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Control Strategies for Nipah Virus

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Abstract

Recently Nipah virus (NiV) appeared in coastal area of Kerala. This virus attacks both animals and human populations through the fruits bats of Pteropodidae Family. It is transmitted from bats to human, bats to bats and human to human. To study the dynamics of NiV, two layered compartmental model is formulated. The two compartments considered here are that of bat and human. Model is constructed using non-linear ordinary differential equations for both the layers. Stability analysis has been carried out at NiV free, bat infectious free and endemic equilibrium point. The model advocates controls as preventive measures in terms of buried, infected bats and dead bodies of human, spraying insecticides on infected bats and in time hospitalization for infected human. Numerical simulation is carried out to study impact of control on respective compartments. Sensitivity analysis is carried out to sort out factors responsible for spread of disease.

Keywords: Mathematical model, System of non-linear ordinary differential equations, NiV, Stability, Control

INTRODUCTION

NiV infection in humans has a scope of medical sign, from a condition in which a patient is a carrier for infection but experiences no symptoms to death [11]. Initially infected people develop influenza-like symptoms of fever, headaches, muscle pain, vomiting and pain in throat [10]. This can be followed by drowsiness, dizziness, altered state of consciousness, and neurological signs that indicate acute inflammation (swelling) of the brain. Also some people can experience condition in which breathing is difficult and the oxygen levels in the blood suddenly drop lower than normal including severe lung condition. In severe cases inflammation of the brain and uncontrolled electrical activity in the brain occur, progressing to coma within 24 to 48 hours [12].

Nipah virus infection initial signs and symptoms are nonspecific, and the diagnosis is often not suspected at the time of presentation. This can stop accurate diagnosis and creates challenges in outburst detection, effective and timely infection control measures, and outburst response activities [8]. Currently there are no drugs or vaccines specific for Nipah virus infection. To treat severe respiratory and neurologic complications intensive supportive care is recommended.

Biswas, M. H. A., [2] has done research entitled "Optimal Control of Nipah Virus (NiV) Infections: A Bangladesh Scenario" in which he discussed the disease propagation and control strategy of NiV infections. He used educational campaigns and social distancing as controls to prevent people from being infected by Nipah virus.

In this paper control strategies for NiV have been discussed by considering human and bat population. Human population has been divided into susceptible humans, exposed humans, infected humans, hospitalized humans and death of human [7]. Bat population has been divided into susceptible bats, exposed bats, infected bats and removed bats. Mathematical model for the transmission of NiV has been described in Section 2. Section 3, 4 and 5 includes stability, control and simulation for the compartments. Conclusion is described in Section 6.

MATHEMATICAL MODEL

Here, we formulate a mathematical model for Human-Bat population. The notations with its description for each parameter are described in Table 1.

Table1: Notations and its Parametric Values

Notations	Description	Parametric Values
$S_H(t)$	Number of Susceptible Humans at some instant of time *t*	10
$E_H(t)$	Number of Exposed Humans at some instant of time *t*	6
$I_H(t)$	Number of Infected Humans at some instant of time *t*	7
$H_H(t)$	Number of Hospitalized Humans at some instant of time *t*	5
$D_H(t)$	Number of Human deaths at some instant of time *t*	3
$S_B(t)$	Number of Susceptible Bats at some instant of time *t*	20
$E_B(t)$	Number of Exposed Bats at some instant of time *t*	15
$I_B(t)$	Number of Infected Bats at some instant of time *t*	12
$R_B(t)$	Number of removed Bat at some instant of time *t*	10
B_H	New Recruitment Rate for Human	0.1
B_B	New Recruitment Rate for Bats	0.1
μ_H	Humans Death Rate	0.7

Notations	Description	Parametric Values
μ_B	Bats Death rate	0.7
β_1	Transmission rate of humans from susceptible to exposed	0.01
β_2	Transmission rate of humans from exposed to infected	0.3
β_3	Transmission rate of humans from infected to hospitalized	0.25
β_4	Transmission rate of humans from hospitalized to death	0.8
β_5	Transmission rate of humans from infected to death	0.8
β_6	Transmission rate of humans from death to exposed	0.8
β_7	Transmission rate of humans from infected to susceptible	0.1
α_1	Transmission rate of bats from susceptible to exposed	0.03
α_2	Transmission rate of bats from exposed to infected	0.6
α_3	Transmission rate of bats from infected to removed	0.5
α_4	Transmission rate of bats from infected to susceptible	0.2
α_5	Transmission rate of bats from infected to susceptible humans	0.9
u_1	Control rate in terms of spraying insecticides	[0,1]
u_2	Control rate in terms of buried bats	[0,1]
u_3	Control rate in terms of self-prevention	[0,1]
u_4	Control rate in terms of in time hospitalization	[0,1]

The transmission diagram of NiV from different compartments is shown in Figure 1.

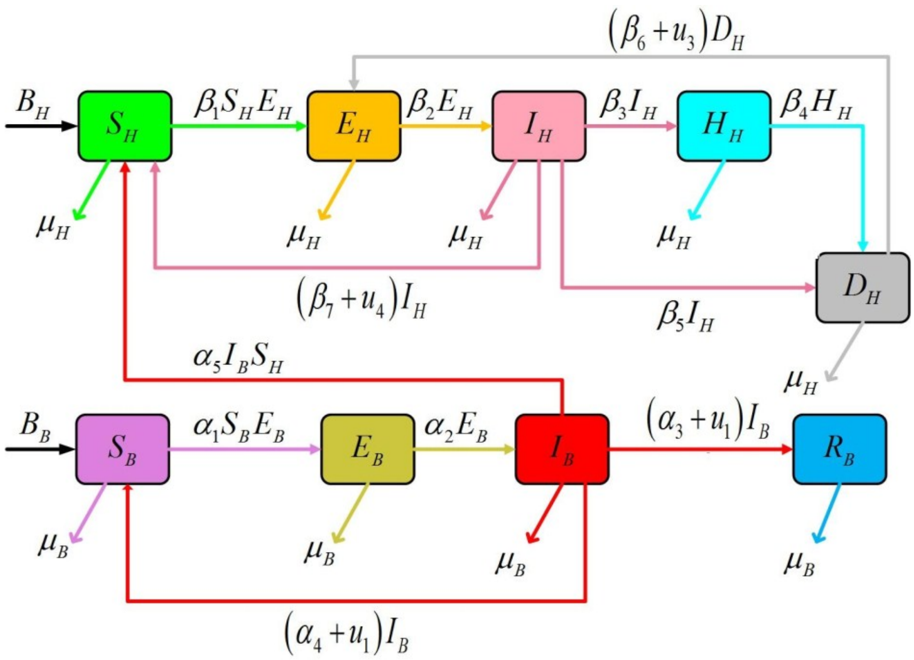

Figure 1: Schematic diagram of spread of NiV in human and bat population

Infected bat (I_B) can infect susceptible bats (S_B) and susceptible human (S_H). Also, infected human (I_H) can infect susceptible human (S_H). So, the disease can spread by two ways: infected bats and infected humans. Susceptible human infects human at the rate β_1. The rate of β_3 infected humans are seeking hospitalization so total individuals in the hospitalized compartment (H_H) are $\beta_3 I_H$. μ_H denotes natural death rate of humans and μ_B is for bats. Control u_1 in terms of spraying insecticides for bat population, u_2 in terms of buried bats, u_3 for self-prevention and u_4 for in time hospitalization.

So, from the above figure 1, we have the following set of nonlinear ordinary differential equations describing the causes of NiV from one compartment to other.

$$\begin{aligned}
\frac{dS_H}{dt} &= \mathrm{B}_H - \beta_1 S_H E_H + \beta_7 I_H + \alpha_5 I_B S_H - \mu_H S_H \\
\frac{dE_H}{dt} &= \beta_1 S_H E_H - \beta_2 E_H + \beta_6 D_H - \mu_H E_H \\
\frac{dI_H}{dt} &= \beta_2 E_H - \beta_3 I_H - \beta_5 I_H - \beta_7 I_H - \mu_H I_H \\
\frac{dH_H}{dt} &= \beta_3 I_H - \beta_4 H_H - \mu_H H_H \\
\frac{dD_H}{dt} &= \beta_4 H_H - \beta_6 D_H + \beta_5 I_H - \mu_H D_H \\
\frac{dS_B}{dt} &= B_B - \alpha_1 S_B E_B + \alpha_4 I_B - \mu_B S_B \\
\frac{dE_B}{dt} &= \alpha_1 S_B E_B - \alpha_2 E_B - \mu_B E_B \\
\frac{dI_B}{dt} &= \alpha_2 E_B - \alpha_3 I_B - \alpha_5 I_B S_H - \alpha_4 I_B - \mu_B I_B \\
\frac{dR_B}{dt} &= \alpha_3 I_B - \mu_B R_B
\end{aligned} \tag{1}$$

The feasible region for the above set of equations (1) is

$$\Lambda = \left\{ \begin{array}{l} \left(S_H + E_H + I_H + H_H + D_H + S_B + E_B + I_B + R_B\right) / \\ S_H + E_H + I_H + H_H + D_H \le \dfrac{\mathrm{B}_H}{\mu_H}, S_B + E_B + I_B + R_B \le \dfrac{\mathrm{B}_B}{\mu_B}; \\ S_H, S_B \ge 0, E_H, I_H, H_H, D_H, E_B, I_B, R_B > 0 \end{array} \right\}$$

On equating set of equations (1) to zero, following equilibrium points are obtained:

i. NiV free equilibrium point $\left(E_0\right)$

$$E_0 = \left(S_H, E_H, I_H, H_H, D_H, S_B, E_B, I_B, R_B\right) = \left(\frac{B_H}{\mu_H}, 0, 0, 0, 0, \frac{B_B}{\mu_B}, 0, 0, 0\right)$$

ii. Bat infectious free equilibrium point $\left(E_1\right)$

$$E_1 = \left(S_{H_1}, E_{H_1}, I_{H_1}, H_{H_1}, D_{H_1}, S_{B_1}, 0, 0, 0\right)$$

where, $$E_{H_1} = \frac{I_{H_1}\beta_6\left(\mu_H\beta_5 + \beta_4\left(\beta_3 + \beta_5\right)\right)}{\mu_H^{\;3} + \mu_H^{\;2}\left(\beta_2 + \beta_4 + \beta_6 - S_{H_1}\beta_1\right) + \mu_H\begin{pmatrix}\left(\beta_4 + \beta_6\right)\left(\beta_2 - S_{H_1}\beta_1\right) \\ +\beta_4\beta_6\end{pmatrix} + \beta_4\beta_6\left(\beta_2 - S_{H_1}\beta_1\right)}$$

$$D_{H_1} = \frac{I_{H_1}\left(\mu_H\beta_5 + \beta_4\left(\beta_3 + \beta_5\right)\right)}{\mu_H\left(\mu_H + \beta_4 + \beta_6\right) + \beta_4\beta_6}, \quad H_{H_1} = \frac{\beta_3 I_{H_1}}{\mu_H + \beta_4}, \quad S_{B_1} = \frac{B_B}{\mu_B}$$

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iii. NiV existence equilibrium point $\left(E^*\right)$

$$E^* = \left(S_H^*, E_H^*, I_H^*, H_H^*, D_H^*, S_B^*, E_B^*, I_B^*, R_B^*\right)$$

where, $$E_H^* = \frac{I_H^*\beta_6\left(\mu_H\beta_5+\beta_3\beta_4+\beta_4\beta_5\right)}{\mu_H^3+\mu_H^2\left(\beta_2+\beta_4+\beta_6-S_H^*\beta_1\right)+\mu_H\begin{pmatrix}\left(\beta_4+\beta_6\right)\left(\beta_2-S_H^*\beta_1\right)\\ +\beta_4\beta_6\end{pmatrix}+\beta_4\beta_6\left(\beta_2-S_H^*\beta_1\right)}$$

$$H_H^* = \frac{\beta_3 I_H^*}{\mu_H+\beta_4},\ D_H^* = \frac{I_H^*\left(\mu_H\beta_5+\beta_4\left(\beta_3+\beta_5\right)\right)}{\mu_H\left(\mu_H+\beta_4+\beta_6\right)+\beta_4\beta_6},\ S_B^* = \frac{\mu_B+\alpha_2}{\alpha_1},$$

$$E_B^* = -\frac{\left(\mu_B^2+\mu_B\alpha_2-B_B\alpha_1\right)\left(S_H^*\alpha_5+\mu_B+\alpha_3+\alpha_4\right)}{\alpha_1\left[\mu_B\left(S_H^*\alpha_5+\mu_B+\alpha_2+\alpha_3+\alpha_4\right)+S_H^*\alpha_2\alpha_5+\alpha_2\alpha_3\right]},$$

$$I_B^* = -\frac{\left(\mu_B^2+\mu_B\alpha_2-B_B\alpha_1\right)\alpha_2}{\alpha_1\left[\mu_B\left(S_H^*\alpha_5+\mu_B+\alpha_2+\alpha_3+\alpha_4\right)+S_H^*\alpha_2\alpha_5+\alpha_2\alpha_3\right]},$$

$$R_B^* = -\frac{\left(\mu_B^2+\mu_B\alpha_2-B_B\alpha_1\right)\alpha_2\alpha_3}{\alpha_1\left[\mu_B\left(S_H^*\alpha_5+\mu_B+\alpha_2+\alpha_3+\alpha_4\right)+S_H^*\alpha_2\alpha_5+\alpha_2\alpha_3\right]}$$

Now, our actual interest lies in calculating the basic reproduction number which is calculated using the next generation matrix method which is defined as FV^{-1} where F and V both are Jacobian matrices of $\Im$ and v evaluated with respect to infected humans, bats at the point E_0.

Let $X=\left(S_H, E_H, I_H, H_H, D_H, S_B, E_B, I_B, R_B\right)$

$$\therefore \frac{dX}{dt} = \Im(X) - v(X)$$

where $\Im(X)$ denotes the rate of newly recruited and $v(X)$ denotes the rate of derived recruited which is given as

$$\Im(X) = \begin{bmatrix} \alpha_5 I_B S_H \\ \beta_1 S_H E_H \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha_1 S_B E_B \\ 0 \\ 0 \end{bmatrix} \text{ and } v\left(X\right) = \begin{bmatrix} -\mathrm{B}_H+\beta_1 S_H E_H-\beta_7 I_H+\mu_H S_H \\ \beta_2 E_H-\beta_6 D_H+\mu_H E_H \\ -\beta_2 E_H+\beta_3 I_H+\beta_5 I_H+\beta_7 I_H+\mu_H I_H \\ -\beta_3 I_H+\beta_4 H_H+\mu_H H_H \\ -\beta_4 H_H+\beta_6 D_H-\beta_5 I_H+\mu_H D_H \\ -B_B+\alpha_1 S_B E_B-\alpha_4 I_B+\mu_B S_B \\ \alpha_2 E_B+\mu_B E_B \\ -\alpha_2 E_B+\alpha_3 I_B+\alpha_5 I_B S_H+\alpha_4 I_B+\mu_B I_B \\ -\alpha_3 I_B+\mu_B R_B \end{bmatrix}$$

Now, the derivative of $\Im$ and v evaluated at a NiV free equilibrium point $\left(E_0\right)$ gives matrices F and V of order 9×9 which is defined as

$$F=\left[\frac{\partial \Im_i(E_0)}{\partial X_j}\right] \quad V=\left[\frac{\partial v_i(E_0)}{\partial X_j}\right] \text{ for } i,j=1,2,3,4,5,6,7,8,9$$

So, $F=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_5\frac{B_H}{\mu_H} & 0 \\ 0 & \beta_1\frac{B_H}{\mu_H} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_1\frac{B_B}{\mu_B} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ and

$$V=\begin{bmatrix} \mu_H & \beta_1\frac{B_H}{\mu_H} & -\beta_7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2+\mu_H & 0 & 0 & -\beta_6 & 0 & 0 & 0 & 0 \\ 0 & -\beta_2 & \begin{pmatrix}\beta_3+\beta_5\\ +\beta_7+\mu_H\end{pmatrix} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_3 & \beta_4+\mu_H & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta_5 & -\beta_4 & \beta_6+\mu_H & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu_B & \alpha_1\frac{B_B}{\mu_B} & -\alpha_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_2+\mu_B & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 & \begin{pmatrix}\alpha_3+\alpha_5\frac{B_H}{\mu_H}\\ +\alpha_4+\mu_B\end{pmatrix} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_3 & \mu_B \end{bmatrix}$$

where V is non-singular matrix. Thus, the basic reproduction number R_0 which is the spectral radius of matrix FV^{-1} is given as

$$R_0=\frac{X}{Y}$$

where, $X=\left(\beta_1\beta_H\begin{pmatrix}\mu_H{}^3+\left(\beta_3+\beta_4+\beta_5+\beta_6+\beta_7\right)\mu_H{}^2\\ +\left(\beta_3\left(\beta_4+\beta_6\right)+\beta_4\left(\beta_5+\beta_6+\beta_7\right)+\beta_6\left(\beta_5+\beta_7\right)\right)\mu_H\\ +\beta_4\beta_6\left(\beta_3+\beta_5+\beta_4\beta_6\right)\end{pmatrix}\right)$ and

$$Y=\left(\mu_H\left(\begin{array}{l}\mu_H{}^4+(\beta_2+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)\mu_H{}^3\\+\left(\beta_2(\beta_3+\beta_4+\beta_5+\beta_6+\beta_7)+\beta_3(\beta_4+\beta_6)+\beta_4(\beta_5+\beta_6+\beta_7)+\beta_6(\beta_5+\beta_7)\right)\mu_H{}^2\\+\left(\beta_2\beta_3(\beta_4+\beta_6)+\beta_2\beta_4(\beta_5+\beta_6+\beta_7+\beta_6\beta_7)+\beta_2\beta_6\beta_7+\beta_4\beta_6(\beta_3+\beta_5+\beta_7)\right)\mu_H\end{array}\right)\right)$$
$$+\frac{\alpha_1 B_B}{\mu_B(\alpha_2+\mu_B)}+\frac{\alpha_5 B_H\alpha_2}{(\alpha_2+\mu_B)(\mu_B\mu_H+\mu_H\alpha_3+\mu_H\alpha_4+B_H\alpha_5)}$$

STABILITY ANALYSIS

In this section, the local and global stability at E_0, E_1 and E^* using the linearization method and matrix analysis are to be studied.

Local Stability

Theorem 1 (stability at E_0) The system is locally asymptotically stable at NiV free

equilibrium point E_0, if

i. $A_4A_2A_1>\beta_5\beta_2\beta_6$

ii. $A_4A_2A_1>\beta_2\beta_6(\beta_5A_3+\beta_3\beta_4)$

Proof: Jacobian Matrix of the system evaluated at point E_0 is

$$J(E_0)=\begin{bmatrix}-A_1 & 0 & 0 & \beta_6 & 0\\ \beta_2 & -A_2 & 0 & 0 & 0\\ 0 & \beta_3 & -A_3 & 0 & 0\\ 0 & \beta_5 & \beta_4 & -A_4 & 0\\ 0 & 0 & 0 & 0 & -A_5\end{bmatrix}$$

where, $A_1=-\beta_1\frac{B_H}{\mu_H}+\beta_2+\mu_H$, $A_2=\beta_3+\beta_5+\beta_7+\mu_H$, $A_3=\beta_4+\mu_H$, $A_4=\beta_6+\mu_H$,

$$A_5=-\alpha_1\frac{B_B}{\mu_B}+\alpha_2+\mu_B$$

The eigenvalues of the characteristic equation are $\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5$ which satisfies the equation $a_0\lambda^5+a_1\lambda^4+a_2\lambda^3+a_3\lambda^2+a_4\lambda+a_5=0$

The coefficients of characteristic polynomial are positive if following conditions are satisfied,

i. $A_4A_2A_1>\beta_5\beta_2\beta_6$

ii. $A_4A_2A_1>\beta_2\beta_6(\beta_5A_3+\beta_3\beta_4)$

Theorem 2 (stability at E_1) The system is locally asymptotically stable at bat infectious free

equilibrium point E_1, if

i. $A_{10}A_8A_7 > \beta_5\beta_2\beta_6$

ii. $A_8\beta_1 S_{H_1} > \beta_2\beta_7$

iii. $A_{10}A_4A_3A_2 > \beta_2\beta_6\left(\beta_5A_9 + \beta_3\beta_4\right)$

Proof: Jacobian Matrix of the system evaluated at point E_1 is

$$J\left(E_1\right) = \begin{bmatrix} -A_6 & -\beta_1 S_{H_1} & \beta_7 & 0 & 0 \\ -\beta_2 E_{H_1} & -A_7 & 0 & 0 & \beta_6 \\ 0 & \beta_2 & -A_8 & 0 & 0 \\ 0 & 0 & \beta_3 & -A_9 & 0 \\ 0 & 0 & \beta_5 & \beta_4 & -A_{10} \end{bmatrix}$$

where, $A_6 = \beta_1 E_{H_1} + \mu_H$, $A_7 = \beta_2 + \mu_H - \beta_1 S_{H_1}$, $A_8 = \beta_3 + \beta_5 + \beta_7 + \mu_H$, $A_9 = \beta_4 + \mu_H$, $A_{10} = \beta_6 + \mu_H$

The eigenvalues of the characteristic equation are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ which satisfies the equation

$$a_0\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0$$

The coefficients of characteristic polynomial are positive if following conditions are satisfied,

i. $A_{10}A_8A_7 > \beta_5\beta_2\beta_6$

ii. $A_8\beta_1 S_{H_1} > \beta_2\beta_7$

iii. $A_{10}A_4A_3A_2 > \beta_2\beta_6\left(\beta_5A_9 + \beta_3\beta_4\right)$

Theorem 3 (stability at E^*) The system is locally asymptotically stable at endemic

equilibrium point E^*, if

Proof: Jacobian Matrix of the system evaluated at point E^* is

$$J\left(E^*\right) = \begin{bmatrix} -A_{11} & -\beta_1 S_H^* & \beta_7 & 0 & 0 & 0 & 0 & \alpha_5 S_H^* \\ -\beta_1 E_H^* & -A_{12} & 0 & 0 & \beta_6 & 0 & 0 & 0 \\ 0 & \beta_2 & -A_{13} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & -A_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_5 & \beta_4 & -A_{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -A_{16} & -\alpha_1 S_B^* & \alpha_4 \\ -\alpha_5 I_B^* & 0 & 0 & 0 & 0 & \alpha_1 E_B^* & -A_{17} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & -A_{18} \end{bmatrix}$$

where, $A_{11} = \beta_1 E_H^* - \alpha_5 I_B^* + \mu_H$, $A_{12} = \beta_2 + \mu_H - \beta_1 S_H^*$, $A_{13} = \beta_3 + \beta_5 + \beta_7 + \mu_H$, $A_{14} = \beta_4 + \mu_H$,

$A_{15} = \beta_6 + \mu_H$, $A_{16} = \mu_B + \alpha_1 E_B^*$, $A_{17} = \mu_B + \alpha_2 - \alpha_1 S_B^*$, $A_{18} = \alpha_3 + \alpha_5 S_H^* + \alpha_4 + \mu_B$

The eigenvalues of the characteristic equation are $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ which satisfies the equation

$$a_0\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0$$

The coefficients of characteristic polynomial are positive if following conditions are satisfied,

i. $A_{13}\alpha_1 S_B^{*} > \alpha_2\alpha_4$

ii. $A_{15}\alpha_1 S_B^{*} > \alpha_2\alpha_4$

iii. $A_{18}\alpha_1 S_B^{*} > \alpha_2\alpha_4$

iv. $A_{13}\beta_1 S_H^{*} > \beta_2\beta_7$

v. $A_{14}\beta_1 S_H^{*} > \beta_2\beta_7$

vi. $A_{18}\beta_1 S_H^{*} > \beta_2\beta_7$

vii. $A_{15}A_{13}A_{12} > \beta_2\beta_5\beta_6$

viii. $A_{15}A_{14}A_{13}A_{12} > \beta_2\beta_6\left(\beta_5 A_{14} + \beta_3\beta_4\right)$

Global Stability

Theorem 4 (stability at E_0) The system is globally asymptotically stable at NiV free

equilibrium point E_0.

Proof: Consider a Lyapunov function

$$L(t) = E_H(t) + I_H(t) + H_H(t) + D_H(t) + E_B(t) + I_B(t) + R_B(t)$$

$$\begin{aligned}
\therefore L'(t) &= \beta_1 S_H E_H - \mu_H E_H - \beta_7 I_H - \mu_H I_H - \mu_H H_H - \mu_H D_H \\
&\quad + \alpha_1 S_B E_B - \mu_B E_B - \alpha_5 I_B S_H - \alpha_4 I_B - \mu_B I_B - \mu_B R_B
\end{aligned}$$

$$\begin{aligned}
\therefore L'(t) &= B_H - \mu_H S_H - \mu_H E_H - \mu_H I_H - \mu_H H_H - \mu_H D_H + B_B - \mu_B S_B - \mu_B E_B - \mu_B I_B - \mu_B R_B \\
&= B_H - \mu_H\left(S_H + E_H + I_H + H_H + D_H\right) + B_B - \mu_B\left(S_B + E_B + I_B + R_B\right) \\
&= B_H - \mu_H\left(\frac{B_H}{\mu_H}\right) - \mu_H\left(E_H + I_H + H_H + D_H\right) + B_B - \mu_B\left(\frac{B_B}{\mu_B}\right) - \mu_B\left(E_B + I_B + R_B\right) \\
&= -\mu_H\left(E_H + I_H + H_H + D_H\right) - \mu_B\left(E_B + I_B + R_B\right) \\
&\le 0
\end{aligned}$$

And if $E_H = I_H = H_H = D_H = E_B = I_B = R_B = 0$ then $L'(t) = 0$.

So, by LaSalle's Invariance Principle, every solution of system (1), with initial conditions in Λ, approaches E_0 as $t \to \infty$.

Thus, a system is globally asymptotically stable at point E_0.

Theorem 5 (stability at E_1) The system is globally asymptotically stable at bat infected equilibrium point E_1.

Proof: Consider a Lyapunov function

$$L(t)=S_H(t)+I_H(t)+H_H(t)+D_H(t)+S_B(t)+I_B(t)+R_B(t)$$

$$\begin{aligned}\therefore L'(t)&=B_H-\beta_1 S_H E_H-\mu_H S_H+\beta_2 E_H-\mu_H I_H-\mu_H H_H-\beta_6 D_H-\mu_H D_H+B_B\\&\quad-\alpha_1 S_B E_B-\mu_B S_B+\alpha_2 E_B-\mu_B I_B-\mu_B R_B\\&=B_H-\mu_H\left(S_H+E_H+I_H+H_H+D_H\right)+B_B-\mu_B\left(S_B+E_B+I_B+R_B\right)\\&=B_H+B_B-\mu_H\left(S_H+E_H+I_H+H_H+D_H\right)-\mu_B\left(E_B+I_B+R_B\right)-\mu_B S_B\end{aligned}$$

At point E_1, $E_B=I_B=R_B=0$

$$\begin{aligned}\therefore L'(t)&=B_H+B_B-\mu_H\left(S_H+E_H+I_H+H_H+D_H\right)-\mu_B S_B\\&\le B_H-\mu_H\left(N_H\right)+B_B-\mu_B S_B\\&\le 0\end{aligned}$$

Thus, a system is globally asymptotically stable at point E_1.

Theorem 6 (stability at E^*) The system is globally asymptotically stable at endemic equilibrium point E^*.

Proof: Consider a Lyapunov function

$$L(t)=\frac{1}{2}\begin{bmatrix}\left\{S_H(t)-S_H^*(t)\right\}+\left\{E_H(t)-E_H^*(t)\right\}+\left\{I_H(t)-I_H^*(t)\right\}\\+\left\{H_H(t)-H_H^*(t)\right\}+\left\{D_H(t)-D_H^*(t)\right\}\\+\left\{S_B(t)-S_B^*(t)\right\}+\left\{E_B(t)-E_B^*(t)\right\}+\left\{I_B(t)-I_B^*(t)\right\}+\left\{R_B(t)-R_B^*(t)\right\}\end{bmatrix}^2$$

$$\therefore L'(t)=\begin{vmatrix}\left\{S_H(t)-S_H^*(t)\right\}+\left\{E_H(t)-E_H^*(t)\right\}+\left\{I_H(t)-I_H^*(t)\right\}\\+\left\{H_H(t)-H_H^*(t)\right\}+\left\{D_H(t)-D_H^*(t)\right\}\\+\left\{S_B(t)-S_B^*(t)\right\}+\left\{E_B(t)-E_B^*(t)\right\}+\left\{I_B(t)-I_B^*(t)\right\}+\left\{R_B(t)-R_B^*(t)\right\}\end{vmatrix}$$
$$\left[B_H+B_B-\mu_H\left(S_H+E_H+I_H+H_H+D_H\right)-\mu_B\left(S_B+E_B+I_B+R_B\right)\right]$$

On using, $\mathrm{B}_H=\mu_H S_H^*+\mu_H E_H^*+\mu_H I_H^*+\mu_H H_H^*+\mu_H D_H^*$, $\mathrm{B}_B=\mu_B S_B^*+\mu_B E_B^*+\mu_B I_B^*+\mu_B R_B^*$

$$\begin{aligned}L'(t)&=-\mu_H\begin{bmatrix}\left\{S_H(t)-S_H^*(t)\right\}+\left\{E_H(t)-E_H^*(t)\right\}+\left\{I_H(t)-I_H^*(t)\right\}\\+\left\{H_H(t)-H_H^*(t)\right\}+\left\{D_H(t)-D_H^*(t)\right\}\end{bmatrix}^2\\&\quad-\mu_B\left[\left\{S_B(t)-S_B^*(t)\right\}+\left\{E_B(t)-E_B^*(t)\right\}+\left\{I_B(t)-I_B^*(t)\right\}+\left\{R_B(t)-R_B^*(t)\right\}\right]^2\\&\le 0\end{aligned}$$

So, the system is globally asymptotically stable at E^*.

OPTIMAL CONTROL MODEL

In this section, a control function has been implemented to decrease the spread of NiV in human population. The objective function along with the optimal control variable is given by

$$J\left(u_i,\Omega\right)=\int_0^T\begin{pmatrix} A_1S_H{}^2+A_2E_H{}^2+A_3I_H{}^2+A_4H_H{}^2+A_5D_H{}^2+A_6S_B{}^2 \\ +A_7E_B{}^2+A_8I_B{}^2+A_9R_B{}^2+w_1u_1{}^2+w_2u_2{}^2+w_3u_3{}^2+w_4u_4{}^2 \end{pmatrix}dt \tag{2}$$

where Ω denotes the set of all compartmental variables, $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9$ denotes non-negative weight constants for the compartments $S_H, E_H, I_H, H_H, D_H, S_B, E_B, I_B, R_B$ respectively and w_1, w_2, w_3, w_4 are the weight constant for the control variable u_1, u_2, u_3, u_4 respectively.

The weights w_1, w_2, w_3 and w_4 which are constant parameters for u_1, u_2, u_3 and u_4 will standardized using the optimal control condition.

Now, we will calculate the value of control variables u_1, u_2, u_3 and u_4 from $t = 0$ to $t = T$ such that

$$J\left(u_1(t),u_2(t),u_3(t),u_4(t)\right)=\min\left\{J\left(u_i^*,\Omega\right)/\,u_1,u_2,u_3,u_4\in\phi\right\}$$

where ϕ = smooth function on the interval $[0,1]$.

Using, Fleming and Rishel results, the optimal control denoted by u_i^* is obtained by collecting all the integrands of the objective function using the lower bounds and upper bounds of the both the control variables respectively.

Using Pontrygin's principle, we construct a Lagrangian function consisting of state equation and adjoint variables $A_V = (\lambda_{S_H}, \lambda_{E_H}\ \lambda_{I_H}, \lambda_{H_H}, \lambda_{D_H}, \lambda_{S_B}, \lambda_{E_B}\lambda_{I_B}, \lambda_{R_B})$ which is as follows:

$$\begin{aligned}
L\left(\Omega,A_V\right)=\ & A_1S_H{}^2+A_2E_H{}^2+A_3I_H{}^2+A_4H_H{}^2+A_5D_H{}^2+A_6S_B{}^2+A_7E_B{}^2+A_8I_B{}^2+A_9R_B{}^2 \\
& +w_1u_1{}^2+w_2u_2{}^2+w_3u_3{}^2+w_4u_4{}^2 \\
& +\lambda_{S_H}\left[\mathrm{B}_H-\beta_1S_HE_H+\beta_7I_H+\alpha_5I_BS_H-\mu_HS_H+u_4I_H\right] \\
& +\lambda_{E_H}\left[\beta_1S_HE_H-\beta_2E_H+\beta_6D_H-\mu_HE_H+u_3D_H\right] \\
& +\lambda_{I_H}\left[\beta_2E_H-\beta_3I_H-\beta_5I_H-\beta_7I_H-\mu_HI_H-u_4I_H\right] \\
& +\lambda_{H_H}\left[\beta_3I_H-\beta_4H_H-\mu_HH_H\right] \\
& +\lambda_{D_H}\left[\beta_4H_H-\beta_6D_H+\beta_5I_H-\mu_HD_H-u_3D_H\right] \\
& +\lambda_{S_B}\left[B_B-\alpha_1S_BE_B+\alpha_4I_B-\mu_BS_B+u_1I_B\right] \\
& +\lambda_{E_B}\left[\alpha_1S_BE_B-\alpha_2E_B-\mu_BE_B\right] \\
& +\lambda_{I_B}\left[\alpha_2E_B-\alpha_3I_B-\alpha_5I_BS_H-\alpha_4I_B-\mu_BI_B-u_1I_B-u_2I_B\right] \\
& +\lambda_{R_B}\left[\alpha_3I_B-\mu_BR_B+u_2I_B\right]
\end{aligned}$$

Now, the partial derivative of the Lagrangian function with respect to each variable of the compartment gives us the adjoint equation such that

$$\dot{\lambda}_{S_H}=-\frac{\partial L}{\partial S_H}=-2A_1S_H+\left(\beta_1E_H\right)\left(\lambda_{S_H}-\lambda_{E_H}\right)+\left(\alpha_5I_B\right)\left(\lambda_{E_H}-\lambda_{S_H}\right)+\mu_H\lambda_{S_H}$$

$$\dot{\lambda}_{E_H}=-\frac{\partial L}{\partial E_H}=-2A_2E_H+\left(\beta_1S_H\right)\left(\lambda_{S_H}-\lambda_{E_H}\right)+\beta_2\left(\lambda_{E_H}-\lambda_{I_H}\right)+\mu_H\lambda_{E_H}$$

$$\dot{\lambda}_{I_H} = -\frac{\partial L}{\partial I_H} = -2A_3 I_H + (\beta_7 + u_4)\left(\lambda_{I_H} - \lambda_{S_H}\right) + \beta_3\left(\lambda_{I_H} - \lambda_{H_H}\right) + \beta_5\left(\lambda_{I_H} - \lambda_{D_H}\right) + \mu_H \lambda_{I_H}$$

$$\dot{\lambda}_{H_H} = -\frac{\partial L}{\partial H_H} = -2A_4 H_H + \beta_4\left(\lambda_{H_H} - \lambda_{D_H}\right) + \mu_H \lambda_{H_H}$$

$$\dot{\lambda}_{D_H} = -\frac{\partial L}{\partial D_H} = -2A_5 D_H + (\beta_6 + u_3)\left(\lambda_{D_H} - \lambda_{E_H}\right) + \mu_H \lambda_{D_H}$$

$$\dot{\lambda}_{S_B} = -\frac{\partial L}{\partial S_B} = -2A_6 S_B + (\alpha_1 E_B)\left(\lambda_{S_B} - \lambda_{E_B}\right) + \mu_B \lambda_{S_B}$$

$$\dot{\lambda}_{E_B} = -\frac{\partial L}{\partial E_B} = -2A_7 E_B + (\alpha_1 S_B)\left(\lambda_{S_B} - \lambda_{E_B}\right) + (\alpha_2)\left(\lambda_{E_B} - \lambda_{I_B}\right) + \mu_B \lambda_{E_B}$$

$$\dot{\lambda}_{I_B} = -\frac{\partial L}{\partial I_B} = -2A_8 I_B + (\alpha_5 S_H)\left(\lambda_{I_B} - \lambda_{S_H}\right) + (\alpha_4 + u_1)\left(\lambda_{I_B} - \lambda_{S_B}\right) + (\alpha_3 + u_2)\left(\lambda_{I_B} - \lambda_{R_B}\right) + \mu_B \lambda_{I_B}$$

$$\dot{\lambda}_{R_B} = -\frac{\partial L}{\partial R_B} = -2A_9 R_B + \mu_B \lambda_{R_B}$$

The necessary conditions for Lagrangian function L to be optimal for control are

$$\frac{\partial L}{\partial u_1} = 2w_1 u_1 + I_B\left(\lambda_{S_B} - \lambda_{I_B}\right) = 0 \tag{3}$$

$$\frac{\partial L}{\partial u_2} = 2w_2 u_2 + I_B\left(\lambda_{R_B} - \lambda_{I_B}\right) = 0 \tag{4}$$

$$\frac{\partial L}{\partial u_3} = 2w_3 u_3 + D_H\left(\lambda_{E_H} - \lambda_{D_H}\right) = 0 \tag{5}$$

$$\frac{\partial L}{\partial u_4} = 2w_4 u_4 + I_H\left(\lambda_{S_H} - \lambda_{I_H}\right) = 0 \tag{6}$$

On solving equation (3), (4), (5) and (6) we get,

$$u_1 = \frac{I_B\left(\lambda_{I_B} - \lambda_{S_B}\right)}{2w_1}, u_2 = \frac{I_B\left(\lambda_{I_B} - \lambda_{R_B}\right)}{2w_2}, u_3 = \frac{D_H\left(\lambda_{D_H} - \lambda_{E_H}\right)}{2w_3} \text{ and } u_4 = \frac{I_H\left(\lambda_{I_H} - \lambda_{S_H}\right)}{2w_4}$$

Hence, the required optimal control condition is obtained as

$$u_1^* = \max\left(a_1, \min\left(b_1, \frac{I_B\left(\lambda_{I_B} - \lambda_{S_B}\right)}{2w_1}\right)\right), u_2^* = \max\left(a_2, \min\left(b_2, \frac{I_B\left(\lambda_{I_B} - \lambda_{R_B}\right)}{2w_2}\right)\right)$$

$$u_3^* = \max\left(a_3, \min\left(b_3, \frac{D_H\left(\lambda_{D_H} - \lambda_{E_H}\right)}{2w_3}\right)\right), u_4^* = \max\left(a_4, \min\left(b_3, \frac{I_H\left(\lambda_{I_H} - \lambda_{S_H}\right)}{2w_4}\right)\right)$$

where a_1, a_2, a_3, a_4 = lower bounds and b_1, b_2, b_3, b_4 = upper bounds of the control variables u_1, u_2, u_3 and u_4 respectively.

NUMERICAL SIMULATION

In this section, we will analyze and study the effect of control on each compartment numerically.

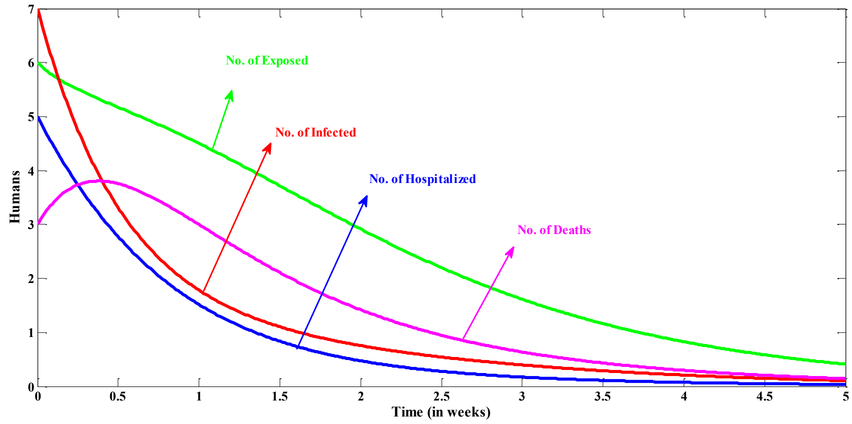

Figure 2(a): Impact of without control on human population

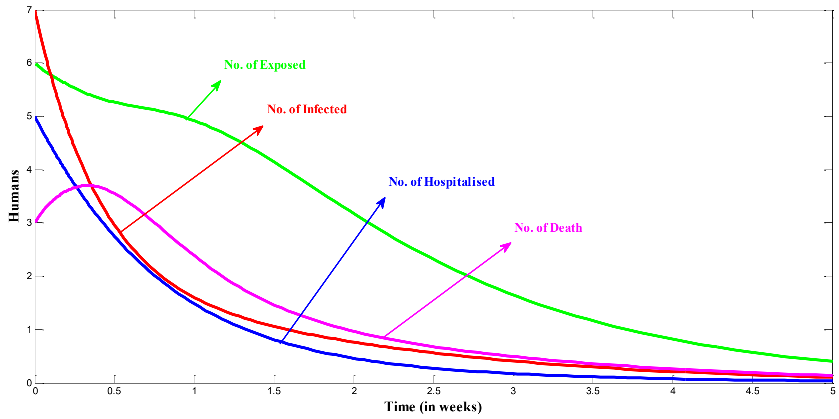

Figure 2(b): Impact of control on human population

Figure 2(a) and 2(b) suggests the impact of without and with control on human population. It can be observed that when control is applied exposed individuals decreases for certain period of time but then after it decreases. Number of infected individuals decreases from approximately 2.5 to 2 in comparison to without control. Hospitalized humans are gradually decreasing in both the cases which means they have met with a death as it can be seen from the figure that number of deaths are more in first half week.

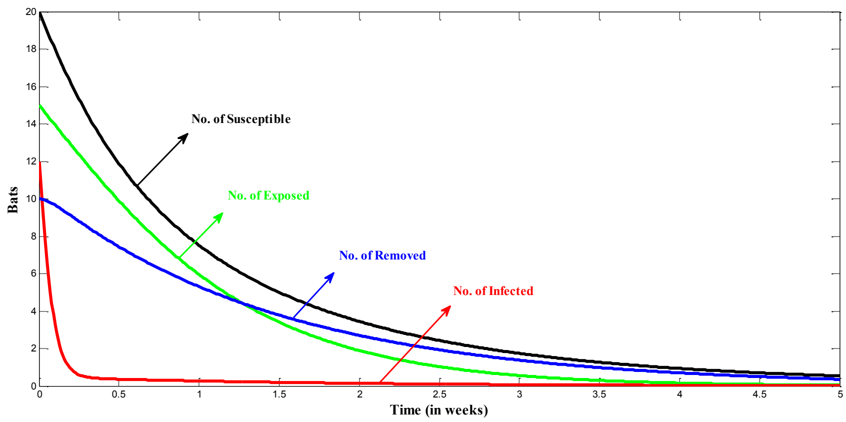

Figure 3(a): Impact of without control on bat population

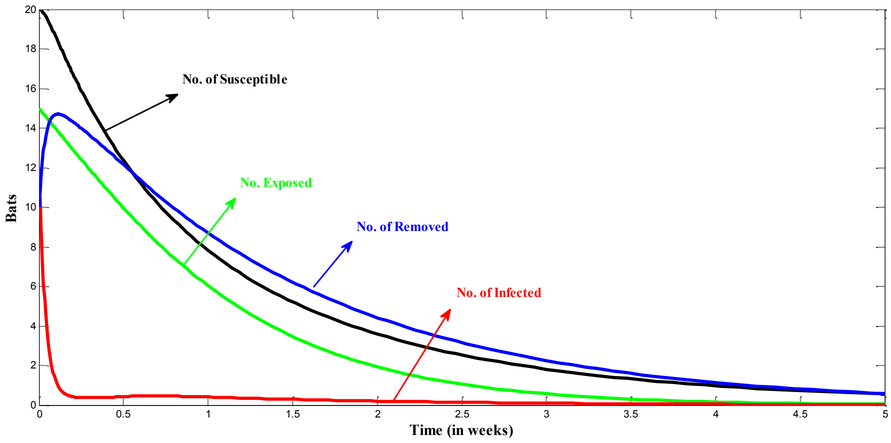

Figure 3(b): Impact of with control on bat population

Figure 3(a) and 3(b) depicts the impact of without and with control on bat population. It is seen from the figure that the susceptible and exposed bats are decreasing in both the cases. Also, it can be observed that when control is applied the infected bats are increasing from first half week to approximately a week but then after it decreases and gets stabilized. Removed bats are more in comparison to that of without control.

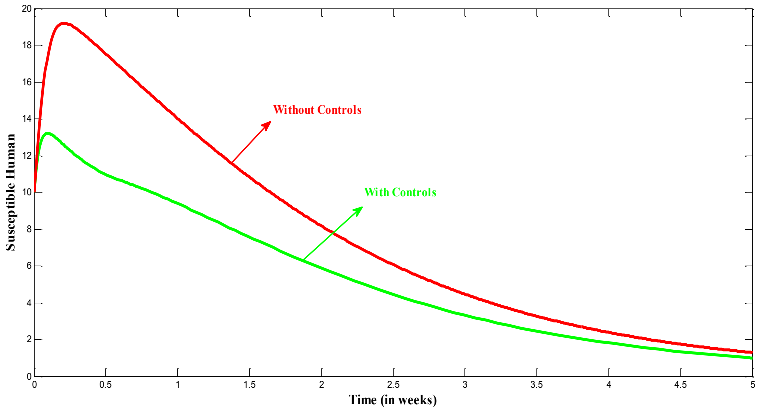

Figure 4: Control in terms of preventive measures on S_H compartment

Figure 4 depicts that when control is not applied, susceptible humans increases from 10 to 19 but when control in terms of preventive measures are taken by them the individual decrease to 13 approximately within a day.

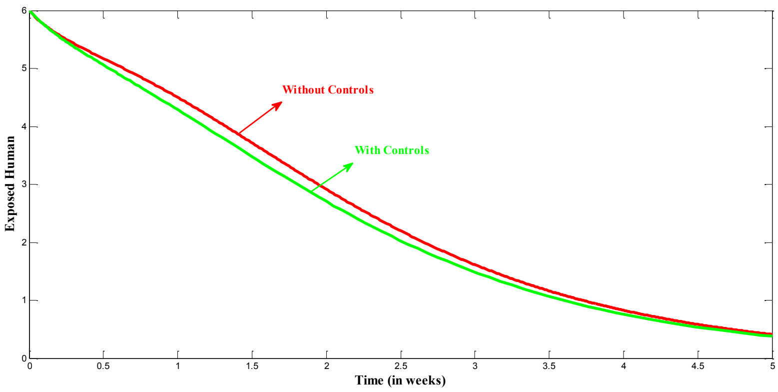

Figure 5: Effect of self-preventive measures on E_H compartment

From figure 5, it can be seen that when an individual gets medical treatment along with preventive measures, then exposed human decreases.

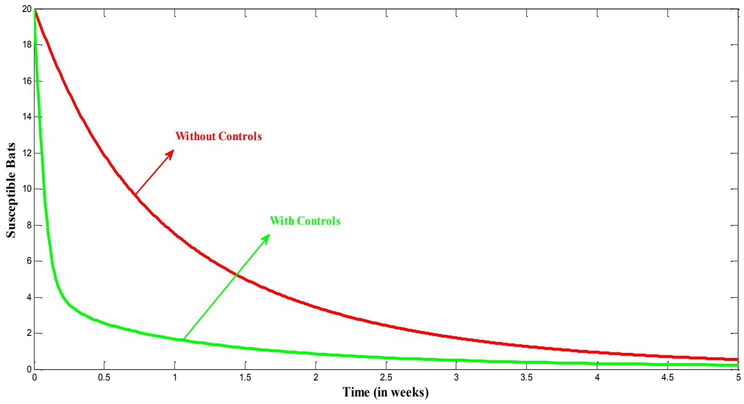

Figure 6: Effect of spraying insecticides on S_B compartment

Figure 6 shows that bats responsible for NiV are removed from the community in a small duration of time when insecticides are used.

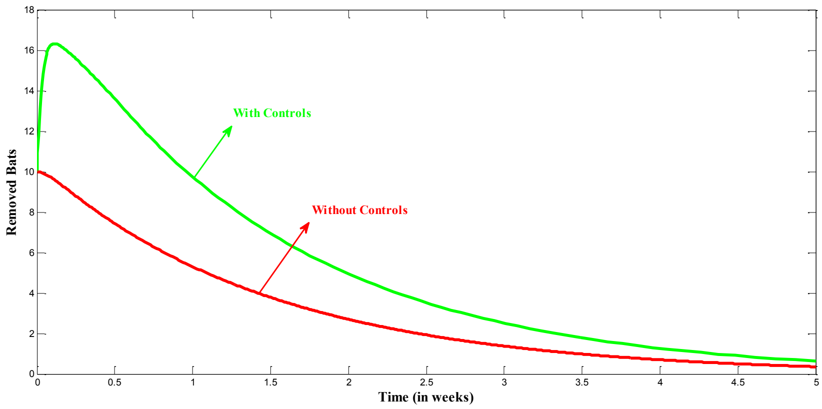

Figure 7: Effect of buried bats on R_B compartment

Figure 7 shows that number of bats responsible for NiV has increased in R_B compartment when they buried.

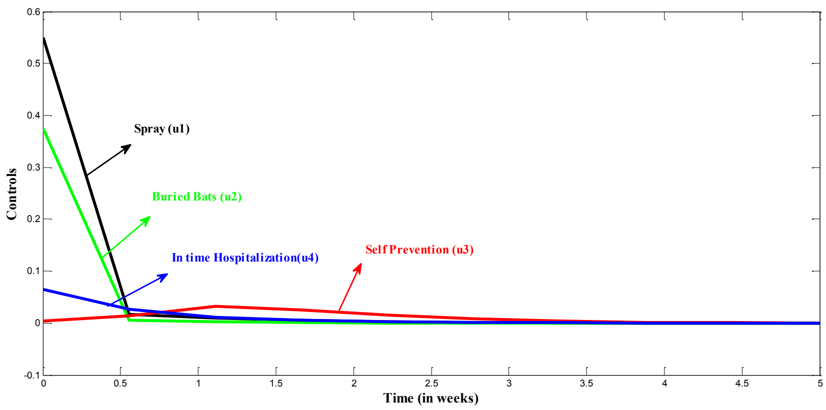

Figure 8: Control Variables versus Time (in weeks)

Figure 8 shows that each control has a vital role in protecting humans from NiV. It is seen that 3% self-preventive measures, 50% use of insecticides, 37% of buried bats and 6% of in time hospitalization are required in the initial days to stop oneself from becoming the victim of this disease.

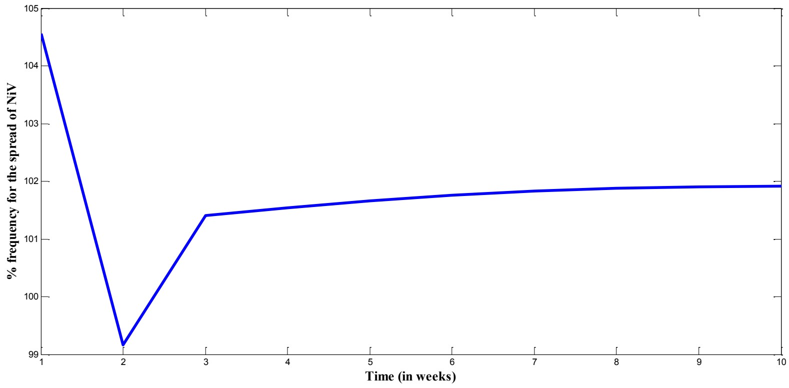

Figure 9: Percentage frequency for the spread of NiV

From figure 9, we can see that initially approximately 104 humans reduce to 99% in two weeks duration but once again it increases to approximately 101in three weeks and when it gets stabilize but not vanishes so preventive measures are advocated to control the spread of NiV.

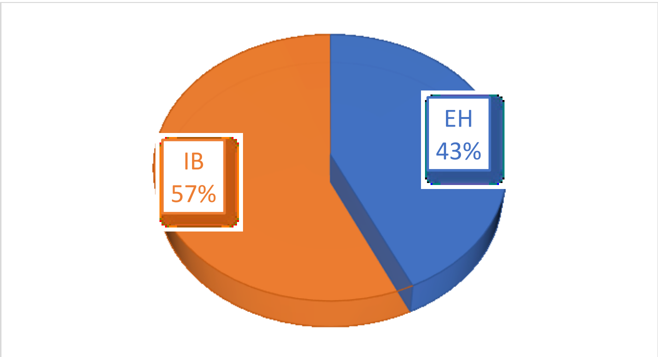

Figure 10: Effect of infected bats on humans

Figure 10 shows that 43% of human gets exposed by 57% of infected bats.

CONCLUSION

In this paper, a mathematical model has been constructed to study the spread of Nipah Virus (NiV). It has been established that this number of NiV affected individual can be reduced by using control on them as well on bats. Three equilibrium points have been found: NiV free equilibrium point, bat infection free equilibrium point and Nipah virus existence equilibrium points. At these points, system proves to be locally asymptotically stable and globally asymptotically stable. Results for different compartments have been calculated numerically which interprets that to reduce overall severity of the NiV, increase in spray of insecticides for the bats and preventive measures at the end of humans are required.

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