On Edge Degree Properties of Middle, Subdivision and Total Bipolar Fuzzy Graphs

Dr. M. Vijaya¹ and V. Mekala²

¹Research Advisor², Research Scholar²
P.G. and Research Department of Mathematics,
Marudupandiyar College, Thanjavur, India.

Abstract
In this paper, we introduce the concepts of Edge Degree of Bipolar Middle fuzzy graph, Bipolar Subdivision fuzzy graph, Bipolar Total Fuzzy Graphs and present some of the properties.

INTRODUCTION
Fuzzy graph theory was introduced by Azriel Rosenfield in 1975 [1]. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and bang have also introduced various connectedness concepts in fuzzy graphs [2]. A Nagoorgani and J. Malarvizhi discussed the concept of subdivision, middle and total fuzzy graphs and its properties [3]. K. Radha and N. Kumaravel (2014) introduced the concept of edge regular fuzzy graphs [4]. In 1994, Zhang [5, 6] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1, 1]. In this paper we study the degree of an edge in subdivision, middle and total bipolar fuzzy graphs and their properties.

BASIC CONCEPTS

Definition: 2.1
A BFG, we mean a pair \( G = (V, E) \) where \( V = (\sigma_B^p, \sigma_B^N) \) is a bipolar fuzzy set and \( E = (\mu_B^p, \mu_B^N) \) is bipolar relation on \( V \) such that
\[
\mu_B^p(ab) \leq \min(\sigma_B^p(a), \sigma_B^p(b))
\]
and
\[
\mu_B^N(ab) \geq \max(\sigma_B^N(a), \sigma_B^N(b))
\]
for all \((a, b) \in E\). Then \( V \) is the vertex set of bipolar fuzzy graph and \( E \) is the edge set of bipolar fuzzy graph.

Definition: 2.2
Let \( G : (\sigma_B, \mu_B) \) be a bipolar fuzzy graph \( G(V, E) \). The positive degree of a vertex \( x \in G \) is defined as \( d_G^p(x) = \sum \mu_B(x, y) \) for \( xy \in E \). The negative degree for a vertex \( x \in G \) is defined as \( d_G^N(x) = \sum \mu_B^N(x, y) \) for \( xy \in E \) and \( \mu_B^N(xy) = \mu_B^N(xy) = 0 \) if \( xy \) not in \( E \). The degree of a vertex \( x \) in a bipolar fuzzy graph is defined as \( d_G(x) = (d_G^p(x), d_G^N(x)) \). The minimum degree of \( G \) is \( \delta_V(G) = \land\{d_G(x) : x \in V\} \). The maximum degree of \( G \) is \( \Delta_V(G) = \lor\{d_G(x) : x \in V\} \).

Definition: 2.3
Let \( G : (\sigma_B, \mu_B) \) be a bipolar fuzzy graph \( G^*(V, E) \). The positive degree of an edge is defined as \( d_G^p(xy) = d_G^p(x) + d_G^p(y) - 2\mu_B^p(xy) \) and the negative degree of an edge is defined as \( d_G^N(xy) = d_G^N(x) + d_G^N(y) - 2\mu_B^N(xy) \). The degree of an edge in bipolar fuzzy graph is defined as
\[ \delta_e(G) = \bigwedge \{d_G(xy) : xy \in E \} \]

The maximum degree of an edge is
\[ \Delta_E(G) = \bigvee \{d_G(xy) : xy \in E \} \]

Definition: 2.4
If \( O(G) = \left( \sum_{x \in V} \sigma^p_B(x), \sum_{x \in V} \sigma^N_B(x) \right) \) is called a Order of a bipolar fuzzy graph.

Definition: 2.5
If \( S(G) = \left( \sum_{xy \in E} \mu^p_B(xy), \sum_{xy \in E} \mu^N_B(xy) \right) \) is called a size of a bipolar fuzzy graph.

Definition: 2.6
A bipolar fuzzy graph with the underlying crisp graph \( G^* : (\sigma^*, \mu^*) \) is defined to be a pair \( G : (\sigma, \mu) \) where \( \sigma = (m^p_\sigma, m^N_\sigma) \), \( \mu = (m^p_\mu, m^N_\mu) \). Let \( G^* \) be \( (V, E) \). The nodes and edges of \( G^* \) are taken together as node set, of the pair \( Bsd(G) : (\sigma_{Bsd}, \mu_{Bsd}) \), where \( \sigma_{Bsd} = (m^p_{\sigma_{Bsd}}, m^N_{\sigma_{Bsd}}) \), \( \mu_{Bsd} = (m^p_{\mu_{Bsd}}, m^N_{\mu_{Bsd}}) \). In \( Bsd(G) \) each edge \('e'\) in \( G \) is replaced by a new vertex and that vertex is made as a neighbour of those vertices which lie on \('e'\) in \( G \). Hence \( \sigma_{Bsd} \) is a bipolar fuzzy subset defined on \( V \cup E \) as
\[
\sigma^p_{Bsd}(x) = \sigma^p(x) \quad \text{if} \quad x \in V
\]
\[
= \mu^p(e) \quad \text{if} \quad e \in V
\]
\[
\sigma^N_{Bsd}(x) = \sigma^N(x) \quad \text{if} \quad x \in V
\]
\[
= \mu^N(e) \quad \text{if} \quad e \in V
\]

The bipolar fuzzy relation \( \mu_{Bsd} \) on \( V \cup E \) is defined as \( \mu^p_{Bsd}(x,e) \leq \sigma^p_{Bsd}(x) \land \sigma^p_{Bsd}(e) \), \( \mu^N_{Bsd}(x,e) \geq \sigma^N_{Bsd}(x) \lor \sigma^N_{Bsd}(e) \) for all \( x,e \) in \( V \cup E \). \( \mu_{Bsd}(x,e) \) is a bipolar fuzzy relation of \( \sigma_{Bsd} \) and hence the pair \( Bsd(G) : (\sigma_{Bsd}, \mu_{Bsd}) \) is a bipolar fuzzy graph. This pair is termed as bipolar subdivision of fuzzy graph \( G \). And the size of bipolar subdivision fuzzy graph is \( S(Bsd(G)) = 2S(BG) \) and the order of bipolar subdivision fuzzy graph is \( O(Bsd(G)) = O(BG) + S(BG) \).

Definition: 2.7
A Bipolar fuzzy graph \( G : (\sigma_B, \mu_B) \) with the underlying graph \( G^* : (\sigma^*, \mu^*) \) be given. Let \( G^* \) be \( (V, E) \). The nodes and set of the pair \( BM(G) : (\sigma_{BM}, \mu_{BM}) \) where \( \sigma_{BM} = (m^p_{BM}, m^N_{BM}) \), \( \mu_{BM} = (m^p_{BM}, m^N_{BM}) \), where
The bipolar fuzzy relation $\sigma_{BM}(x) = \begin{cases} \sigma_B(x), & \text{if } x \in \sigma^* \\ \mu_B(x), & \text{if } x \in \mu^* \\ 0 & \text{Otherwise} \end{cases}$

$\mu_{BM}(e_i,e_j) = \begin{cases} \mu_B(e_i) \land \mu_B(e_j), & \text{if } e_i,e_j \in \mu^* \text{ are adjacent in } G^* \\ 0 & \text{Otherwise} \end{cases}$

$\mu_{BM}(v_i,v_j) = \begin{cases} \mu_B(e_j), & \text{if } v_i \text{ in } \sigma^* \text{ lies on the edge } e_j \in \mu^* \\ 0, & \text{Otherwise} \end{cases}$

As $\sigma_{BM}$ is defined only through the values of $\sigma$ and $\mu$, $\sigma_{BM}^p : V \cup E \to [0,1]$ and $\sigma_{BM}^N : V \cup E \to [-1,0]$ is well defined fuzzy subset on $V \cup E$. Also $\mu_{BM}$ is a bipolar fuzzy relation on $\sigma_{BM}$ and

$\mu_{BM}^p(x,y) \leq \sigma_{BM}^p(x) \land \sigma_{BM}^p(y) \quad \forall x, y \in V \cup E.$

$\mu_{BM}^N(x,y) \geq \sigma_{BM}^N(x) \lor \sigma_{BM}^N(y) \quad \forall x, y \in V \cup E.$

Hence the pair $B(M(G)) : (\sigma_{BM}, \mu_{BM})$ where $\sigma_{BM} = (m_{BM}^p, m_{BM}^N)$. $\mu_{BM} = (m_{BM}^p, m_{BM}^N)$ is a bipolar fuzzy graph called bipolar middle fuzzy graph of $G$. And the size of middle bipolar fuzzy graph is

$$\text{size}(BM(G)) = 2 \text{Size}(BG) + \sum_{e_i,e_j \in \mu^*} ((\mu_B^p(e_i) \land \mu_B^p(e_j), \mu_B^N(e_i) \lor \mu_B^N(e_j))).$$

and the order of bipolar middle fuzzy graph is

$$0(BM(G)) = \text{Order}(BG) + \text{Size}(BG).$$

**Definition: 2.8**

A bipolar fuzzy graph with an underlying set $V$ is defined to be a pair $G : (\sigma, \mu)$, where $\sigma = (m_\sigma^p, m_\sigma^N)$, $\mu = (m_\mu^p, m_\mu^N)$ and crisp graph $G^* : (\sigma^*, \mu^*)$. The pair $BT(G) : (\sigma_{BT}, \mu_{BT})$ of $G$ where $\sigma_{BT} = (m_{BT}^p, m_{BT}^N)$, $\mu_{BT} = (m_{\mu_{BT}}^p, m_{\mu_{BT}}^N)$. We call $\sigma_{BT}$ the bipolar total vertex set of $V$, $\mu_{BT}$ the bipolar fuzzy edge set of $E$ respectively. The bipolar fuzzy subset $\sigma_{BT}$ is defined on $V \cup E$ as

$$\sigma_{BT}^p(x) = \sigma_B^p(x) \quad \text{if } x \in V$$

$$= \mu_B^p(e) \quad \text{if } e \in V$$

$$\sigma_{BT}^N(x) = \sigma_B^N(x) \quad \text{if } x \in V$$

$$= \mu_B^N(e) \quad \text{if } e \in V$$

The bipolar fuzzy relation $\mu_{BT}$ is defined as

$$\mu_{BT}^p(x,y) = \mu_B^p(x,y) \quad \text{if } x, y \in V$$

$$\mu_{BT}^N(x,y) = \mu_B^N(x,y) \quad \text{if } x, y \in V$$
By the definition \( \mu_{BT}^p(x, y) \leq \sigma_{BT}^p(x) \land \mu_{BT}^p(y) \), \( \mu_{BT}^N(x, y) \geq \sigma_{BT}^N(x) \lor \mu_{BT}^N(y) \) for all \( x, y \) in \( V \cup E \). Hence \( \mu_{BT} \) is a bipolar fuzzy relation on the bipolar fuzzy subset \( \sigma_{BT} \). Hence the pair \( BT(G) : (\sigma_{BT}, \mu_{BT}) \) is a bipolar fuzzy graph, and is termed as bipolar total fuzzy graph of \( G \). And the size of total fuzzy graph is
\[
S(BT(G)) = 3S(BG) + \sum_{e_i, e_j \in E} ((\mu_{B}^p(e_i) \land \mu_{B}^p(e_j)),(\mu_{B}^N(e_i) \lor \mu_{B}^N(e_j)))
\]
and the order of bipolar total fuzzy graph is \( 0(BT(G)) = 0(BG) + S(BG) \).

**EDGE DEGREE PROPERTIES OF MIDDLE, SUBDIVISION AND TOTAL BIPOLAR FUZZY GRAPHS**

**Theorem 3.1**

Let \( G : (\sigma_B, \mu_B) \) be a bipolar fuzzy graph on \( G^* : (V, E) \). Let \( e_1 = ab \in E \) be any edge in \( G^* \). Then the edge degree of the edge \( ae_1 \in E_{Bsd} \) is given by \( d_{Bsd}(G)(ae_1^+, ae_1^+) = d_G(a^+, a^-) \).

**Proof**

Let \( ae_1 \in E_{Bsd} \) be any edge in \( Bsd(G) \). Then \( e_1 = ab \in E \), for some \( b \in V \). Therefore \( a \) and \( b \) are the only vertices adjacent to \( e_1 \) in \( G \).

By definition,
\[
d_{Bsd}(G)(ae_1^+, ae_1^+) = \sum_{ae_2 \in E_{Bsd}, e_2 \neq e_1} \mu_{Bsd}(ae_1^+, ae_2^+) + \sum_{e_2 \in E_{Bsd}, b \neq a} \mu_{Bsd}(e_1^+, b^+, e_2^+) \\
\]
\[
d_{Bsd}(G)(ae_1^+, ae_1^+) = \sum_{e_2 \in E, e_2 \neq e_1} ((\sigma_B^p(a) \land \mu_B^p(e_2)), \sigma_B^N(a) \lor \mu_B^N(e_2)) + \mu_{Bsd}(e_1^+, b^+, e_2^+) \\
\]
\[
S_B^N(a) \lor \mu_B^N(e_2^+)) + \mu_{Bsd}(e_1^+, b^+, e_2^+) \\
\]
\[
= \sum_{ae \in E, e \neq b} ((\sigma_B^p(a) \land \mu_B^p(ac)), (\sigma_B^N(a) \lor \mu_B^N(ac)) \\
+ ((\mu_B^p(e_1^+) \land \mu_B^p(b)), (\mu_B^N(e_1^+) \lor \mu_B^N(b)))
\]
\[ d_{\text{BM}}(G)(ae) = \sum_{a \in V, e \in E} \mu_B(ac^p, ac^N) + \mu_B(ab^p, ab^N) \]

\[ d_{\text{BM}}(G)(ae) = \sum_{a \in V} \mu_B(ac^p, ac^N) \]

\[ d_{\text{BM}}(G)(ae) = d_G(a^p, a^N) \]

**Theorem 3.2**

Let \( G : (\sigma_B, \mu_B) \) be a bipolar fuzzy graph on \( G^* : (V, E) \). Then the edge degree in its bipolar middle fuzzy graph is given by

\[ d_{\text{BM}}(G)(ae_1, ae_2) = d_G(a^p, a^N) + \sum_{e_1 e_2 \in E_{\text{BM}}} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_2))) \]

and

\[ d_{\text{BM}}(G)(e_1 e_2^p, e_1 e_2^N) = 2((\mu_B^p(e_1, e_1^N) + \mu_B^p(e_2, e_2^N)) + \sum_{e_1 e_2 \in E_{\text{BM}}} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_2))) + \sum_{e_2 e_3 \in E_{\text{BM}}} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3))) \]

where \( a \in V, e_1, e_2 \in E \).

**Proof**

Let the edge \( ae_1 \) in \( BM(G) \), where \( a \in V \) and \( e_1 = ab \in E \) with \( b \in V \). Then the edges adjacent to \( ae_1 \) are of the form \( ae_2 \), where \( e_2 = ac \in E \) (or) \( e_1 b \) (or) \( e_3 e_4 \), where the edges \( e_1 \) and \( e_3 \) are adjacent in \( G \).

\[ d_{\text{BM}}(G)(ae_1, ae_2) = \sum_{ae_2 \in E_{\text{BM}}, e_2 \neq e_1} \mu_B(ae_2, ae_2^N) + \sum_{e_1 e_2 \in E_{\text{BM}}} \mu_B(e_1, b^p, e_1 b^N) + \sum_{e_1 e_2 \in E_{\text{BM}}} \mu_B(e_1, e_1 e_2^N) \]

\[ = \sum_{ae_2 \in E_{\text{BM}}, e_2 \neq e_1} \mu_B(ae_2^p, ae_2^N) + \mu_B(e_1, b^p, e_1 b^N) \]

\[ + \sum_{e_1 e_2 \in E_{\text{BM}}} \mu_B(e_1, e_1 e_2^N) \]

\[ = \sum_{ae_2 \in E_{\text{BM}}, e_2 \neq e_1} ((\sigma_B^p(a) \land \mu_B^p(e_2), \sigma_B^N(a) \lor \mu_B^N(e_2))) \]

\[ + ((\mu_B^p(e_1) \land \sigma_B^p(b), (\mu_B^N(e_1) \lor \sigma_B^N(b))) + \sum_{e_1 e_2 \in E_{\text{BM}}} ((\mu_B^p(e_1) \land \mu_B^p(e_3)), (\mu_B^N(e_1) \lor \mu_B^N(e_3))) \]
\[
= \sum_{a \in E} \left( (\mu_B^p(e_2), \mu_B^N(e_2)) + (\mu_B^N(e_1), \mu_B^N(e_1)) \right) + \\
\sum_{e_1, e_2 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_2)))
\]
\[
= \sum_{a \in E} \mu_B^p(ac^p, ac^N) + \\
\sum_{e_1, e_2 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_2)))
\]
\[
=d_G(a^p, a^N) + \sum_{e_1, e_2 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_2))).
\]

Now, consider the edge \(e_1e_2\) in \(BM(G)\), where \(e_1 = ab\) and \(e_2 = bc \in E\) with \(a, b, c \in V\). Then the edges adjacent to \(e_1e_2\) are of the form \(xe_1\), where the edges \(e_2\) and \(e_3\) are adjacent in \(G\).
\[
\therefore d_{BM(G)}(e_1e_2, e_1e_3) = \sum_{x \in E} \mu_{BM}(xe_1, xe_1^N) + \sum_{e_1, e_2, e_3 \in E} \mu_{BM}(e_1e_3, e_1e_3^N) + \\
\sum_{x \in E} \mu_{BM}(xe_2^p, xe_2^N) + \sum_{e_1, e_2, e_3 \in E} \mu_{BM}(e_2e_3, e_2e_3^N)
\]
\[
= \mu_{BM}(ae_1^p, ae_1^N) + \mu_{BM}(be_1^p, be_1^N) + \sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_3)))
\]
\[
+ \mu_{BM}(be_2^p, be_2^N) + \mu_{BM}(ce_2^p, ce_2^N) + \sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3)))
\]
\[
= 2\mu_B(e_1^p, e_1^N) + \sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_3))) + 2\mu_B(e_2^p, e_2^N) + \\
\sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3)))
\]
\[
= 2(\mu_B(e_1^p, e_1^N) + \mu_B(e_2^p, e_2^N)) + \sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_3)), (\mu_B^N(e_1) \lor \mu_B^N(e_3)))
\]
\[
+ \sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3)))
\]

**Theorem: 3.3**

Let \(G: (\sigma_B, \mu_B)\) be a bipolar fuzzy graph on \(G^*: (V, E)\). Then the edge degree in its bipolar total fuzzy graph is given by
\[
d_{BT(G)}(ae_1^p, ae_1^N) = 2d_G(a^p, a^N) + \sum_{e_1, e_2 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_2)), (\mu_B^N(e_1) \lor \mu_B^N(e_2)))
\]
\[
d_{BT(G)}(e_1e_2^p, e_1e_2^N) = 2((\mu_B(e_1^p, e_1^N) + \mu_B(e_2^p, e_2^N))
\]
\[
+ \sum_{e_1, e_2, e_3 \in E} ((\mu_B^p(e_1) \land \mu_B^p(e_3)), (\mu_B^N(e_1) \lor \mu_B^N(e_3)))
\]
\[ + \sum_{e_2,e_1 \in E_{BT}, e_1 \neq e_2} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3))) \]

and

\[ d_{BT(G)}(ab^p, ab^N) = 2(d_G(ab^p, ab^N) + \mu_B(ab^p, ab^N) \]

where \( a, b \in V \) and \( e_1, e_2 \in E \).

**Proof**

Let the edge \( ae_1 \) in \( BT(G) \), where \( a \in V \) and \( e_1 = ab \in E \) with \( b \in V \). Then the edges adjacent to \( ae_1 \) are of the form \( ax \), where \( ae_1 \), where \( e_2 = ac \in E \) (or) \( e_1b \) (or) \( e_1e_3 \), where the edges \( e_1 \) and \( e_3 \) are adjacent in \( G \).

\[ \therefore d_{BT(G)}(ae_1) = \sum_{ab \in E} \mu_B(ab^p, ab^N) + \sum_{ae_2 \in E_{BT}, e_2 \neq e_1} \mu_B(BT, ae_2) + \mu_B(\mu_B^p(e_1), \mu_B^p(e_3)) + \mu_B(\mu_B^N(e_1), \mu_B^N(e_3)) \]

\[ = d_G(a^p, a^N) + \sum_{ae_2 \in E_{BT}, e_2 \neq e_1} \mu_B(e_1b^p, e_1b^N) + \mu_B(\mu_B^p(e_1), \mu_B^p(e_3)) + \mu_B(\mu_B^N(e_1), \mu_B^N(e_3)) \]

\[ = d_G(a^p, a^N) + \sum_{ae_2 \in E_{BT}, e_2 \neq e_1} \mu_B(e_1b^p, e_1b^N) + \mu_B(\mu_B^p(e_1), \mu_B^p(e_3)) + \mu_B(\mu_B^N(e_1), \mu_B^N(e_3)) \]

Now, consider the edge \( e_1e_2 \) in \( BT(G) \), where \( e_1 = ab \) and \( e_2 = bc \in E \) with \( a, b \) and \( c \in V \). Then the edges adjacent to \( e_1e_2 \) are of the form \( xe_1 \), where \( x = a \) or \( b \) (or) \( e_1e_3 \), where edges \( e_3 \) and \( e_1 \) are adjacent in \( G \) (or) \( e_2x \), where \( x = b \) (or) \( c \) (or) \( e_2e_3 \), where the edges \( e_2 \) and \( e_3 \) are adjacent in \( G \).

\[ \therefore d_{BT(G)}(e_1e_2^p, e_1e_2^N) = \sum_{xe_1 \in E_BT} \mu_B(xe_1, xe_1^N) + \sum_{e_1e_2 \in E_BT, e_2 \neq e_1} \mu_B(e_1e_2^p, e_1e_2^N) + \sum_{xe_2 \in E_BT} \mu_B(xe_2, xe_2^N) + \sum_{e_1e_3 \in E_BT, e_2 \neq e_1} \mu_B(e_1e_3^p, e_1e_3^N) \]

where \( x, e_1, e_2 \) and \( e_3 \) are adjacent in \( G \).
\[
\sum_{e_2e_3 \in E_{BT}, e_3 \neq e_1} \mu_{BT}(e_be_c, e_be_c)
\]

\[
= \mu_{BT}(ae_1^P, ae_1^N) + \mu_{BT}(be_1^P, be_1^N) + \\
\sum_{e_1e_3 \in E_{BT}, e_3 \neq e_2} ((\mu_B^p(e_1) \land \mu_B^p(e_3)), (\mu_B^N(e_1) \lor \mu_B^N(e_3))), \\
+ \mu_{BT}(be_2^P, be_2^N) + \mu_{BT}(ce_2^P, ce_2^N) + \\
\sum_{e_2e_3 \in E_{BT}, e_3 \neq e_1} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3)))
\]

\[
= 2\mu_B(e_1^P, e_1^N) + \sum_{e_1e_3 \in E_{BT}, e_3 \neq e_2} ((\mu_B^p(e_1) \land \mu_B^p(e_3)))
\]

\[
(\mu_B^N(e_1) \lor \mu_B^N(e_3)) + 2\mu_B(e_2^P, e_2^N) + \\
\sum_{e_2e_3 \in E_{BT}, e_3 \neq e_1} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3)))
\]

\[
= 2\mu_B(e_1^P, e_1^N) + \mu_B(e_2^P, e_2^N) + \\
\sum_{e_1e_3 \in E_{BT}, e_3 \neq e_2} ((\mu_B^p(e_1) \land \mu_B^p(e_3)), (\mu_B^N(e_1) \lor \mu_B^N(e_3))) + \\
\sum_{e_2e_3 \in E_{BT}, e_3 \neq e_1} ((\mu_B^p(e_2) \land \mu_B^p(e_3)), (\mu_B^N(e_2) \lor \mu_B^N(e_3)))
\]

Finally, let the edge \(ab\) in \(BT(G)\), where \(a, b \in V\). Then the edges adjacent to \(ab\) are of the form \(ac\), where \(c \in V\) (or) \(ae_1\), where \(e_1 = ac \in E\) with \(c \in V\) (or) \(bc\), where \(c \in V\) (or) \(be_2\), where \(c_2 = bc \in E\) with \(c \in V\).

\[
\therefore d_{BT(G)}(ab) = \sum_{ac \in E, c \neq b} \mu_B(ac^P, ac^N) + \sum_{ae_1 \in E_{BT}} \mu_{BT}(ae_1^P, ae_1^N) + \\
\sum_{bc \in E, c \neq a} \mu_B(bc^P, bc^N) + \sum_{be_2 \in E_{BT}} \mu_{BT}(be_2^P, be_2^N)
\]

\[
= d_G(ab^P, ab^N) + \sum_{ac \in E, c \neq b} ((\sigma_B^p(a) \land \mu_B^p(e_1)), (\sigma_B^N(a) \lor \mu_B^N(e_1))) + \\
\sum_{bc \in E, c \neq a} ((\sigma_B^p(b) \land \mu_B^p(e_2)), (\sigma_B^N(b) \lor \mu_B^N(e_2)))
\]

\[
= d_G(ab^P, ab^N) + \sum_{ac \in E} \mu_B(ac^P, ac^N) + \sum_{bc \in E} \mu_B(bc^P, bc^N)
\]

\[
= d_G(ab^P, ab^N) + d_G(a^P, a^N) + d_B(b^P, b^N)
\]

\[
= 2(d_G(ab^P, ab^N) + \mu_B(ab^P, ab^N)).
\]
CONCLUSION
We have introduced the concept of Edge degree of Middle, Subdivision and Total bipolar fuzzy graphs in this paper.

REFERENCES