absolute velocities of the frame $\vec{V}$ and the proof-mass $\vec{V}_p$ are defined by following expressions:

$$\vec{V}_p = \vec{r}_n + \vec{\Omega} \times \vec{r}_n = \{x \Omega_z, -x \Omega_y\}$$

$$\vec{V} = \vec{r} + \vec{\Omega} \times \vec{r} = \{x - y \Omega_z, y \Omega_x, y \Omega_y - x \Omega_z\}$$

(2)

Total kinetic energy of the system will be presented as following expression:

$$T = \frac{m}{2} \vec{V}_p^2 + \frac{m}{2} \vec{V}^2$$

$$= \frac{m}{2} \left[ x^2 + x^2 \Omega_z^2 + x^2 \Omega_y^2 \right] +$$

$$+ \frac{m}{2} \left[ (x - y \Omega_z)^2 + (y + x \Omega_z)^2 + (y \Omega_y - x \Omega_z)^2 \right]$$

(3)

Total potential energy of the springs in the flexible suspension of the system will be presented as following expression:

$$\Pi = \frac{k_x}{2} x^2 + \frac{k_y}{2} y^2$$

(4)

where $k_x$ and $k_y$ is equivalent stiffness of the flexible beams in drive and sense direction, respectively.

Total dissipative energy of the damping element in the suspension system can be expressed as:

$$\Phi = \frac{c_x}{2} x^2 + \frac{c_y}{2} y^2$$

(5)

where $c_x$ and $c_y$ is equivalent damping coefficient of the system in driving and sensing direction, respectively.
Lagrange equation of the second kind for this system is given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad (i = 1, 2) (6)$$

Here \( L = T - \Pi \) is the Lagrange function, \( Q \) are generalized forces acting on the elements.

Combining expressions (3), (4) and (5) into the Lagrange function (6), simplifying gives us the following system of two differential equations, describing the motion of the MVG system:

\[
\begin{align*}
(m + m_g)\ddot{x} + c_1\dot{x} + k_1 - (m + m_g)\Omega_x^2\dot{x} - \Omega_y^2\dot{y} = Q_x \\
(m + m_g)\ddot{y} + c_1\dot{y} + k_1 - (m + m_g)\Omega_y^2\dot{y} - \Omega_x^2\dot{x} = Q_y
\end{align*}
\]

These equations can be rewritten by dividing both parts of the equations by corresponding coefficients \((m + m_g)\) for the first equation and \(m\) for the second one. The result is as follows:

\[
\begin{align*}
\ddot{x} + 2\mu_1\dot{x} + [\omega_x^2 - (\Omega_x^2 + \Omega_y^2)]x - 2d\Omega_x\dot{y} + d(\Omega_x\dot{y} - \Omega_y\dot{x})y = a_1 \\
\ddot{y} + 2\mu_1\dot{y} + [\omega_y^2 - (\Omega_x^2 + \Omega_y^2)]y + 2\Omega_x\dot{x} + (\Omega_x\dot{y} + \Omega_y\dot{x})x = a_2
\end{align*}
\]

where \(\omega_x = k_1/(m + m_g)\) and \(\omega_y = k_1/m\) are natural frequencies of drive and sense modes, \(d = m/(m + m_g)\) is dimensionless inertia asymmetry factor \((d = 1\) for the first equation and \(m\) for the second one). The equation system (8) becomes motion equations for the single mass MVG without decoupling frame, \(a_1 = Q_x/(m + m_g)\) and \(a_2 = Q_y/m\) are generalized accelerations from external forces, \(\mu_1 = c_1/(2m + m_g)\) and \(\mu_1 = c_1/2m\) are damping factors in the drive and sense direction, respectively.

The components of the angular velocity \((\Omega_x, \Omega_y)\) cause any displacement of proof mass in motion plane. It is only \(\Omega_x\) in terms of Coriolis forces \((2m\Omega_x\dot{x} + 2m\Omega_y\dot{y})\) in equations (7) that can cause the translation energy between two oscillations in-plane. Therefore, in general case, the vector angular velocity can be assumed as they are constants or quasi-constants and defined by the simpler form as \(\Omega = \{0, 0, \Omega_z\}\). The equation system (8) can be rewritten as:

\[
\begin{align*}
\ddot{x} + 2\mu_1\dot{x} + [\omega_x^2 - \Omega^2]x - 2d\Omega_x\dot{y} = a_1 \\
\ddot{y} + 2\mu_1\dot{y} + [\omega_y^2 - \Omega^2]y + 2\Omega_x\dot{x} = a_2
\end{align*}
\]

In this MVG model, the external forces apply on the elements including electrostatic force in drive direction \((F_a, F_s)\) in \(x\)-direction), the system (8) can be simplified as:

\[
\begin{align*}
\ddot{x} + 2\mu_1\dot{x} + (\omega_x^2 - \Omega^2)x - 2d\Omega_x\dot{y} = a_d \\
\ddot{y} + 2\mu_1\dot{y} + (\omega_y^2 - \Omega^2)y + 2\Omega_x\dot{x} = a_s
\end{align*}
\]

where \(a_d = F_d/(m + m_s)\) is generalized acceleration from the drive force.

If exciting force of the drive oscillation is harmonic with the form as \(a_d = Re\{ae^{i\omega t}\}\) where \(\omega\) is the exciting frequency and initial phase is assumed as zero, the solutions of system (10) can be determined by using Kramer’s method with assumed form for harmonic oscillation of the elements as:

\[
x(t) = Re\{A_ie^{i\omega t}\} \quad A_i = A_{i0}e^{i\phi_{i0}} \\
y(t) = Re\{A_ie^{i\omega t}\} \quad A_i = A_{s0}e^{i\phi_{s0}}
\]

where \(A_{i0}, A_{s0}\) and \(\phi_{i0}, \phi_{s0}\) are amplitudes and initial phases of the drive and sense oscillation, respectively. The equations which express the relation of the amplitudes are written as following:

\[
\begin{align*}
(a_x^2 - \Omega^2 - \omega^2 + 2\mu_1\omega)A_x - 2d\Omega_xA_y = a_d \\
2\Omega_xA_y + (a_x^2 - \Omega^2 - \omega^2 + 2\mu_1\omega)A_x = A_s
\end{align*}
\]

The solution of the system (12) is:

\[
A_x = \frac{a_0(a_x^2 - \Omega^2 - \omega^2 + 2\mu_1\omega)}{\Delta}; \quad A_s = \frac{2a_0\Omega_x\phi_x}{\Delta}
\]

where \(\Delta = (a_x^2 - \Omega^2 - \omega^2)(a_x^2 - \Omega^2 - \omega^2) - 4\omega^2(2\omega d\Omega_x\mu_1) + 4\omega^2]\nu_x(a_x^2 - \Omega^2 - \omega^2)\]

The value of amplitude and phase of oscillations can be determined by following:

\[
A_{i0} = \left|A_i\right| = \sqrt{Re(A_i)^2 + Im(A_i)^2} \\
\phi_{i0} = \tan^{-1}\left(\frac{Im(A_i)}{Re(A_i)}\right) \quad (i = 1, 2)
\]

Therefore, the expressions for amplitude and phase are:

\[
\begin{align*}
A_{i0} = \frac{a_0(\omega_x^2 - \Omega^2 - \omega^2)^2 + 4\mu_1^2\omega_x^2}{\Delta}; \quad A_{s0} = \frac{2a_0\Omega_x\omega_x}{\Delta} \\
\phi_{i0} = \frac{tan^{-1}\left(2\omega\left[\mu_1B_x - (\omega_x^2 - \Omega^2 - \omega^2)B_y\right] + 4\mu_1\omega_x^2B_z\right)}{\Delta} \\
\phi_{s0} = \frac{tan^{-1}\left(B_x/2\omega B_z\right)}{\Delta}
\end{align*}
\]

\[\Delta = ([\omega_x^2 - \Omega^2 - \omega^2][\omega_x^2 - \Omega^2 - \omega^2] - 4\omega^2(2\omega d\Omega_x\mu_1))^2 + 4\omega^2]\nu_x(\omega_x^2 - \Omega^2 - \omega^2) + \mu_1(\omega_x^2 - \Omega^2 - \omega^2)\]

\[B_x = (\omega_x^2 - \Omega^2 - \omega^2)(\omega_x^2 - \Omega^2 - \omega^2) - 4\omega^2(\omega d\Omega_x^2 + \mu_1\mu_s)\]

\[B_z = \mu_1(\omega_x^2 - \Omega^2 - \omega^2) + \mu_s(\omega_x^2 - \Omega^2 - \omega^2)\]

The expressions (15), (16) represent the amplitude and phase of drive and sense oscillation as the functions of both excited
frequency and angular velocity. These relations are shown in Figures 5 and 6.

One can observe that the sensing amplitude is proportional to angular velocity. This relation is linear when the angular velocity is small enough (under 100 rad/s) and the driving amplitude is quasi-constant. Besides, the driving and sensing frequencies change insignificantly and the mismatched frequency between driving and sensing modes is constant. By increasing the angular velocity the sensing and driving amplitudes and the mismatched frequency increase nonlinearly. This characteristic allows to determine the measuring range of the MVG.

![Figure 5: The dependence of sensing amplitude on exciting frequency and angular velocity](image)

**Figure 5:** The dependence of sensing amplitude on exciting frequency and angular velocity

**FEM ANALYSIS**

In this paper, the ANSYS Workbench is used to create, analyze and simulate the proposed model in Figure 2a. The mesh model for beams are controlled by using element size of 20 μm, whereas the rest of this MVG is defined by using maple face meshing with element size of 400 μm and 600 μm to reduce element quantity. The finite element type in this model is SOLID 187 (element 3D with 10 nodes). The element quantity for MVG is 9133. The MVG after mesh is shown in Figure 2b.

The fundamental motion equation of system is defined as:

$$\begin{bmatrix} M \end{bmatrix}\ddot{u} + [C]\dot{u} + [K]u = \{P_{ex}\}$$

$$\quad (17)$$

where \([M]\) - the mass matrix; \([C]\) - the damping matrix; \([K]\) - the spring matrix; \(\{u\}\) - the displacement vector and \(\{P_{ex}\}\) - the external force vector.

In MEMS devices, the viscous effects of the air between the masses and the substrate are discussed. The damping coefficients are defined to Rayleigh function \([20]\):

$$[C] = \alpha[M] + \beta[K]$$

$$\quad (18)$$

where \(\alpha\) and \(\beta\) are the Rayleigh damping coefficients and defined as constants.

**Modal analysis**

It is important to find out the desired resonant frequencies, where the displacement of the proof-mass in driving and sensing directions is maximum.

In general, this structure needs to have some advantages. First of all, the drive and sense modes should be appeared before the other ones. The drive and sense modes are major modes, whereas the others are parasitic modes. These major frequencies should be far different from the parasitic ones \([13÷17]\). The modal analysis is carried out to determine the natural frequencies of structures by solving equation (1) without external force. Six frequencies of the proposed MVG are shown in Table 3. Some natural frequencies of this MVG are shown in Figure 7.
The resonant frequencies of drive and sense modes are determined to be about 8.76 kHz and 9.09 kHz, respectively. This range of frequency ensures to reduce the effect of vibration and acoustic. In fact, the frequencies of two desired modes (drive and sense modes) should be matched to amplify the output signal on sense mode. But it may make a longer response time. Besides, when the difference between two modes is decreased, the mechanical coupling increases, which causes an increase the zero-rate output shift and the devise operation becomes more unstable [10], [11], [16]. The larger width is changed in range 450 ÷ 550 µm, 16 ÷ 22 µm for driving and sensing springs, respectively. The relationships between mismatched frequency and these parameters are determined in Response Surface ANSYS and shown in Figure 8. In each graph of Figure 8, two parameters are fixed, whereas two other are changed in the considered range. Moreover, it can be seen that the mismatched frequency strongly depends on the length of these beams. The widths of them affect complicatedly this frequency.

<table>
<thead>
<tr>
<th>Vibratory modes</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive mode</td>
<td>8757</td>
</tr>
<tr>
<td>Sense mode</td>
<td>9095</td>
</tr>
<tr>
<td>Third mode</td>
<td>12799</td>
</tr>
<tr>
<td>Fourth mode</td>
<td>18688</td>
</tr>
<tr>
<td>Fifth mode</td>
<td>19487</td>
</tr>
<tr>
<td>Sixth mode</td>
<td>27729</td>
</tr>
</tbody>
</table>

The major frequencies strongly depend on the dimensions of structure. In order to match two wanted frequencies with supposed bandwidth, the investigation of relationship between the bandwidth (mismatched) frequency and the parameters of the structure should be carried out. In this paper, the authors focus on the length and width of the flexible beams in two directions. In this proposed structure, the width of the beams is designed to be much smaller than the thickness. The length and width is changed in range 450 ÷ 550 µm, 16 ÷ 22 µm and 440 ÷ 500 µm, 14 ÷ 20 µm for driving and sensing springs, respectively. The relationships between mismatched frequency and these parameters are determined in Response Surface ANSYS and shown in Figure 8. In each graph of Figure 8, two parameters are fixed, whereas two other are changed in the considered range. Moreover, it can be seen that the mismatched frequency strongly depends on the length of these beams. The widths of them affect complicatedly this frequency.

Table 4: The mismatched frequency corresponding to the specific value of folded beams

<table>
<thead>
<tr>
<th>Length (µm)</th>
<th>Width (µm)</th>
<th>Length (µm)</th>
<th>Width (µm)</th>
<th>Drive mode (Hz)</th>
<th>Sense mode (Hz)</th>
<th>Mismatched (Hz)</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>490</td>
<td>19</td>
<td>460</td>
<td>15</td>
<td>8396</td>
<td>8342</td>
<td>54</td>
<td>0.65</td>
</tr>
<tr>
<td>450</td>
<td>19</td>
<td>470</td>
<td>17</td>
<td>9467</td>
<td>9527</td>
<td>60</td>
<td>0.63</td>
</tr>
<tr>
<td>480</td>
<td>20</td>
<td>450</td>
<td>16</td>
<td>9266</td>
<td>9335</td>
<td>69</td>
<td>0.74</td>
</tr>
<tr>
<td>480</td>
<td>19</td>
<td>450</td>
<td>15</td>
<td>8647</td>
<td>8572</td>
<td>75</td>
<td>0.87</td>
</tr>
<tr>
<td><strong>490</strong></td>
<td><strong>20</strong></td>
<td><strong>460</strong></td>
<td><strong>16</strong></td>
<td><strong>8998</strong></td>
<td><strong>9098</strong></td>
<td><strong>100</strong></td>
<td><strong>1.1</strong></td>
</tr>
<tr>
<td>500</td>
<td>20</td>
<td>450</td>
<td>15</td>
<td>8743</td>
<td>8584</td>
<td>159</td>
<td>1.85</td>
</tr>
<tr>
<td>500</td>
<td>19</td>
<td>500</td>
<td>17</td>
<td>8178</td>
<td>8770</td>
<td>592</td>
<td>6.75</td>
</tr>
</tbody>
</table>

By using these results, we can determine the appropriate parameters for elastic beams with any desired mismatched frequency. The typical value of parameters can be consulted in Table 4. So, the minimum value of mismatched frequency is 54 Hz when the driving and sensing springs are about 490 µm, 460 µm for length and 19 µm, 15 µm for width, respectively. In order to have the 100 Hz bandwidth, the parameters should be chosen as 490 µm length, 20 µm width for driving beams and 460 µm length, 16 µm width for sensing beams, respectively. If the length of these beams is fixed, the width of them can be chosen from Figure 8 corresponding the desired mismatched frequency.

The above-chosen parameters are used to recalculate the characteristic value of system. Now, the equivalent stiffness coefficients in driving and sensing directions are respectively $k_d = 1162 \text{ N/m}$ and $k_s = 769 \text{ N/m}$. In this case, the value of damping coefficient only includes slide damping of gas around system. Experimental data show that the effect of air damping is almost constant when the air pressure is near the atmospheric pressure. For example, the viscosity of air is $1.79 \times 10^{-5} \text{ Ns/m at 1 atm}$ and the viscosity reduces to $1.61 \times 10^{-5} \text{ Ns/m at 0.5 atm}$.
Hence, the calculated frequencies of single gyroscope are $\omega_x = 8998 \text{ Hz}$ and $\omega_y = 9098 \text{ Hz}$ for drive mode and sense mode, respectively. The mismatched frequency is 100 Hz (1.1%). The next higher parasitic mode has further frequency (12931 Hz) with 42% difference from two lower modes. It means that the crosstalk effect can be suppressed and the amplitude along sensing direction achieves maximum [21].

Figure 8: The effect of the dimension parameters of flexible beams to mismatched frequency

**Amplitude-Frequency Response**

When applying external force in drive direction, the desired response of gyroscope is that the drive vibration has stabilizing and maximum amplitude. It is necessary to determine the resonant frequency ($f$) of exciting force. The function of this force is defined as follow:

$$F = F_0 \sin(2\pi ft)$$

(19)

In order to determine this frequency, the amplitude of exciting force is fixed at $F_0 = 1 \mu N$, while the frequency $f$ changes in range $8800 \div 9200 \text{ Hz}$ to reduce the calculation time and ensure the calculation accuracy. The response of amplitude and phase to frequency is shown in Figure 9. Therefore, the resonant frequency is determined as $8998 \text{ Hz}$ for drive oscillation and $9098 \text{ Hz}$ for sense oscillation. These values are equal to the frequencies in modal analysis in ANSYS Workbench. So, the exciting frequency $f$ could be defined as: $f = 8998 \text{ Hz}$.

To compare with analytical method, according to expressions (15), (16) in above section, the resonant frequencies in driving and sensing directions are respectively $8968 \text{ Hz}$ and $9085 \text{ Hz}$ (Figure 10). The difference in two methods is negligible 0.33% and can be explained that the mass of flexible driving and sensing beams was not mentioned in analytical issue. Because of that the value of resonant frequencies in analytical method is slightly smaller than in simulation method. This result demonstrates that the analytical solution is fairly accurate.

Figure 9: Simulated frequency responses

Figure 10: Analytical amplitude and phase responses versus exciting frequency
CONCLUSION

This paper presents a methodology to solve the dynamic analysis issue for a proposed two-degree-of-freedom MVG with two masses. In this MVG, the masses are connected together and to substrate by the U-shaped flexible beams. By using ANSYS Workbench software, the 3D and mesh model of this MVG is designed and then the modal analysis is carried out to define the frequency of two major modes of this gyroscope. The mismatched resonant frequency between these two modes is determined according to dimension parameters of U-shaped beams in two major directions. The equivalent stiffness coefficients of the flexible beams in sense and drive direction determined by both analytical and simulation method are matching with small errors as 3%. These values were used as the input parameter of the analytical problem. The frequency of drive and sense modes in both two methods were determined with slight error as 0.33%. The geometric parameters of U-shaped beams are optimized with the desired mismatched frequency as 0.33%. The geometric parameters of U-drive and sense modes in both two methods were determined by both analytical and simulation method as 3%. These values were used as input parameter of the analytical problem. The frequency of drive and sense modes in both two methods were determined with slight error as 0.33%. The geometric parameters of U-shaped beams are optimized with the desired mismatched frequency as 100 Hz. The above obtained results are the basis to solve dynamic analysis issue for the single and double MVG in subsequent studies.

REFERENCES