Methods and Algorithms of Fuzzy Models Construction Assessing the State of the Low-Formalized Processes

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Abstract
This research aims to examine the hybrid methods of fuzzy models construction of the intellectual analysis of the state of weakly formalized processes. Developed an algorithm for constructing a fuzzy logical model of intellectual analysis — classification, estimation and forecasting of the state of weakly formalized processes based on the fuzzy clustering method. The possibility of obtaining a fuzzy solution in constructing a fuzzy logical model for the classification, estimation and forecasting of the state of weakly formalized processes is shown on the basis of tuning the model parameters by neural networks and the bees algorithm for various membership functions.

Keywords: Theory of fuzzy sets, neural networks, bees algorithm

INTRODUCTION
Identification of hidden regularities, classification of data, forecasting of processes, as well as analysis of the process under study with large amounts of data describing the state of weakly formalized processes, and in conditions of their uncertainty are the main tasks of data mining. For such processes, characterized by vague situations of external and internal environment, i.e. non-stochasticity and incompleteness of the initial information, it is usually impossible to construct a simple adequate mathematical model. Information about the parameters of such processes is usually expressed by experts in the form of words and sentences, i.e. in a linguistic form.

In such cases, in modeling, decision-making and management, it is advisable to use the means of soft computing technology [1-7].

A hybrid system obtained on the basis of combining the components of soft computing technology - fuzzy logic, neural networks and evolutionary algorithms, will have the same intellectual property as the application of knowledge in natural language. In this connection, the development of algorithms and programs for constructing fuzzy models of classification, estimation, prediction and decision-making problems based on fuzzy inference rules, neural networks and evolutionary algorithms is an actual task [1-4].

Therefore, there is a need to develop methods and algorithms for solving data mining problems, that is, tasks of constructing fuzzy models for classification, estimation and forecasting based on the fuzzy clustering method, adjusting model parameters with the help of neural networks and evolutionary algorithms, as well as fuzzy multi criteria optimization arising from building a model [8-10].

To date, large-scale scientific research in the field of modeling of poorly formalized objects and processes, pattern recognition under uncertainty, the construction of intelligent systems, support for decision-making and the knowledge base and management in robotics, fuzzy modeling, theory are being carried out in scientific centers and higher educational institutions of developed countries of the world fuzzy logical conclusions, application of methods and means of "soft computing" in various applied systems, neural networks and evolutionary calculations, in particular, Apple, IBM, E.piphany, SAS, Silicon Graphics, SPSS, Oracle Data Mining, Oracle, UNICA, SQL Server Data Mining (USA), Angoss, Cognos, STATISTICA Data Miner (Canada), humanIT Software GmbH, BonaVista Systems, mentegrafica infovis solutions (Italy), Acknosoft, SIPINA (France), GR Brains (India), Marcom (China), NeuroShell, PolyAnalyst (Russia), Jooble (Ukraine), AL.11, AL.12, AL.13, ESPLAN (Azerbaijan) [11-14].

The carried out researches on development and practical application of fuzzy models of the intellectual analysis have shown that in the field of development of methods and algorithms of models of classification, estimation and forecasting in conditions of fuzzy knowledge in the world, in particular, the following results are obtained: Methods of classification and forecasting in the conditions of fuzzy knowledge "Attar Software Ltd." (Great Britain); methods of evolutionary programming in the system "PolyAnalyst, Unica" (USA); A method for structuring a problem in the form of a graph has been developed that allows to classify data or carry out an analysis of the consequences of solutions, and this method was implemented in C5.0, RuleQuest (Australia), a method for constructing logical prediction models based on fuzzy clustering in Clementine, Integral Solutions (United Kingdom); in "SIPINA, University of Lyon" (France), the task of multi criteria optimization in conditions of uncertainty is solved; adjustment of parameters of fuzzy model with the help of neural networks is realized in "IDIS, Information Discovery" (USA); methods for constructing models for the intellectual analysis of the state of weakly formalized processes using neural networks in "NeuroShell, Ward Systems Group" (Russia) [10-12].

In the field of development of systems for the intellectual analysis of the state of weakly formalized processes, studies
are currently conducted in such prioritized areas as the use of nonstandard approaches based on soft computing technology, the processing of large data, the transition from numerical computations to linguistic calculations, the theory of fuzzy modelling and fuzzy logic output and the definition of their application, hybrid methods based on neural networks and evolutionary algorithms.

The tasks of fuzzy logic, the development of algorithms for tuning neural networks and evolutionary algorithms, the transition from numerical computations to linguistic calculations and the use of natural languages in data processing, decision making and control, the theory of possibilities and its application in intelligent systems, the theory of fuzzy modeling and fuzzy logical inferences, application of soft computing, neural networks and evolutionary computations in industrial systems, development of decision-making methods in conditions of uncertainty, situational management and intellectual hybrid systems are considered in the works of a number of scientists: L. Zade, A. Dyubua, A. Prada, E. Mamdani, M. Sugeno, T. Takahi, M.Jamshidi, N.N. Moiseev, S.A. Orlovsky, E. Muschkin, O. Larichev, G.S. Pospelov, D.A. Pospelov, R.A. Aliyev and others. [1-13]

Domestic scientists also made a great contribution to the development of the theory of data mining, fuzzy sets, indistinct stochastic mathematical modeling: Kamilov M.M., Bekmuratov T.F., Abutaliev F.B., Marakhamov A.R., Nursratov T.S., Rakhmatullaev M.A., Ignatiev N.A., Usmanov R.N., Mukhamedieva D.T.. Despite this, the problems of developing algorithms for constructing fuzzy models for the intelligent analysis of the state of weakly formalizable processes in the case of nonlinear dependence of input and output data, the construction of multi-agent optimization of the parameters of the fuzzy model being created, in particular, the algorithm for tuning parameters using a hybrid method based on neural networks and evolutionary bees algorithm.

At the same time, the development of modified hybrid methods and algorithms for constructing logical models for the intelligent analysis of weakly formalized processes based on fuzzy inference rules, using the fuzzy clustering method with the use of neural networks and the bees algorithm for adjusting the parameters of these models has not been sufficiently investigated.

**FORMULATION OF THE PROBLEM.**

When solving practical problems for constructing a system of classification, estimation and forecasting in conditions of uncertainty, the necessary fuzzy information with no stochastic characteristics can be divided into two parts: numerical (quantitative) and linguistic (qualitative) part obtained from the expert. Most fuzzy systems use knowledge of the second type - most often data described as bases of fuzzy inference rules, which are combined into systems of fuzzy inferences. The algorithms for constructing fuzzy logic models based on the rules of fuzzy inference play a major role in solving problems of classification, estimation and forecasting under conditions of uncertainty of input data.

Formation of the rules of fuzzy in the construction of models for the classification, evaluation and prediction of the state of weakly formalized processes determine the importance of an optimal reduction in the number of rules.

Correct use of current information about the object in the process of modelling, that is, determining the adequacy of the model, is important. In this plan, the main problems of the development of models of weakly formalized processes are formulated.

Traditional fuzzy systems have some drawbacks, so it is imperative to involve experts from one or another field to formulate rules and functions of ownership. This in turn is a factor in the occurrence of a number of inconveniences. Adaptive fuzzy systems solve this problem. In such systems, during the training, their parameters are adjusted on the basis of experimental data. The process of adapting fuzzy systems consists of two stages: 1) the creation of linguistic rules; 2) adjustment of model parameters. To create fuzzy rules, you need the appropriate functions, and to make a fuzzy conclusion, you need rules [1-4].

The questions of constructing fuzzy models of classification, estimation and forecasting can be expressed as a multi criterion optimization problem with four objective functions $f_1(S)\rightarrow\max, f_2(S)\rightarrow\min, f_3(S)\rightarrow\min, f_4(S)\rightarrow\min$.

Here $f_1(S)$ - the number of correctly classified objects using a set of rules $S$; $f_2(S)$ - the number of fuzzy rules in the set of rules $S$; $f_3(S)$ - the total number of elements of the set $S$ and $f_4(S)$ - the root-mean-square error between the received and expected results of the model. Thus, the problem reduces to solving the multicriteria optimization problem.

Also in this chapter an anatomic review of existing methods of classification, estimation and forecasting of weakly formalized processes is given, and approaches of fuzzy sets for solving the multi criteria optimization problem are considered. As a result of the analysis, the main problems and problems solved in this paper are determined.

The main problem that needs to be solved is overcome by constructing a logical model based on the rules of fuzzy inference, using the method of fuzzy clustering [2-3].

The difference between the proposed approach and traditional approaches is the use of modern technologies for intellectual data analysis (knowledge base, components of Soft Computing - neural networks, bees algorithms) for the development of algorithmic and software tools for constructing logical models based on the fuzzy clustering of classification, estimation and forecasting problems [1-7].
CONSTRUCTION OF FUZZY LOGIC MODEL

Models of classification, estimation and forecasting of states of weakly formalized processes, which are investigated in scientific works of scientists such as L. Zade, A. Rotstein and R. Aliyev, are determined by the following rules of fuzzy inference [1-4]:

\[ x_i = a_{i,jp}, \text{ with weight } w_{jp} \rightarrow y_j = f_j(x_1, x_2, \ldots, x_n) \hat{\mu} \]  \hspace{1cm} (1)

Here \( a_{i,jp} \) expresses the linguistic term of the variable \( x_i \) of the line \( jp \);

\( w_{jp} \) is the weight coefficient of the variable \( x_n \); \( y_j = f_j(x_1, x_2, \ldots, x_n) \) - fuzzy rule of inference.

Three types of fuzzy models of intellectual analysis of the state of weakly formalized processes, described with the help of fuzzy inference rules, are developed.

1. Fuzzy model of classification, estimation and forecasting of states of weakly formalized processes in the form of a nonlinear connection

\[ x_i = a_{i,jp}, \text{ with weight } w_{jp} \rightarrow y_j = b_{j0} + \sum_{k=1}^{n} b_{jk} x_k \]  \hspace{1cm} (2)

2. Fuzzy model of the classification, estimation and prediction of the states of processes in the form of a linear connection

\[ x_i = a_{i,jp}, \text{ with weight } w_{jp} \hat{\mu} \rightarrow y_j = b_{j1} x_1 + \ldots + b_{jn} x_n \]  \hspace{1cm} (3)

3. Fuzzy model of the classification, estimation and prediction of process states in the form of fuzzy terms output

\[ x_i = a_{i,jp}, \text{ with weight } w_{jp} \hat{\mu} \rightarrow y_j = r_j, \ j = 1, M \]  \hspace{1cm} (4)

In the construction of a logical model for the classification, estimation and prediction of weakly formalized processes, an algorithm for fuzzy clustering, consisting of seven steps, has been developed.

When constructing a fuzzy model in the case of different types of membership functions, the model parameters are adjusted on the basis of neural networks and a bees algorithm, that is, the problem of learning a logical fuzzy model is solved. The essence of training consists in solving the optimization task of minimizing the differences between the real properties of the object and the results of fuzzy approximation.

Adjusting the parameters of a fuzzy logic model based on neural networks and bees algorithms increases the adequacy of this model. As a result of this adjustment, the resulting model acquires intellectual characteristics. The article deals with the process of adjusting the parameters of membership functions in the form of Gauss, parabola, triangle, trapezoid and bell-shaped form on the basis of neural networks and bees algorithms.

1. Configuring parameters of the mining model for weakly formalized processes based on neural networks [1-4]:

The system of recurrence relations for different types of membership functions is used to minimize the criterion

\[ E = \frac{1}{2} \sum_{j=1}^{m} (y_j - \hat{y}_j)^2 \rightarrow \min, \]  \hspace{1cm} (5)

used for training in the theory of neural networks (Table 1).

<table>
<thead>
<tr>
<th>Membership function</th>
<th>Recurrence relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>[ c_i^{\mu}(t + 1) = c_i^{\mu}(t) - \eta(y_i - \hat{y}<em>i) w</em>{jp} \sum_{j=1}^{M} \mu_{j}^{\mu}(x_j) \times ]  [ \frac{1}{\sigma_i^{\mu}} \left( \sum_{j=1}^{M} \mu_{j}^{\mu}(y_j) \right)^2 ]  \hspace{1cm} (6)</td>
</tr>
<tr>
<td></td>
<td>[ \sigma_i^{\mu}(t + 1) = \sigma_i^{\mu}(t) - \eta(y_i - \hat{y}<em>i) w</em>{jp} \sum_{j=1}^{M} \mu_{j}^{\mu}(x_j) \times ]  [ \frac{1}{\sigma_i^{\mu}} \left( \sum_{j=1}^{M} \mu_{j}^{\mu}(y_j) \right)^2 ]  \hspace{1cm} (7)</td>
</tr>
</tbody>
</table>

Table 1. Recurrence relations of configuring parameters for different types of membership functions

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Normal distribution (The bell-shape):
\[
\mu(x) = \frac{1}{1+\left(\frac{x-c}{\sigma}\right)^2}
\]
\[
c_i^{jp}(t+1) = c_i^{jp}(t) - \eta(y_i - y_{i,j})w_{jp} \frac{\prod_{i=1}^{n} \mu^{jp}(x_i)}{\mu^{jp}(x_i)} \times \\
\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y) \times \\
\frac{2\sigma_i^{jp}(x_i^* - c_i^{jp})}{(\sigma_i^{jp})^2 + (x_i^* - c_i^{jp})^2}
\]
\[
\sigma_i^{jp}(t+1) = \sigma_i^{jp}(t) - \eta(y_i - y_{i,j})w_{jp} \frac{\prod_{i=1}^{n} \mu^{jp}(x_i)}{\mu^{jp}(x_i)} \times \\
\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y) \times \\
\frac{2\sigma_i^{jp}(x_i^* - c_i^{jp})^2}{(\sigma_i^{jp})^2 + (x_i^* - c_i^{jp})^2}
\]

In the form of a parabola:
\[
\mu(x) = 1 - \left(\frac{x-c}{\sigma}\right)^2
\]
\[
c_i^{jp}(t+1) = c_i^{jp}(t) - \eta(y_i - y_{i,j})w_{jp} \frac{\prod_{i=1}^{n} \mu^{jp}(x_i)}{\mu^{jp}(x_i)} \times \\
\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y) \times \\
\frac{2(x_i^* - c_i^{jp})}{(\sigma_i^{jp})^2}
\]
\[
\sigma_i^{jp}(t+1) = \sigma_i^{jp}(t) - \eta(y_i - y_{i,j})w_{jp} \frac{\prod_{i=1}^{n} \mu^{jp}(x_i)}{\mu^{jp}(x_i)} \times \\
\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y) \times \\
\frac{2(x_i^* - c_i^{jp})^2}{(\sigma_i^{jp})^2}
\]

In the form of a triangle:
\[
\mu(x) = \begin{cases} 
  \frac{x-a}{b-a}, & a \leq x \leq b, \\
  \frac{x-c}{b-c}, & b \leq x \leq c, \\
  0, & \text{in other cases.}
\end{cases}
\]
\[
a_i^{jp}(t+1) = a_i^{jp}(t) - \eta(y_i - y_{i,j})w_{jp} \frac{\prod_{i=1}^{n} \mu^{jp}(x_i)}{\mu^{jp}(x_i)} \times \\
\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y) \times \\
\frac{x_i^* - b_i^{jp}}{(b_i^{jp} - a_i^{jp})^2}
\]
\[ c_i^{jp} (t + 1) = c_i^{jp} (t) - \eta(y_i - y_i)w_{jp}^{i} \frac{\prod_{i=1}^{n} \mu^{rp}(x_i)}{\mu^{jp}(x_i)} \times \]
\[ \frac{\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y)}{\left( \sum_{j=1}^{m} \mu^{d_j}(y) \right)^2} \frac{x_i - b_i^{jp}}{(b_i^{jp} - c_i^{jp})^2} \]

If \( a \leq x \leq b \), then

\[ b_i^{jp} (t + 1) = b_i^{jp} (t) - \eta(y_i - y_i)w_{jp}^{i} \frac{\prod_{i=1}^{n} \mu^{rp}(x_i)}{\mu^{jp}(x_i)} \times \]
\[ \frac{\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y)}{\left( \sum_{j=1}^{m} \mu^{d_j}(y) \right)^2} \frac{a_i^{jp} - x_i}{(b_i^{jp} - a_i^{jp})^2}, \]

if \( b \leq x \leq c \), then

\[ b_i^{jp} (t + 1) = b_i^{jp} (t) - \eta(y_i - y_i)w_{jp}^{i} \frac{\prod_{i=1}^{n} \mu^{rp}(x_i)}{\mu^{jp}(x_i)} \times \]
\[ \frac{\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y)}{\left( \sum_{j=1}^{m} \mu^{d_j}(y) \right)^2} \frac{c_i^{jp} - x_i}{(b_i^{jp} - c_i^{jp})^2}. \]

\[ w_{jp} (t + 1) = w_{jp} (t) - \mu(y_i - y_i)w_{jp}^{i} \frac{\bar{d}_j \sum_{j=1}^{m} \mu^{d_j}(y) - \sum_{j=1}^{m} \bar{d}_j \mu^{d_j}(y)}{\left( \sum_{j=1}^{m} \mu^{d_j}(y) \right)^2} w_{jp} \prod_{i=1}^{n} \mu^{rp}(x_i) \]

2. The process of setting parameters of the fuzzy model of intellectual analysis of weakly formalized processes based on evolutionary algorithm – bees algorithm. This algorithm is developed by analogy with the behaviour of wasps in a colony of bees [14-25].

The main essence of parameters adjustment of the model, based on the bees algorithm, is to select parameter values that minimize the difference between the real object properties and the model output results. This algorithm consists of the following steps [14-27].

Step 1. Initialisation. Here totalNumberBees – the number of bees, numberInactive – the number of inactive bees, numberScout – number of bees-scouts, maxNumberVisits – number of visits to the sources of nectar, maxNumberCycles - the number of iterations, determined by the intervals of values of the parameters a, b, c and w.

Step 2. Scouts examine the area around the hive in search of new sources of nectar. In this case, the initial parameter values are determined and the results are stored in the BS matrix.

Step 3. Waggle dance – waggle dance of observing bees. Here, the most optimal nectar sources (in which there is a lot of nectar or the closest) are transferred from the BS matrix to the WG matrix.
The duration of Waggle dance is determined by the formula \( D_i = d_i A \). Here A is the scalability factor; the value showing the relative usefulness, quality, and volume of nectar found by the \( d_i \) dancing i - bee scout.

**Step 4.** The worker bee starts flying to the nectar after selecting the desired source of the nectar.

Based on the WG matrix obtained from spying bees, worker bees carry nectar and find new sources (parameter values) around the source of this nectar. The found information is entered into the NW matrix.

**Step 5.** The bees-scouts carry nectar based on the WG information, the result giving the optimal values is determined, which is assigned to the variable best. The obtained results are entered into the matrix NV.

**Step 6.** The formation of an archive of decisions based on existing matrices NW, NB, WG.

**Step 7.** Under the conditions of the criterion

\[
E = \frac{1}{2} \sum_{i=1}^{m} (y - \hat{y})^2 \rightarrow \min
\]

of a specific iteration to maxNumberOfCycles, the optimal values of parameters from WG are determined.

Here \( f_j(w,a,b,c,d) \) - the output of the model, \( w \) is the weight of the rules, \( \hat{y}_j \) - the real characteristics of the object, \( a, b \) and \( c \) are the parameters of membership functions. These parameters are determined according to the type of membership function. If the membership function is in the form of Gauss, parabola, bell-shaped, then the function parameters will be \( a \) and \( b \). If the membership function is in trapeze form, the function parameters will be in \( a, b, c \), and \( d \) form.

**Step 8.** If the conditions of the corresponding criterion are not met, the transition to step 2 is carried out.

**CONFIGURING THE PARAMETERS OF FUZZY LOGIC MODEL**

Configuring the parameters of fuzzy logic model consists of two stages. In the first step the model values (Y) are defined.

At the second stage, the error value (E) is determined and the values of the parameters of the membership functions are calculated.

In this process, a model, which consists of fuzzy inference rules (2) to (4), is created by dint of the membership functions that result in the highest values. Here it is required to find the values of the coefficients \( b_{ji} \). \( (i=0,1,2,...,t; j=1,...,m) \).

Here, if the model is of linear type, then \( t = n \), and \( t = 2n \) if the model is nonlinear.

The values of the obtained coefficients (5) are considered the values that minimize the quadratic deviation.

The input vector \( X_r = (x_{r,1}, x_{r,2}, ..., x_{r,n}) \) has the following fuzzy output: \( y_r = \sum_{j=1}^{m} \mu_{d_j} \cdot (x_r) \cdot y_j \). The execution level of the fuzzy output rule \( j \) is determined by using expression \( \mu_{d_j} (x_r) = \mu_{j_1}^{k_1} (x_{r_1}) \cdot \mu_{j_2}^{k_2} (x_{r_2}) \cdot ... \cdot \mu_{j_n}^{k_n} (x_{r_n}) \).

Using expression \( \beta_{ji} = \frac{\mu_{d_j} (x_r)}{\sum_{k=1}^{m} \mu_{d_k} (x_r)} \) for the input vector \( X_r \), the relative execution level of the fuzzy output rule \( j \) is determined.

Then:

a) with linear dependence of the output:

\( y_r = \sum_{j=1}^{m} \beta_{ji} y_j = \sum_{j=1}^{m} (\beta_{r_1} b_{j_1} + \beta_{r_2} b_{j_2} \cdot x_{r_2} + ... + \beta_{r_n} b_{j_n} \cdot x_{r_n}) \).

b) with nonlinear dependence of the output:

\( y_r = \sum_{j=1}^{m} \beta_{ji} y_j = \sum_{j=1}^{m} (\beta_{r_1} b_{j_1} + \beta_{r_2} b_{j_2} \cdot x_{r_2} + ... + \beta_{r_n} b_{j_n} \cdot x_{r_n} + ...eta_{r_{n-1}} b_{j_{n-1}} \cdot x_{r_{n-1}} + ... + \beta_{r_n} b_{j_n} \cdot x_{r_n}) \).

The \( \beta_{r_i} \) parameter values are determined according to the type of membership function (table 2):

<table>
<thead>
<tr>
<th>Membership function</th>
<th>Value of the parameter ( \beta_{r_i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>( \mu(x) = \exp \left( -\left( \frac{x - c}{\sigma} \right)^2 \right) ) shape: ( \beta_{r_i} = \exp \left[ -\frac{1}{2} \sum_{j=1}^{m} \left( \frac{x_{r_i} - c_{i j}}{\sigma_{i j}} \right)^2 \right] / \sum_{k=1}^{m} \exp \left[ -\frac{1}{2} \sum_{i=1}^{m} \left( \frac{x_{r_i} - c_{i k}}{\sigma_{i k}} \right)^2 \right] ).</td>
</tr>
</tbody>
</table>

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**Table 1.** The value of the parameter \( \beta_{r_i} \) for different types of membership functions

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Normal distribution (The bell-shape):
\[ \mu(x) = \frac{1}{1 + \left(\frac{x - c}{\sigma}\right)^2} \]

In the form of a parabola:
\[ \mu(x) = 1 - \left(\frac{x - c}{\sigma}\right)^2 \]

In the form of a triangle:
\[ \mu(x) = \begin{cases} \frac{x - a}{b - a}, & a \leq x \leq b, \\ \frac{x - c}{b - c}, & b \leq x \leq c, \\ 0, & \text{in other cases.} \end{cases} \]

Let the following symbols be introduced:
\[ \begin{align*} Y &= (y_1, y_2, \ldots, y_M)^T, \\ \hat{Y} &= (\hat{y}_1, \hat{y}_2, \ldots, \hat{y}_M)^T. \end{align*} \]

Then the problem (5) is reduced to a matrix form: it is necessary to find a vector \( B \) that meets the following requirements:
\[ E = (Y - \hat{Y})^T \cdot (Y - \hat{Y}) \rightarrow \min. \]  
\[ (6) \]

The solution of the problem (6) is reduced to the solution of the following equation:
\[ Y = A \cdot B \]

The problem of multicriteria optimization arises while constructing a model of intellectual analysis of weakly formalized processes.

In general, the problem of multicriteria optimization is as follows:
\[ f(x) = [f_1(x), f_2(x), \ldots, f_q(x)]^T \rightarrow \min, \quad x \in X \]

where
\[ f_k(x) = \sum_{j=1}^{n} c_{kj} x_j, \]
\( k \in Q = \{1, 2, \ldots, q\}, \)
\( x = \{x \in \mathbb{R}^n \mid Ax \subseteq K, x \geq 0\}, \)
\( K = \{y \mid y \in \mathbb{R}^m, y \leq b\} \)

The multi-objective optimization problem with fuzzy goal implies the presence of \( x \) satisfying the following conditions:
\[ \tilde{g}_k = f_z \mid z \in \mathbb{R}^q, z \leq g_k, \]
\( f_k(x) \subseteq \tilde{g}_k, \quad k = 1, 2, \ldots, Q, x \in X, \]  
\[ (8) \]
The solution of fuzzy problem (8) in each section $\lambda$ can be transformed to the solution of a clear problem:

$$\lambda \rightarrow \text{max},$$

$$\mu_i(f_i(x)) \geq \lambda,$$

$$x \in X.$$

CONDUCTING COMPUTATIONAL EXPERIMENTS

For the implementation of the pilot study, the application of the neuro-fuzzy approach was chosen as the main goal and the following tasks were solved:

- creating a base of fuzzy rules and reducing the set of rules by adjusting the parameters of fuzzy models fuzzy rule base using neural network and bees algorithm, as well as construction of a fuzzy model highly effective rating (high-identification rate);
- conducting a comparative analysis of the results of various tasks in the form of tables and graphs.

The known model tasks, located at the electronic address: [http://www.ics.uci.edu/~mlearn/databases/](http://www.ics.uci.edu/~mlearn/databases/), are taken in order to conduct a comparative analysis. These include the following tasks: Iris Data Set, Glass Identification Data Set, Pima Indians Diabetes, Ecoli Data Set, Haberman’s Survival Data Set, Wine Data Set, Liver.

Table 3 below shows the parameters of the tasks listed.

<table>
<thead>
<tr>
<th>Task name</th>
<th>Number of classes</th>
<th>Number of characteristics</th>
<th>Number of objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>7</td>
<td>9</td>
<td>214</td>
</tr>
<tr>
<td>Haberman</td>
<td>2</td>
<td>4</td>
<td>306</td>
</tr>
<tr>
<td>Iris</td>
<td>3</td>
<td>4</td>
<td>150</td>
</tr>
<tr>
<td>Ecoli</td>
<td>8</td>
<td>7</td>
<td>336</td>
</tr>
<tr>
<td>Pima</td>
<td>2</td>
<td>8</td>
<td>768</td>
</tr>
<tr>
<td>Wine</td>
<td>3</td>
<td>13</td>
<td>178</td>
</tr>
<tr>
<td>Liver</td>
<td>2</td>
<td>6</td>
<td>345</td>
</tr>
</tbody>
</table>

For comparison, table 4 shows the results of solving some model problems based on various known and proposed algorithms.

### Table 4. The results of the proposed and existing algorithms

<table>
<thead>
<tr>
<th>Task name</th>
<th>The proposed algorithm</th>
<th>GBC</th>
<th>SGF</th>
<th>SVM</th>
<th>1NN</th>
<th>KNN</th>
<th>Conventional RBF network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>88.75</td>
<td>84.27</td>
<td>75.74</td>
<td>71.50</td>
<td>72.01</td>
<td>72.01</td>
<td>69.16</td>
</tr>
<tr>
<td>Iris</td>
<td>98.3</td>
<td>98.00</td>
<td>97.33</td>
<td>97.33</td>
<td>96.00</td>
<td>95.33</td>
<td>95.33</td>
</tr>
<tr>
<td>Wine</td>
<td>98.88</td>
<td>100</td>
<td>99.44</td>
<td>99.44</td>
<td>95.52</td>
<td>96.07</td>
<td>98.89</td>
</tr>
</tbody>
</table>

The table shows the best results of the algorithms under consideration. The best results for a specific task are highlighted separately.

Checking the operation of each algorithm is carried out using the cross-validation method (10x10 cross-validation).

The results of the proposed method compared with the results of other classification methods (in particular, classifiers CBA, GBC, SGF, SAMGA, C4.5-type, 1NN, KNN, Fuzzy integral based perceptron, Conventional RBF network, GARC and SVM) proved to be more accurate.

CONCLUSIONS

1. The system analysis of the problem of constructing fuzzy models of classification, estimation and forecasting problems in weakly formalized systems proved the urgency and necessity of developing methods for their solution. Based on mathematical analysis, the importance of ensuring the adequacy of the model, that is, how well the current information about the subject of the study is used in the modeling process, is shown. The algorithm of construction of fuzzy model of intellectual analysis of the state of processes allows to solve problems of classification, estimation and forecasting of states of processes in the weakly formalized conditions and uncertainty of information on these processes.

2. The conducted experimental studies have shown higher efficiency of the developed algorithms in comparison with the known algorithms in solving model problems of classification, estimation and forecasting.

REFERENCES


[2] L. A. Zade. Основы нового подхода к анализу сложных систем и процессов принятия решений [Fundamentals of a new approach to the analysis of complex systems and decision-making processes]. // -


