Verification of Over-Speed and Burst Margin Limits In aero Engine Rotor Coupling Along with Estimation of Low Cycle Fatigue Life

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Abstract
Turbine of an aero engine is considered as a key knowledge for development in overall engine performance. High performance turbines with augmented operational temperature require sophisticated design concepts. Weight reduction, increased strength and stiffness are the major requirements for highly stressed turbine disc in forthcoming aircraft engines. A major challenge during the design process of a modern low weight turbine disc is to find rotor disc geometries that meet both, static and fatigue requirement for both mechanical and thermal loads. The important functions of flexible coupling in turbomachines are, transmit mechanical power from one shaft to another with constant velocity, reimburse for mis-alignment without affecting structural integrity, without generating excessive thrust on either shaft for axial movement with minimum power loss. Aero engine rotor burst evaluation is one of the most important problems to be taken care off, whenever it comes to designing a turbo machinery disc. The consequences of a failure can be intense, since the disc fragments into multiple pieces and they are hurled away in all the possible direction at high speeds. In present work evaluation of safety limits and low-cycle fatigue (LCF) life estimation of an aero engine flange coupled disc through classical methods. By blending the terminologies with simulation engineering to arrive at a probable interpretation of number of duty cycles is carried out. The methodology compares the fatigue parameters involved in evaluation of disc life at off-design condition through sensitivity analysis. The design tool closely connects the flight certification regulating agencies for safety in air transportation vehicles. The off-design speed regulations through API and MIL handbook for material specification are considered to carry out finite element analysis.

Keywords: Flexible couplings, rotor burst, low-cycle fatigue, API & MIL handbook, finite element analysis

INTRODUCTION
The functional necessities and features of flexing coupling is to transmit rated torque without undergoing buckling, permanent deformation i.e. to possess with high torsional rigidity. However, under conditions of misalignment, flexing coupling element must have necessary flexibility to accommodate these situations without inducing excessive force and moment on shoulder shaft, bearing and bolts. Misalignment is compensated using laminated disc sets. Both of the requirements should be achieved by maintaining stress levels which are safely in the range of fatigue limit of flexing material. Metal-flexing coupling is identified to show irregular large-amplitude vibrations in axial direction when excited at natural frequency of coupling.

Considering various loads acting upon the disc and variation of the loads with respect to time are the factors that add to the complexity of turbine disc design. Weight of disc plays a vital role in improving efficiency of the gas turbine[1]. Hence, allows component to operate under plastic zone assuring the safety of the component with design limits. In present work an attempt is made to understand design criteria’s used for the design of gas turbine disc running at speed of 12000 RPM and operating at a temperature of 500°C. The finite element analyses were carried out to check the mechanical and structural integrity of the disc in a systematic order using the commercially validated FE package software. This includes,

- A sensitivity study for material model and its behaviour at in-service condition.
- 3-D analysis for estimation of over-speed margin evaluation with safety factor
- Estimation of over-speed margin in rotating aero rotor disc as per international authorities for integrity, blending the classical approach with FEA
- Application of 3-D elasto-plastic strain to conventional equations to arrive at fatigue life of disc; Coffin-Manson method

DESIGN CONSTRAINTS
The major loads acting on turbine blades are, centrifugal and thermo-mechanical loads. Thermal load is dominated by centrifugal loads for every cycle; Hence, importance is given for mechanical loads. In a bladed disk assembly, the disk happens to be the stronger section compared to disk. Replacing blade under failure is cost effective than replacing disks. During operating at design speed and over-speed circumstances, the average stresses obtained at cross sectional areas has to be well within allowable design limits as per design rule for both blade and disk. To sidestep all these complexities an integrated bladed rotor coupled assembly is considered for analysis. The constraints for present analysis is sequenced as follows
1. Design Parameters
2. Behavior constraints
• **Design parameters**
  1. At the Bore, permissible hoop and equivalent stress is 95% of 0.2% proof stress.
  2. At the web, permissible hoop stress is 85% of 0.2% proof stress.
  3. At the web, permissible radial and equivalent stress is 80% of 0.2% proof stress.
  4. At the rim, permissible hoop and equivalent stress is 67% of 0.2% proof stress.
  5. Axial and radial growth \( \leq \) specified tolerance i.e., 1 mm

• **Behaviour constraints**
  1. The average section stress at cross section area of disk must not exceed allowable design limits.
  2. Average cross sectional stress should not cross the limit of 10% of blade root neck average stress.
  3. Permissible AWMHS< 72% of 0.2% proof stress at peak temperature in the disk. [3]
  4. Permissible AWMHS< 64% of UTS at peak temperature in the disc. [3]
  5. Disc burst-speed \( \geq \) 125% of maximum permissible steady state speed for 12000 rpm.
  6. Disc over-speed \( \geq \)118% of maximum permissible steady state speed for 12000rpm.

These constraints are not readily available in the program like ANSYS and these are the constraints which are used in aero industry.

**MATERIAL MODEL**

The aero engine discs are subjected to high centrifugal and thermal loads Inconel is the material preferred for their manufacturing because of corrosion resistant property. Hence, for blade and disk material Inconel [2] is considered.

<table>
<thead>
<tr>
<th>Table 1: Material property of INCONEL718</th>
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</thead>
<tbody>
<tr>
<td>Density</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Young’s modulus</td>
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<tr>
<td>Tensile strength</td>
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<tr>
<td>Co-efficient of thermal expansion</td>
</tr>
</tbody>
</table>

**BOLT-PRETENSION**

Conventional method of designing a mechanical component is used for the preliminary design considerations of the turbine rotor.

- Petersons chart is used to calculate the Kt effect at the discontinuities which is finally used to decide on the initial fillet radius to start with. [4]
- Bolt pretention loads are calculated to prevent the loosening of bolts during operation.
- No of bolt holes are decided based on the flange opening between the discs and the torque getting transmitted through the discs.

Mathematical representation of bolt pre-tension is given by

\[ F_i = C \times A_s \times S_p \]  
**Eqn.1**

**LOAD CONSIDERATIONS**

1. **Rotational velocity Range**

Rotational or angular velocity is defined as rate of change of angular displacement and is a vector quantity which specifies the rotational speed (angular speed) of an object and the axis rotation.

2. **Influence of the blades**

The blades on the external ring of the disc causes an additional traction load in the radial direction, since they behave like concentrated masses under the effect of the centrifugal field.

**Formulas for blade pressure:**

\[ F_c = m \omega^2 r_c \]  
**Eqn.2**

\[ P_B = \frac{F_c + N_B}{\pi D_t} \]  
**Eqn.3**
3. Thermal load

Loads due to change in temperature are produced by the non-uniform distribution of temperature in the disc under service conditions. The external region of the rotor that is closer to the hot gasses, in fact, is hotter than the internal area. This non-uniform temperature gradient causes a deformation of the material, along with the coefficient of thermal expansion.

Service conditions for disc:

Turbine disc experiences centrifugal loads, thermal loads and blade pressure under operating conditions. The loading conditions of the disc for the present work are classified into three cases: first case is by considering centrifugal force only, second case is combination of centrifugal and blade pressure, third case is by considering all three loads as mentioned above. Figure 2 represents the schematic diagram of disc loading.

From the figure 3 it is observed that maximum at mid span of the bolt. The bolt pretention is provided in the bolts to increase the contact stress in the flange retaining the tensile load parallel to the axis of the rotor. Due to centrifugal pull, each bolt is subjected to shear loading since each stage of the rotor tries to radially grow outward resulting in combination of bolt bending and shear. The first principal stress in bolt indicates the shear loading dominance in the bolt and the equivalent stress indicates the stress intensity due to bolt bending. The obtained stresses are well with the design limits therefore the design is safe under these conditions.

RESULT AND DISCUSSIONS

The above figure 4 shows the total deformation obtained for the cyclic symmetry analysis for the given Boundary condition. The radial growth is 0.534 mm which is safer under allowable growth.

From figure 5 it is evident that hoop stress (maximum at the disc bore) induced in the disc sector decrease as the length of disc increase from bore to rim. The radial stress (minimum at the bore and maximum in the web) increase in the bore region and decreases in web region as the length increases. Since, the hoop stress is concentrated at disc bore and plays a major role.
in disc failure. Hoop stress considered for evaluating the safety margins. The deformation induced in the turbine disc is less than ‘1 mm’ which is desired result.

EVALUATION OF OVER-SPEED AND BURST MARGINS

Over-speed is a situation where the aero rotor disc is forced to rotate after its design limit. The significances of running the disk speed differ from engine type, model etc. The duration of over-speed is main factor which speed of disc is dependent upon.

Generally, in aero engine discs, even a moment of over-speed results in reducing the engine life or leads to catastrophic failure. Speed at which disc undergoes catastrophic failure is known as burst speed.

The constraints for rotating speed of aero engine turbine disc given by the two international agencies is given below [EASA CS-E 840 AND FAR 33.27] [7]

a. 120% of maximum allowable rotor speeds related with any of ratings of excluding OEI ratings < 2 ½ minutes.

b. 115% of the maximum allowable rotor speeds associated with any OEI ratings < 2 ½ minutes.

c. 105% of the highest rotor speed that is outcome from the failure of part or system which is the most crucial with regard to over-speeding.

d. The burst speed is a highest speed that result from failure of any component or system in a respective installation of engine.

Gross-yielding approach at critical locations is followed for evaluating the safety margins.

Failure criteria for rotating disc

The determination of the burst-speed has brought to the formulation of several theories and criteria that prescribe the procedure to determine the rotational speed that causes failure. Among them, two are used in this work: The Robinson’s Criteria also called average hoop stress criterion; and Hallinan’s criteria.

Robinson’s Criteria [8]:

The Robinson criteria is a method which is developed for calculating the burst speed in the hoop mode knowing the ultimate tensile strength, $\sigma_{UTS}$ and the mean hoop stress, $\sigma_{c, mean}$. As per this criterion “burst occurs when the mean hoop stress on a disc section becomes equal to the nominal tensile strength of the material, determined from a uniaxial tensile stress”.

The mathematical formulation of Robinson criteria,

$$\omega_{burst} = \omega \sqrt{\frac{\sigma_{UTS}}{\sigma_{c, mean}}}$$  \hspace{1cm} Eqn.4

It is important to know that ultimate strength considered by Robinson Criteria is the engineering stress that differs from true stress. ANSYS applies engineering stress to perform the analysis since; the evolution of safety margins is to be performed by considering true stress. Due to this relation between these stresses is to be performed and utilized for evaluating the safety margins. When passing from engineering stress to true stress, the following relations must be applied:

$$true = \ln(1 + \varepsilon_{eng})$$  \hspace{1cm} Eqn.5

$$\sigma_{true} = \sigma_{eng}(1 + \varepsilon_{eng})$$  \hspace{1cm} Eqn.6

From Eqn. 4 and Eqn. 5 a relationship between true stress and engineering stress was obtained. By utilizing these relations, a graph was plotted between stress and strain which is represented in figure 6.

Figure 6: comparison between engineering stress and true stress [2]

Robinson criteria fail to explain the relation between the true stress and engineering stress. It also fails to explain the effect of maximum hoop stress in the disc failure. Hence this criterion is modified in several ways and one of the forms is Hallinan criteria.

Hallinan criteria:

This criteria extends the use of the Robinson criteria considering the maximum hoop stress, $\sigma_{c, max}$ introducing a factor $S$ [8] (it is the ratio of true stress to the engineering stress) to weight the influence of the maximum stress over the mean stress. The ratio $S$ is obtained from the graph figure 3 by comparing true and engineering stress.

The mathematical form of Hallinan criteria

$$\omega_{burst} = 0.95\omega \left[S \left( \frac{\sigma_{UTS}}{\sigma_{c, mean}} - \frac{\sigma_{UTS}}{\sigma_{c, max}} \right) + \frac{\sigma_{UTS}}{\sigma_{c, max}} \right]$$  \hspace{1cm} Eqn. 7

These two criterions are similar to each other if the value of $S$ is close to 1, such as for discs made of ductile materials.

Therefore, the equation becomes,

$$\omega_{burst} = 0.95\omega \frac{\sigma_{UTS}}{\sigma_{c, mean}}$$  \hspace{1cm} Eqn. 8

In general, the concept of calculating the average stress is associated with the ductility of material and no difference
occurs between the two formulations; while if the disc is made of brittle material it is more appropriate to weight the influence of the maximum stress by applying the Hallinan Formula.

1. Linear Static Analysis:
From the behavior constraints mentioned over-speed \( \geq 118\% \) of the 100% speed. Hence, for evaluating the safety margins 121% speed (16940 rpm) is considered as per API standards,

\[
\text{Over speed Margin} = \sqrt{\frac{\sigma_{\text{yield}}}{\sigma_{c,\text{mean}}}} \quad \text{Eqn.9}
\]

For burst speed,

\[
\omega_{\text{burst}} = \omega \sqrt{\frac{\sigma_{\text{UTS}}}{\sigma_{c,\text{mean}}}}
\]

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<td>1.29</td>
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<tr>
<td>Burst Speed</td>
<td>190.9%</td>
<td>158.7%</td>
<td>157.50%</td>
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Similarly, for other cases and methods of analysis similar approach is carried out for calculating the over-speed margin, the burst-speed.

Linearity is just an assumption which simplifies the modeling. Every problem is bi-linear, but solving these models time consumption is more when compare to the linear models. Hence, analysis of linear models is carried out whenever possible. However, the assumptions built into linearity must be considered with every model. It is assumed here that the ultimate stress is within the elastic limit and there is no yield i.e. the stiffness of the material remains constant in linear analysis which is not real. But, the burst takes place in the disc when the induced stress is equivalent to ultimate stress i.e. the linearity of the material is lost. Therefore, the assumptions made are not applicable for evaluating the burst speed. Hence, the burst speed obtained from this approach is in-valid.

2. Bi-linear Isotropic Analysis:
In isotropic hardening when the material is subjected to loading tension or compression the material deforms linearly till the yield limit. Once the material is crossed the yield limit the material doesn’t deforms uniformly. But in this case the deformation of the material takes place by maintaining its centre constant. This implies that stress while tension and compression remains same in value but opposite in signs, which is not possible in the real behaviour of the material. Hence the obtained burst speeds by this approach are also unacceptable.

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<td>186.14%</td>
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3. Bi-linear Kinematic Analysis:
The isotropic model indicates that, yield strength in tension and compression are same in the beginning, i.e. the yield surface is symmetric about the stress axes; they remain equal as the yield surface develops within plastic strain. In Isotropic hardening the material hardens till it responds elastically. To overcome this, alternate laws i.e. kinematic hardening laws is introduced. As per hardening laws, the material become softer in compression and thus yield surface remains same shape and size but turns in stress space, which gives the real behavior of the material. Hence, this approach gives the real behavior burst speed obtained are acceptable.

The evaluated safety margins is plotted in the graph with respect to the change in speed. For over-speed (121% as per API standards) condition the saftey margins obatined for the disc are in the range of 32-34% which is as shown in figure 7.

![Figure 7: OSM for different approaches with respect to change in speed](image1)

![Figure 8: Burst-margin for different approaches with respect to change in speed](image2)
The graph plotted figure 8 is by considering CASE 3 loading condition. The burst-speed obtained for the disc from different approaches is in the range of 152.73-149.7% of the operating speed which is as shown in figure 8.

**Comparison of Burst speed between Robinson and Hallinan criteria:**

Hallinan criteria = 0.95*Robinson criteria [by using Eqn. (6) & Eqn. (1)] [2]

Therefore,

Burst speed in Hallinan criteria = 0.95* 190.9% = 181.355%

Similarly, the above is used to calculate for all other cases.

**FATIGUE LIFE ASSESSMENT**

Fatigue is a critical failure phenomenon as it is the reason to more than 90% of all service failures of machine components. Therefore, fatigue life estimation is extremely important in extremely loaded components design. The turbo machines are the most critical parts of gas turbine engines. These parts have colossal kinetic energies that are at the peak during maximum thrust conditions. Such processes incite intense cyclic stresses in turbo machines. Hence, the absence of life prediction leads to low cycle fatigue failures. The expected life of small gas engine is much less than large engines. Hence certain degree of plastic deformation is permissible in this class of engines. Manson- Coffin’s method (Strain approach) [9] is the best suited life estimation method for the case under study. For such application the strain approach is better than the stress approach which basically ignores plasticity.

From Manson’s-Coffin’s Approach

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_p}{2} + \frac{\Delta \varepsilon_f}{2} = \frac{\sigma'_f}{2E} (2N_f)^b + \varepsilon'_f (2N_f)^c$$

$$N_f = 296319.69 \text{ cycles}$$

**CONCLUSION**

- The sensitivity analysis and design checks conducted through blending the classical equations and methodologies has thrown light on various aspects namely design considerations, parametric and design constraints along with behavior constraints to achieve design goal. A non-linear analysis is must for evaluating the safety limits as per regulations to ensure integrity.

- From the present study for evaluating the safety limits for disc a bi-linear kinematic approach is recommended. The safety limits obtained from this analysis provides a conservative result in a range of 6-8% when compared to other two approaches

- The burst limits obtained from Hallinan’s criteria is considered since, the results are 5-7% conservative when compare to Robinsons criteria. By incorporating this criterion in FE code would help in evaluating burst limits.

- The estimated fatigue life of the disc obtained from Coffin-Manson and Universal Slope method is greater than $1 \times 10^6$ cycles. For, estimating the fatigue life of disc Coffin-Manson method is optimum since, it provides a factor of safety 1.66 when compare to universal slope method. Universal Slope method is utilized for approximation of fatigue life when there is lack of material data.
REFERENCES


[3] K. Kumar, S.L Ajit Prasad and Shivarudhraiah “Strength evaluation in turbo machinery Blade disk assembly at constant speed”- IJAME 2010


