On three Dimensional Oscillating Flow of Magneto-micropolar Fluid Past an Inclined Plate with Radiation Absorption, Chemical Reaction and Heat Generation

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Abstract

The aim of the present paper is to investigate the effect of heat generation and thermal radiation on the three dimensional free convective heat and mass transfer flow of an incompressible, magnetohydrodynamic (MHD) micropolar fluid over an inclined oscillating porous plate in the presence of first-order chemical reaction. The governing dimensionless equations are solved analytically by using perturbation technique. The effects of the governing physical parameters like chemical reaction, thermal radiation, magnetic field parameter, vortex viscosity parameter, suction parameter on the velocity, temperature and concentration fields are discussed in terms of graphs and tables. The results reveal that, the concentration profile decreases as the value of chemical reaction parameter increases, while an opposite trend is observed for linear velocity profile. It is also observed that both the linear velocity and temperature profile is in increasing trend due to rise in the values of radiation absorption parameter. A comparative analysis ensures that the skin frictional effect is less in micropolar fluid than the corresponding Newtonian fluid.

Keywords: MHD Micropolar fluid, thermal radiation, radiation absorption, chemical reaction, heat generation.

INTRODUCTION

Micropolar fluids constitute an important branch of the micromorphic fluid theory. Micropolar fluids are fluids with microstructure belonging to a class of fluids non-symmetrical stress tensor. In micropolar fluids, rigid particles in a small volume element can rotate about the centroid of the volume element. The micro polar fluids include colloidal fluids ferro liquids, polymers with suspensions animal blood and synovial fluid which consists of a long chain of polymers that flows in liquids, polymers with suspensions animal blood and synovial element. The micro polar fluids include colloidal fluids ferro liquids, polymers with suspensions animal blood and synovial element. The micro polar fluids include colloidal fluids ferro liquids, polymers with suspensions animal blood and synovial element. The micro polar fluids include colloidal fluids ferro liquids, polymers with suspensions animal blood and synovial element. In the works of Eringen [7] and Lukaszewicz [14] many interesting aspects of micropolar fluid theory are found. Qasim et al. [19] studied the effect of heat transfer in a micropolar fluid over a stretching sheet with Newtonian heating. Sengupta and Deb [24] recently studied free convection heat and mass transfer flow of micropolar fluid in porous media with fluctuating wall temperature.

The study of magnetohydrodynamics (MHD) has important applications found in metrology, planetary magnetospheres, solar physics and in motion of the earth’s core. MHD flow may be used to find solution of problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field. Ganesan and Palani [8] applied finite difference technique on unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. Kim [12] studied heat and mass transfer effect in MHD micropolar flow over a vertical moving porous plate in a porous medium. Ahmed et al. [2] analyzed the MHD free convection mass transfer flow past an oscillating plate embedded in a porous medium with Soret effect. Sengupta [26] conducted modeling with applications on heat and mass transfer phenomena in MHD flow. Yadav et al. [31] made numerical analysis of MHD flow of viscous fluid between parallel porous bounding walls.

In recent period, the importance of radiative transfer in industrial and technological field have attracted the attention of many researchers in view of its applications in developments of hypersonic flights, missile re-entry rocket combustion chambers, gas cooled nuclear reactors and power plants for inter planetary flight. The significance of radiative heat transfer is found especially in electronics equipment and many processes in industry, which occur at very high temperatures. The effect of radiation on unsteady free convection flow bounded by an oscillating plate with variable wall temperature were examined by Pathak et al. [17]. Hayat and Qasim [10] considered the effect of thermal radiation on unsteady MHD flow of a micropolar fluid with heat and mass transfer. Oahimire and Olajuwon [15] studied the effects of radiation absorption and thermo-diffusion on MHD heat and mass transfer flow of a micro-polar fluid in the presence of a heat source. Sengupta [22] carried out study on free convective chemically absorption fluid Past an impulsively accelerated plate with thermal radiation variable wall temperature and concentrations. Gul et al [9] studied the effect of thermal radiation on heat transmission in the liquid film flow of micropolar fluid in a porous medium over a stretching sheet.

In many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid.
which moves due to stretching or otherwise of a surface. If the rate of chemical reaction is directly proportional to the concentration and it occurs as a single phase volume reaction the chemical reaction is then said to be of first order and homogenous. Afify [1] considered MHD free convection flow and mass transfer over a stretching sheet with chemical reaction. Chaudhury and Jha [5] studied the effects of chemical reactions on MHD micropolar fluid past a vertical plate in slip flow regime. Patil and Kulkarni [18] investigated the effects of chemical reaction flow of a polar fluid through porous medium in the presence of internal heat generation. Rahman and Al-Lawatia [20] made an analysis to obtain the effects of higher order chemical reaction on micropolar fluid flow past a permeable stretched sheet in a porous medium. Sharma et al. [28] examined the influence of chemical reaction and radiation on unsteady MHD free convection flow and mass transfer through viscous incompressible fluid of porous medium in the presence of heat source. Pal et al. [16] analyzed the effect of chemical reaction and Hall current on oscillatory mixed convection flow of a micropolar fluid in a rotating system. Sheri and Shamshuddin [29] conducted research on heat and mass transfer effect on the MHD flow of micropolar fluid in the presence of viscous dissipation and chemical reaction. Kiran Kumar et al. [13] examined the effects of diffusion-thermo and chemical reaction for heat and mass transfer phenomena in MHD micropolar fluid. Recently, Sharma et al. studied [27] heat source and Soret effects on MHD fluid flow with variable permeability and chemical reaction.

In recent years, radiative mass transfer flow problems with heat absorptions/generations are of great importance in many processes, and have received a considerable amount of attention from scientists and researchers. Chamkha et al. [4] studied unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate by taking the effect of heat absorption. Khedr et al. [11] studied MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption. Sudheer Babu and Satya Narayana [30] considered the case of radiation absorption effect on free convection flow through porous medium with variable suction in the presence of uniform magnetic field and chemical reaction. Sengupta and Sen [25] studied free convective heat and mass transfer flow past an oscillating plate with heat generation, thermal radiation and thermo-diffusion effects. Sengupta and Ahmed [23] investigated MHD free convection mass transfer flow of radiative uniform heat generation/absorption fluid in the presence of Soret and Dufor effects through a wavy permeable channel.

Some of the theoretical works on three dimensional oscillating flow of micropolar fluid is investigated by various authors in recent times. Bakar [3] considered the aforesaid flow without porous medium and in absence of thermal radiation, radiation absorption and first order chemical reaction. The absence of theses physical effects makes the study a bit incomplete as these effects are significant in many engineering and technological point of view. Though Satya Narayana et al. [21] under took theses effects, but the present authors firmly put the logic that, in micropolar theory, the Darci’s law should be replaced by the generalized Darci’s form, which includes both the viscosities and due to the presence of the Darcian porous medium, the strength of the applied magnetic field can’t be so strong to include the Hall’s effect in the flow. Following the significance of the study and inadequacy in the consideration, the present paper efforts to overcome from all of these and considered the study of a free convection oscillating flow of incompressible micropolar fluid past an inclined plate is investigated in the presence of uniform magnetic field. The physical effects like thermal radiation, radiation absorption, first-order chemical reactions and heat generation are also studied. It is considered that the plate velocity is oscillating with time about a constant non-zero value. The governing dimensionless equations are solved by using perturbation technique. This work may find its possible applications in the field of chemical and nuclear reactors.

MATHMATICAL FORMULATION OF THE PROBLEM:

A three dimensional, unsteady free convective flow of a micropolar fluid over an infinite, inclined moving porous plate in the presence of radiation absorption, a first-order chemical reactions and heat generation is considered for study. In this problem, the plate is considered to oscillate with velocity \( \ddot{u} = u_0 (1 + \varepsilon \cos n \omega t) \) in time \( t \) with frequency \( n_0 \) and with a constant heat source. It is considered that the flow is to be in the \( \vec{x} \) direction, which is taken along the plate in the upward direction. \( \vec{y} \)-axis is taken normal to the plate and \( \vec{z} \)-axis is taken along the width of the plate. We assume that the fluid as well as the plate is initially at rest but for time \( \tilde{t} \geq 0 \) the whole system is allowed to rotate with a constant rotating frame \( \tilde{\Omega} \) in a micropolar fluid about the \( \vec{y} \)-axis. A uniform magnetic field \( B_0 \) is to be acting along the \( \vec{y} \)-axis. We assume that the plate is infinite in extent and all physical quantities thus depend on \( \tilde{y} \) and \( \tilde{t} \) only.
(1) \[
\frac{\partial \bar{v}}{\partial y} = 0
\]
\[
\frac{\partial \bar{u}}{\partial t} + \frac{\partial \bar{v}}{\partial y} - 2\bar{\Omega} \bar{w} = (\nu + \bar{v}) \frac{\partial \bar{u}}{\partial y} - \bar{v} \frac{\partial \bar{u}}{\partial y} + g\beta_c (\bar{T} - \bar{T}_m) \cos \alpha + g\beta_c (\bar{C} - \bar{C}_m) \cos \alpha - \frac{1}{K} (\nu + \bar{v}) \frac{\partial \bar{B}}{\partial y} - \frac{\sigma B}{\rho} \frac{\partial \bar{B}}{\partial y} - \frac{1}{K} (\nu + \bar{v}) \frac{\partial \bar{w}}{\partial y} + \frac{\sigma B}{\rho} \frac{\partial \bar{w}}{\partial y} + 2\bar{\Omega} \bar{w} = (\nu + \bar{v}) \frac{\partial \bar{w}}{\partial y} + \bar{v} \frac{\partial \bar{w}}{\partial y} - \frac{1}{K} (\nu + \bar{v}) \frac{\partial \bar{w}}{\partial y} + \frac{\sigma B}{\rho} \frac{\partial \bar{w}}{\partial y}
\]
\[
\rho \frac{\partial (\frac{\partial \bar{n}}{\partial t} + \frac{\partial \bar{n}}{\partial y})}{\partial y} = \eta \frac{\partial^2 \bar{n}}{\partial y^2}
\]
\[
\rho \frac{\partial (\frac{\partial \bar{n}}{\partial t} + \frac{\partial \bar{n}}{\partial y})}{\partial y} = \eta \frac{\partial^2 \bar{n}}{\partial y^2}
\]
\[
\frac{\partial \bar{C}}{\partial t} + \frac{\partial \bar{C}}{\partial y} = D_u \frac{\partial^2 \bar{C}}{\partial y^2} - K_i (\bar{C} - \bar{C}_m)
\]

Set of boundary conditions defined as:
\[
\bar{u} = 0, \bar{v} = 0, \bar{w} = 0, \bar{C} = \bar{C}_m, \bar{C} = \bar{C}_m \text{ for every } \bar{y}
\]
when \( \bar{i} \leq 0 \),
\[
\bar{u} = u_i \left(1 + \frac{\bar{y}}{2} \left(\exp(i\bar{m}\bar{y}) + \exp(-i\bar{m}\bar{y})\right)\right), \bar{C} = \bar{C}_m
\]
\[
\bar{C} = \bar{C}_m \text{ for } \bar{y} = 0 \text{ when } \bar{i} > 0
\]
where \( \bar{x}, \bar{y}, \bar{z} \) are velocity components along \( \bar{x}, \bar{y}, \bar{z} \)-axis, respectively. \( \bar{n}_x \) and \( \bar{n}_y \) are the angular velocity components along \( \bar{x} \) and \( \bar{y} \)-axis, respectively. \( \beta_y \) and \( \beta_c \) are the coefficient of volume expansion and volume expansion with concentration, \( c_p \) is the specific heat at

**Figure 1:** Physical model and coordinate system

1. Concentration boundary layer
2. Thermal boundary layer
3. Velocity boundary layer
constant pressure, $\rho$ is the density of the fluid, $v$ is the kinematics viscosity, $\sigma$ is the electrical conductivity of the fluid, $K_i$ is the chemical reaction parameter, $Q_h$ is the additional heat source, $\alpha$ is the plate inclination parameter and $D_m$ is the molecular diffusivity.

We now consider an admissible solution of Eq. (1) as

$$v = -v_0$$

(9)

where the constant $v_0$ represents the normal velocity at the plate which is positive for suction and negative for blowing.

We introduce the following non-dimensional quantities as:

$$y = \frac{\bar{u}_y}{v}, \quad u(y,t) = \frac{\bar{u}(y,t)}{u_0}, \quad w(y,t) = \frac{\bar{w}(y,t)}{u_0}, \quad t = \frac{u_0^2 t}{v},$$

$$n_s = \frac{v_n}{u_0^2}, \quad n_w = \frac{v_w}{u_0^2}, \quad \phi = \frac{v}{u_0}, \quad \eta = \frac{v}{u_0}, \quad s_0 = \frac{v_0}{u_0},$$

$$K = \frac{K_0 u_0^2}{v^2}, \quad Gr = \frac{\sigma B^2 v}{\rho u_0^2}, \quad Grm = \frac{\sigma B^2 v}{\rho u_0^2}, \quad M^2 = \frac{\sigma B^2 v}{\rho u_0^2}, \quad K = \frac{K_0 u_0^2}{v^2}, \quad \xi = \frac{v}{v}, \quad s = \frac{v}{v}, \quad s_0 = \frac{v_0}{u_0},$$

$$Ra = \frac{Q_h (C_n - C_s)}{\rho C_p (T_m - T_m)}, \quad \Omega = \frac{Q_h (C_n - C_s)}{\rho C_p (T_m - T_m)}, \quad Pr = \frac{\nu}{\nu}, \quad Sc = \frac{v}{D_u}, \quad Sr = \frac{D_u D_q q_s^*}{q_s^{*^2} k_v}$$

(10)

Where $\Omega$ is the rotational parameter, $S_0$ is suction parameter, $U_i$ is a uniform reference velocity, $Gr$ is the Grashof number, $Sc$ is Schmidt number, $Pr$ is the Prandtl number, $M$ is the magnetic parameter and $Q_h$ is heat source parameter.

In view of above non-dimensional quantities, we obtain the following dimensionless differential equations from the equations (2)–(7) as:

$$\frac{\partial u}{\partial t} - \lambda_u \frac{\partial u}{\partial y} - 2 \Delta w = (1 + \xi) \frac{\partial v}{\partial y} - \frac{\partial n_s}{\partial y} + \frac{Gr \theta}{u_0} + Grm \phi - \frac{(1 + \xi)}{K} \frac{\partial u}{\partial y}$$

(11)

$$\frac{\partial w}{\partial t} - \lambda_w \frac{\partial w}{\partial y} + 2 \Delta w = (1 + \xi) \frac{\partial v}{\partial y} + \frac{\partial n_w}{\partial y} - M^2 w - \frac{(1 + \xi)}{K} w$$

(12)

$$\frac{\partial \theta}{\partial t} - \lambda_\theta \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial \theta}{\partial y} + \frac{1}{Pr} Q_h \theta + Ra \phi$$

(13)

$$\frac{\partial u}{\partial t} - \lambda_u \frac{\partial u}{\partial y} - 2 \Delta w = \frac{\partial v}{\partial y} - \frac{\partial n_s}{\partial y} + \frac{Gr \theta}{u_0} + Grm \phi$$

(14)

$$\frac{\partial \theta}{\partial t} - \lambda_\theta \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial \theta}{\partial y} + \frac{1}{Pr} Q_h \theta + Ra \phi$$

(15)

Subject to the non-dimensional boundary conditions as:

$$u(0,y,t) = 1, \quad v(0,y,t) = 0, \quad \theta(y,t) = 1, \quad \phi(y,t) = 1$$

(16)

(17.1) for every $y$ and $t \geq 0$.

$$u(y,t) = 1, \quad v(y,t) = 0, \quad \theta(y,t) = 0, \quad \phi(y,t) = 0$$

(17.2)

Thus, we have

$$u(0,y,t) = 1, \quad v(0,y,t) = 0, \quad \theta(y,t) = 0, \quad \phi(y,t) = 0$$

(17.3)

To obtain desired solutions, we now simplify equations (11) - (16) by putting the fluid velocity and angular velocity in the complex form as:

$$U(y,t) = u(y,t) + iw(y,t), \quad \phi(y,t) = \lambda \phi(y,t) + \frac{1}{Pr} Q_h \theta + Ra \phi$$

(18)

(19)

(20)

(21)

With refined boundary conditions as

$$U(0,y,t) = U_i(y), \quad \phi(0,y,t) = 0$$

(22.1)

$$u(y,t) = 1, \quad v(y,t) = 0, \quad \theta(y,t) = 0, \quad \phi(y,t) = 0$$

(22.2)

Now,

$$N(y,t) = N_i(y), \quad \phi(y,t) = 0$$

(24)

(25)
\[ \theta(y,t) = \theta_0(y) + \frac{\varepsilon}{2} e^{\alpha_0 y} \theta_0(y) + e^{\alpha_1 y} \phi_0(y), \]  
(26)

\[ \phi(y,t) = \phi_0(y) + \frac{\varepsilon}{2} e^{\alpha_0 y} \phi_0(y) + e^{\alpha_1 y} \phi_0(y), \]  
(27)

Then substituting from (24)–(27) in the equations (18)–(21), equating the coefficients of the harmonic and an harmonic terms, neglecting the terms of \( e^{-y^2} \), we get the following set of equations:

\[ (1 + z^2) \frac{d^2 U_q}{dy^2} + S_0 \frac{d U_q}{dy} - (M_1 + 2\Omega) U_q + Gr \theta \cos \alpha \]

\[ + Gm \cos \alpha + i \varepsilon \frac{d N_1}{dy} = 0 \]  
(28)

\[ (1 + z^2) \frac{d^2 U_i}{dy^2} + S_0 \frac{d U_i}{dy} - (M_1 + i \Omega + i n_b) U_i + Gr \theta \]

\[ + Gm \cos \alpha + i \varepsilon \frac{d N_1}{dy} = 0 \]  
(29)

\[ \frac{1}{s} \frac{d^2 N_0}{dy^2} + S_0 \frac{d N_0}{dy} = 0 \]  
(30)

\[ \frac{1}{s} \frac{d N_1}{dy} = 0 \]  
(31)

The corresponding boundary conditions can be written as

\[ U_q = 1, \theta_0 = 1, \phi_0 = 1, N_1 = \frac{1}{2} \frac{d U_q}{dy} \quad \text{at} \quad y = 0 \]  
(39.1)

\[ U_q = 0, \theta_0 = 0, \phi_0 = 0, N_1 = 0 \quad \text{at} \quad y \rightarrow \infty \]  
(39.2)

\[ U_i = 1, \theta_0 = 0, \phi_0 = 0, N_1 = \frac{i}{2} \frac{d U_i}{dy} \quad \text{at} \quad y = 0 \]  
(39.3)

Solving equations (28)–(38) under the boundary conditions equations (39.1)–(39.3) and substituting the solutions into equations (24)–(27), we obtain

\[ U_q = \exp(-\alpha_1 y) \exp(-\alpha_3 y) (a_0 \cos \alpha y + a_2 \sin \beta y) \]

\[ + \exp(-\alpha_1 y) (a_1 \cos \alpha y + a_2 \sin \beta y) + \frac{\varepsilon}{2} \exp(-\alpha_1 y) \cos(n_b \beta y) \]

\[ + \frac{\varepsilon}{2} \exp(-\alpha_3 y) \cos(n_f \beta y) - \exp(-\alpha_3 y) a_0 \sin(n_f \beta y) \]

\[ + \frac{\varepsilon}{2} \exp(-\alpha_1 y) a_0 \cos(n_f \beta y) \]

\[ \theta_R = -\exp(-\alpha_3 y) (a_0 \cos \beta_3 y + a_10 \sin \beta_3 y) \]

\[ - \exp(-\alpha_3 y) (a_0 \cos \beta_3 y - a_10 \sin \beta_3 y) \]

\[ \phi_R = \exp(-\alpha_3 y) \cos \beta_3 y \]

\[ N_R = c_1 \exp(-\gamma_0 y) + \frac{\varepsilon}{2} \exp(-\alpha_3 y) [c_1 \cos(\beta y - n_f \beta) + c_2 \sin(\beta y - n_f \beta)] \]

\[ + \frac{\varepsilon}{2} \exp(-\alpha_1 y) [c_0 \cos(n_f \beta y) - c_1 \sin(n_f \beta y) - c_2 \sin(n_f \beta y)] \]

Some quantities of engineering interest are discussed as follows:

(i) Nusselt number:

\[ Nu_y = - \frac{1}{Pr} \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = - \frac{1}{Pr} (-\alpha_3 a_0 - \alpha_10 \beta_3 + \alpha\alpha_{a_0} + \alpha_{a_0} \beta_3) \]

(ii) Sherwood number:

\[ Sh_y = - \frac{1}{Sc} \left( \frac{\partial \phi}{\partial y} \right)_{y=0} = \frac{\alpha_3}{Sc} \]
(iii) Skin friction coefficient:-

\[ C_{f} = 2 \left( 1 + \sqrt{\frac{\nu}{
u_{*}}} \right) \frac{2U_{*}}{\delta} \]

\[ = 2 \left( 1 + \sqrt{\frac{\nu}{
u_{*}}} \right) \left( -a_{1} + a_{2} \beta_{1} + a_{3} \beta_{2} + a_{4} \beta_{3} + a_{5} \beta_{4} + a_{6} \beta_{5} - a_{7} \beta_{6} \right) \]

\[ - \left( 1 + \sqrt{\frac{\nu}{
u_{*}}} \right) \left( \sin \beta_{1} \left( a_{7} - a_{8} \beta_{1} + a_{9} \beta_{2} + a_{10} \beta_{3} + a_{11} \beta_{4} + a_{12} \beta_{5} \right) \right) \]

\[ - \left( 1 + \sqrt{\frac{\nu}{
u_{*}}} \right) \left( \sin \beta_{2} \left( a_{7} + a_{8} \beta_{1} - a_{9} \beta_{2} + a_{10} \beta_{3} + a_{11} \beta_{4} + a_{12} \beta_{5} \right) \right) \]

(iv) Couple stress coefficient:-

\[ C_{m_{a}} = 1 + \sqrt{\frac{\nu}{
u_{*}}} N_{*} (0) \]

\[ = \left( s_{*} \right) c_{1} + \frac{\nu}{2} \left( -a_{1} c_{1} \cos n_{t} + a_{2} c_{1} \cos n_{t} - a_{3} c_{1} - a_{4} c_{1} \cos n_{t} \right) \]

\[ + \frac{\nu}{2} \left( -a_{5} c_{1} \sin n_{t} + a_{6} c_{1} \cos n_{t} + a_{7} c_{1} \cos n_{t} + a_{8} c_{1} \cos n_{t} \right) \]

**RESULTS AND DISCUSSION**

A Numerical computations for the problem of unsteady three dimensional oscillating flow of magneto-micropolar fluid past an inclined plate in the presence of radiation absorption, chemical reaction and heat generation effect have been performed. The solutions are obtained by using perturbation technique for different values of governed parameters and the results are illustrated through graphs and in tables. The numerical calculations are presented in the form of non-dimensional temperature velocity and concentration profiles. Discussions are also made on the skin friction and couple stress coefficient as well as the rate of heat and mass transfers. In the present problem, Prandtl number \((Pr)\) is taken as 7.0 at 25°C or 298K which resembles physically water \((H_{2}O)\) and Schmidt number is taken as 0.60 which represents water vapour at 20°C. We have also chosen the value of \(n_{0} = 10, n_{t} = 1.57143\) and \(\alpha = 0.78575\) (until and unless specified) for the study. The values of solutal Grashof numbers taken are both physically from the free convection point of view. The values of other physical parameters are taken as arbitrarily.

The values of other physical parameters such as Prandtl number \((Pr)\), radiation parameter \((\tau)\), non-dimensional temperature profiles \((\theta_{0}, \tau)\) are depicted graphically in figures 2 and 3 respectively. It is observed that the fluid temperature decreases due to increase in values of \(Pr\), while a reverse effect is found in presence of \(R\). Due to increase in values of \(Pr\) the thickness of the thermal boundary layer decreases for which the temperature near the plate surface decreases while an increase in values of \(R\) accelerates the temperature wave within the thermal boundary layer, thereby increases the temperature near the plate surface. In figure 4, the parametric influence of radiation parameter \((R)\) on the linear as well as angular velocity is presented by graphically. Due to rise in values of \(R\), the thermal diffusivity of the medium increases thereby helps in raising the thermal buoyancy force. This is why, the linear fluid velocity is seen to increase.

The parametric influence of chemical reaction parameter \((K_{r})\) on the non-dimensional concentration profiles \((\phi_{0}, \gamma)\) and linear velocity profiles \((U_{0}, \gamma)\) are depicted graphically in figures 5 and 6 respectively. The rise in values of \(K_{r}\) is shown to decrease the concentration of the fluid particles near the plate while a reverse effect is observed on linear velocity. Due to increasing chemical reaction parameter, the constituents from higher concentration zone (plate region) transmit towards the lower concentration zone (free stream region), thereby reducing the concentration gradient within the boundary layer. This minimizes the concentration near the plate region. On account of reduction in concentration level near the plate surface, a deceleration in the flow rate observed due to reduction in solutal buoyancy effects, which is declining the axial velocity.

Figures 7 to 10 represent the parametric effect of thermal Grashof number \((Gr)\) and the solutal Grashof number \((Gm)\) on the linear velocity profile \((U_{0}, \gamma)\) and angular velocity profile \((N_{*}, \gamma)\) respectively. The increasing value of \(Gr\) and \(Gm\) increases the value of \(U_{0}\) while a reverse effect is observed in case of \(N_{*}\). The increase in values of \(Gr\) and \(Gm\) respectively increases the thermal buoyancy as well as mass buoyancy forces while producing a resistive effect by generating extra stress to rotational flow.

Figures 11 and 12 demonstrate the effect of vortex viscosity parameter \((\xi)\) on the linear velocity profile \((U_{0}, \gamma)\) as well as on the angular velocity profile \((N_{*}, \gamma)\) respectively. From these figures it is found that an increase in values of viscosity parameter results in increasing the linear velocity while the rotational velocity decreases. The rise in vortex viscosity parameter, physically decreases the rotational motion and thereby complimented to increase the linear velocity.

The parametric influence of magnetic parameter on the linear velocity profile \((U_{0}, \gamma)\) and on \((N_{*}, \gamma)\) are graphically reflected in figures 13 and 14 respectively. Due to increase in magnetic field parameter \((\gamma)\), a resistive force in terms of Lorentz force generates in the flow which decelerates the linear flow rate while accelerates the rotational flow rate. This is why, the values of \(U_{0}\) decrease while the values of \(N_{*}\) increase.

The influence of plate inclination parameter \((\alpha)\) on the linear velocity profile \((U_{0}, \gamma)\) as well as on the angular velocity \((N_{*}, \gamma)\) is shown in figures 15 and 16 respectively. Due to rise in values of the inclination of magnetic field parameter, the resistive effect of the magnetic field on the main flow velocity ceases. This is why, the values of \(U_{0}\) increases gradually while that of \(N_{*}\) decreases. As the radiation absorption parameter emitted by the surrounding as such it is found to increase the temperature fluxes within the medium which results in increase the temperature near the plate region. Due to increasing temperature, the kinetic energy of the constituents rises which thus increases the flow rate and ultimately the values of linear velocity. In Table 1, a comparative study of the theory of micropolar fluid with that of Newtonian fluid has been made by taking into account the parametric influence of suction parameter, thermal radiation
CONCLUSION:-

In the present paper, the effect of heat generation and thermal radiation on the three dimensional free convective heat and mass transfer flow of an incompressible, magnetohydrodynamic micropolar fluid over a vertical oscillating porous plate in the presence of first order chemical reaction is theoretically studied. The governing non-linear partial differential equations are transformed into a system of non-linear ordinary differential equations and then solved numerically using the Perturbation method. The outcome of the study can be be summarized as follows:

The temperature profile decreases due to rise in the values of the Prandtl numbers while for an increase in the values of thermal radiation parameter, temperature increases. The linear flow rate is found increasing for an increase in values of the Prandtl number. The concentration of fluid particles is found deceasing for an increase in values of chemical reaction parameter while a reverse trend is observed in case of the linear fluid velocity. The investigation reveals that the linear velocity increases as the magnetic field inclination parameter as well as the thermal and solutal Grashof numbers increase, while the angular velocity of the fluid particles shows just an opposite results for the parametric increase of the said non-dimensional parameters. Again as expected an increase in values of vortex viscosity parameter and magnetic field parameter increases the angular velocity but decreases the linear velocity of the fluid particles. The presence of parameters like thermal radiation parameter, suction parameter, chemical reaction parameter and rotational parameter is found to regulate the skin friction coefficient. The skin friction coefficient, by using micropolar fluids is found less compare to Newtonian fluids. It is interesting to observe that the skin friction coefficient can be regulated by controlling the values of $s_0$, $K_l$ and $\alpha$ which is clearly observed in Table 3. The numerical data relating to the variation in values of Nusselt number due to arbitrary change in values of $s_0$, $Ra$, $K_l$ and $Q_s$ is presented. Also, in table 4, the data related to Sherwood number is numerically obtained for arbitrary change in values of $s_0$ and $K_l$. It is interesting to observe that Nusselt number as well as Sherwood number are found to be influenced by $s_0$ and $K_l$.

Figure 1: Graph of temperature $\theta$ versus normal distance $y$ for fixed values of $t=0.1$, $Ra=1.0$, $Q_s=5.0$, $Sc=0.60$, $s_0=0.3$, $K_l=0.50$, $Pr=7.0$, $\varepsilon = 0.001$

Figure 2: Graph of temperature $\theta$ versus normal distance $y$ for fixed values of $t=0.1$, $Ra=1.0$, $Q_s=5.0$, $Sc=0.60$, $s_0=0.3$, $K_l=0.50$, $Pr=7.0$, $\varepsilon = 0.02$

Figure 3: Graph of velocity $U_R$ versus normal distance $y$ for fixed values of $K=2.0$, $\xi = 1.0$, $Gr=10.0$, $Gm=4.0$, $Pr=7.0$, $\varepsilon = 0.001$, $Sc=0.60$, $M=0.6$, $\alpha = 0.7857$, $Ra=1.0$, $Q_s=5.0$, $s_0=0.3$, $K_l=0.2$, $\Omega = 0.5$
Figure 5: Graph of concentration $\phi$ versus normal distance $y$ for fixed values of $Sc=0.60$, $s_0=0.5$.

Figure 6: Graph of velocity $U_R$ versus normal distance $y$ for fixed values of $K=2.0$, $\xi=1.0$, $Gr=10.0$, $Gm=4.0$, $Pr=7.0$, $Sr=1.5$, $\varepsilon=0.01$, $Sc=0.60$, $M=0.5$, $\alpha=0.7857$, $Ra=0.8$, $Q_s=5.0$, $K_l=0.2$, $s_0=0.2$, $Pr=7.0$, $R=3.0$, $\Omega=0.2$.

Figure 7: Graph of velocity $U_R$ versus normal distance $y$ for fixed values of $K=2.0$, $\xi=1.0$, $Gm=4.0$, $Pr=7.0$, $Sr=1.5$, $\varepsilon=0.01$, $Sc=0.60$, $M=0.5$, $\alpha=0.7857$, $Ra=0.8$, $Q_s=5.0$, $K_l=0.2$, $s_0=0.2$, $R=3.0$, $\Omega=0.5$.

Figure 8: Graph of angular velocity $\bar{N}_R$ versus normal distance $y$ for fixed values of $K=0.5$, $\xi=0.5$, $Gm=4.0$, $Pr=7.0$, $\varepsilon=0.001$, $Sc=0.60$, $M=0.5$, $\alpha=0.7857$, $K_l=0.5$, $s_0=1.0$, $Pr=7.0$, $\Omega=0.2$.

Figure 9: Graph of velocity $U_R$ versus normal distance $y$ for fixed values of $K=0.5$, $\xi=0.5$, $Gr=10.0$, $Pr=7.0$, $Sr=1.5$, $\varepsilon=0.001$, $M=0.5$, $\alpha=0.7857$, $Ra=0.6$, $Q_s=5.0$, $Sc=0.60$, $K_l=0.5$, $s_0=1.0$, $R=3.0$, $\Omega=0.5$.

Figure 10: Graph of angular velocity $\bar{N}_R$ versus normal distance $y$ for fixed values of $K=1.0$, $\xi=0.5$, $Gr=10.0$, $Pr=7.0$, $\varepsilon=0.001$, $Sc=0.60$, $M=0.5$, $\alpha=0.7857$, $Sc=0.60$, $K_l=0.5$, $s_0=1.0$, $Pr=7.0$, $\Omega=0.2$. 
Figure 11: Graph of velocity $U_n$ versus normal distance $y$ for fixed values of $Gr=10.0, Gm=8.0, Pr=7.0, Sr=1.5, n=0.001, M=0.5, \alpha=0.7857, K=0.1, Ra=0.6, Q_r=5.0, Sc=0.60, K_1=2.0, s_0=0.5, Pr=7.0, R=1.0, \Omega=0.5$.

Figure 12: Graph of angular velocity $N_n$ versus normal distance $y$ for fixed values of $Gr=10.0, Gm=8.0, \varepsilon=0.02, Sc=0.60, M=0.5, \alpha=0.7857, Sc=0.60, K_1=2.0, s_0=0.5, Pr=7.0, \Omega=0.5$.

Figure 13: Graph of velocity $U_n$ versus normal distance $y$ for fixed values of $Gr=10.0, Gm=8.0, Pr=7.0, \varepsilon=0.001, Sc=0.60, M=0.5, \alpha=0.7857, K=0.1, Ra=0.6, Q_r=5.0, Sc=0.60, K_1=2.0, s_0=2.0, Pr=7.0, R=1.0, \Omega=0.5$.

Figure 14: Graph of angular velocity $N_n$ versus normal distance $y$ for fixed values of $Gr=10.0, Gm=8.0, Pr=7.0, \varepsilon=0.02, Sc=0.60, M=0.5, \alpha=0.7857, Sc=0.60, K_1=2.0, s_0=0.5, Pr=7.0, \Omega=0.5$.

Figure 15: Graph of velocity $U_n$ versus normal distance $y$ for fixed values of $Gr=10.0, Gm=4.0, Pr=7.0, \varepsilon=0.002, Sc=0.60, M=0.5, \alpha=0.7857, K=2.0, Ra=0.5, Q_r=5.0, Sc=0.60, K_1=2.0, s_0=1.0, Pr=7.0, n_0=1.57143, R=1.0, \xi=0.8, \Omega=0.5$.

Figure 16: Graph of angular velocity $N_n$ versus normal distance $y$ for fixed values of $Gr=10.0, Gm=8.0, Pr=7.0, \varepsilon=0.002, Sc=0.60, M=0.5, K=2.0, n_0=10.0, \alpha=0.7857, \xi=1.8, Sc=0.60, K_1=0.6, s_0=0.5, Pr=7.0, n_0=1.57143, \Omega=0.5$. 

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Table 2:- Numerical values of skin friction coefficient at the plate for various values of physical parameters when \( t=0.2, \varepsilon = 0.01, K=0.5, Q=5.0, R=0.5 \).

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Table 3:- Numerical values of couple stress coefficient at the plate for various values of physical parameters when \( t=0.2, \varepsilon = 0.01, K=0.5, Q=5.0, K=0.5 \).

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Table 1:- Numerical values of skin friction coefficient at the plate for various values of physical parameters when \( t=0.2, \varepsilon = 0.01, \alpha = 0.7857, S_0=0.5, Gr=5.0, Gm=5.0, R=0.5, K=0.5, M=10.0 \).

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Table 4: Numerical values of Nusselt number for various values of physical parameters when \( t=0.2 \), \( \varepsilon = 0.01 \), \( Pr=7.0, Sc=0.60, \varepsilon=0.01, R=1.0 \).

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<td>1.0</td>
<td>6.0</td>
<td>0.3737</td>
</tr>
</tbody>
</table>

Table 5: Numerical values of Sherwood number for various values of physical parameters for \( t=0.2 \), \( \varepsilon = 0.01 \), \( K=0.5, K=0.5, Q=5.0 \).

<table>
<thead>
<tr>
<th>S0</th>
<th>K1</th>
<th>ShR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.0</td>
<td>1.6296</td>
</tr>
<tr>
<td>0.6</td>
<td>1.0</td>
<td>1.4961</td>
</tr>
<tr>
<td>0.7</td>
<td>1.0</td>
<td>1.3952</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.3330</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>1.4013</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
<td>1.4642</td>
</tr>
</tbody>
</table>

APPENDIX:

\[
x_1 = \frac{1}{\sqrt{2}} \left( (s_0)^2 + \sqrt{(s_0)^2 - 16n^2s^2} \right)^{1/2}, \quad a_1 = \frac{x_1 + s_0s}{2},
\]

\[
\beta_1 = \frac{1}{\sqrt{2}} \left| - (s_0)^2 + \sqrt{(s_0)^2 - 16n^2s^2} \right|^{1/2},
\]

\[
x_3 = \frac{1}{\sqrt{2}} \left[ \left( (s_0)^2 + 4ScK \right) + \sqrt{(s_0)^2 + 4ScK} \right]^{1/2}, \quad a_3 = \frac{x_3 + s_0Sc}{2},
\]

\[
\beta_3 = \frac{1}{\sqrt{2}} \left[ - (s_0)^2 + ScK \right] + \sqrt{(s_0)^2 + ScK} \right]^{1/2}, \quad a_3 = \frac{(1 + \xi)}{K} + M - M_4, \quad a_3 = (1 + \xi) (\alpha_3^2 - \beta_3^2) - s_0 \alpha_3 - M_4,
\]

\[
a_2 = 2 (1 + \xi) (\alpha_3 \beta_3) - s_0 \beta_3 - 2 \Omega, \quad a_3 = (1 + \xi) (\alpha_3^2 - \beta_3^2) - s_0 \alpha_3 - M_4,
\]

\[
a_4 = 2 (1 + \xi) (\alpha_3 \beta_3) - s_0 \beta_3 - 2 \Omega, \quad a_4 = (1 + \xi) (\alpha_4^2 - \beta_4^2) - s_0 \alpha_4 - M_4,
\]

\[
a_5 = 2 (1 + \xi) (\alpha_4 \beta_4) - s_0 \beta_4 - n_0 - \Omega, \quad a_5 = -Ra \left( \alpha_5^2 - \beta_5^2 \right) + s_0 \Pr \alpha_5 R - QS Ra,
\]

\[
a_{10} = 2 Ra \alpha_5 \beta_5 - s_0 \Pr \beta_5 R a,
\]

\[
a_{10p} = 2 (1 + \xi) (\alpha_5^2 - \beta_5^2) - s_0 \alpha_5 - M_4,
\]

\[
a_{10p} = 2 (1 + \xi) \alpha_5 \beta_5 + s_0 \beta_5 - (n_0 + \Omega),
\]

\[
a_1 = \frac{Gr \cos \alpha (a_{10} - a_{100})}{a_1^2 + a_2^2},
\]

\[
a_2 = \frac{Gr \cos \alpha (a_{10} + a_{100})}{a_1^2 + a_2^2},
\]

\[
a_3 = \frac{-Gr \cos \alpha (a_{10} - a_{100})}{a_1^2 + a_2^2},
\]

\[
a_4 = \frac{-Gr \cos \alpha (a_{10} + a_{100})}{a_1^2 + a_2^2},
\]

\[
c_1 = Gr \cos \alpha + \cos \alpha \sin \alpha + M_1, \quad c_2 = 2 \Omega,
\]

\[
c_3 = -(2 \Omega) \beta_5 + 2 \xi \alpha_5, \quad c_4 = -(2 \Omega) \beta_5 - 2 \xi \alpha_5, \quad c_5 = c_3 \c_4 + c_{n0} (n + \Omega), \quad c_6 = c_3 \c_4 - c_{n0} (n + \Omega),
\]

\[
a_{15} = \frac{\xi (a_5 \alpha_5 - c_3 \alpha_5)}{a_1^2 + a_1^2}, \quad a_{16} = \frac{\xi (a_5 \alpha_5 + c_3 \alpha_5)}{a_1^2 + a_1^2}, \quad a_{17} = \frac{\xi (a_5 \alpha_5 - c_3 \alpha_5)}{a_1^2 + a_1^2}, \quad a_{18} = \frac{\xi (a_5 \alpha_5 + c_3 \alpha_5)}{a_1^2 + a_1^2}, \quad a_{19} = \frac{\xi (a_5 \alpha_5 - c_3 \alpha_5)}{a_1^2 + a_1^2}, \quad a_{20} = \frac{\xi (a_5 \alpha_5 + c_3 \alpha_5)}{a_1^2 + a_1^2},
\]

\[
x_5 = \frac{1}{\sqrt{2}} \left[ \left( (s_0)^2 + 4 \lambda Q \right)^{1/2} + \sqrt{(s_0)^2 - 4 \lambda Q} \right]^{1/2}, \quad a_5 = \frac{x_5 + s_0}{2},
\]

\[
\beta_5 = \frac{1}{\sqrt{2}} \left[ - (s_0)^2 + \sqrt{(s_0)^2 - 4 \lambda Q} \right]^{1/2},
\]

\[
x_5 = \frac{1}{\sqrt{2}} \left[ \left( (s_0)^2 + 4M \left( 1 + \xi \right)^2 \right) + \sqrt{(s_0)^2 + 4M \left( 1 + \xi \right)^2} \right]^{1/2}, \quad a_5 = \frac{x_5 + s_0}{2},
\]

\[
\beta_5 = \frac{1}{\sqrt{2}} \left[ - (s_0)^2 + \sqrt{(s_0)^2 - 4 \lambda Q} \right]^{1/2},
\]

\[
x_5 = \frac{1}{\sqrt{2}} \left[ \left( (s_0)^2 + 4M \left( 1 + \xi \right)^2 \right) + \sqrt{(s_0)^2 + 4M \left( 1 + \xi \right)^2} \right]^{1/2}, \quad a_5 = \frac{x_5 + s_0}{2},
\]

\[
\beta_5 = \frac{1}{\sqrt{2}} \left[ - (s_0)^2 + \sqrt{(s_0)^2 - 4 \lambda Q} \right]^{1/2},
\]

\[
x_5 = \frac{1}{\sqrt{2}} \left[ \left( (s_0)^2 + 4M \left( 1 + \xi \right)^2 \right) + \sqrt{(s_0)^2 + 4M \left( 1 + \xi \right)^2} \right]^{1/2}, \quad a_5 = \frac{x_5 + s_0}{2},
\]

\[
\beta_5 = \frac{1}{\sqrt{2}} \left[ - (s_0)^2 + \sqrt{(s_0)^2 - 4 \lambda Q} \right]^{1/2},
\]

\[
x_5 = \frac{1}{\sqrt{2}} \left[ \left( (s_0)^2 + 4M \left( 1 + \xi \right)^2 \right) + \sqrt{(s_0)^2 + 4M \left( 1 + \xi \right)^2} \right]^{1/2}, \quad a_5 = \frac{x_5 + s_0}{2},
\]
REFERENCES:


