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Fuzzy Connectedness in Fuzzy Tri Topological Space

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Abstract

In this paper, we introduce fuzzy connectedness in fuzzy tri topological spaces and also defined separation properties in fuzzy tri topological spaces.

Keywords: Fuzzy tri topological space, fuzzy connectedness, fuzzy disconnectedness and fuzzy separation.

INTRODUCTION

J.C Kelly [7] initiate the concept of bitopological space. W. J. Pervin [10] was define connectedness in a bitopological space. I.L. Reilly [11] ,J. Swart [13] and T. Birsan [1] studied connectedness in bitopological spaces .B. Dvalishi [3] studied connectedness in bitopological space. Martin Kovar [8] introduced tri topological space .S. Palanimmal [9] investigated tri toplogical spaces in 2011. N.F. Hameed and Moh. Yahya Abid [4] gives the definition of 123 open set in tri topological spaces .

In 1968 Change C.L. [2] introduced the concept of fuzzy topological spaces. K.S. Sethupathy

Raja and S. Lakshmivarahan [12] introduced connectedness in fuzzy topological space.

Kandil A. [6] [7] introduced fuzzy bitopological spaces. In this paper, we introduce fuzzy connectedness and fuzzy separated sets in fuzzy tri topological space.

PRELIMINARIES

Definition 2.1[9]: **"**Let X be a nonempty set and T_1, T_2 and T_3 are three topologies on X .The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3)".

Definition 2.2[13]: **"**A subset A of a topological space X is called 123 open set if $A \in T_1 \cup T_2 \cup T_3$ and complement of 123 open set is 123 closed set".

Definition 2.4[1]: **"**A bitopological space is (X, T_1, T_2) said to be connected if and only if X cannot be expressed as the union of two non empty disjoint sets A and B such that A is T_1 open and B is T open. When X can be so expressed ,we write $X = A/B$ and called call this a separation of X ".

FUZZY SEPARATED SETS IN FUZZY TRI TOPOLOGICAL SPACE

Definition 3.1: Suppose $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space, two non-empty fuzzy subsets $\chi_\lambda, \chi_\delta$ of X are called fuzzy tri separated if and only if $\chi_\lambda \wedge Ftricl(\chi_\delta) = \tilde{0}_X$ and $Ftricl(\chi_\lambda) \wedge \chi_\delta = \tilde{0}_X$.

Both conditions are equivalent to one condition $(\chi_\lambda \wedge Ftricl(\chi_\delta)) \vee (Ftricl(\chi_\lambda) \wedge \chi_\delta) = \tilde{0}_X$.

Example 3.2: Suppose $X = \{1, 2, 3, 4\}$ be a non-empty fuzzy set.

Consider three fuzzy topologies on X

$$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1,3\}}, \chi_{\{4\}}, \chi_{\{1,3,4\}}\},$$

$$\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{2,3\}}, \chi_{\{3\}}, \chi_{\{1,3,4\}}\},$$

$$\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{2\}}\},$$

Fuzzy open sets in fuzzy tri topological space are union of all three topologies.

Fuzzy tri open sets of X

$$X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1,3\}}, \chi_{\{2\}}, \chi_{\{3\}}, \chi_{\{4\}}, \chi_{\{2,4\}}, \chi_{\{1,2,3\}}, \chi_{\{1,4,3\}}\}.$$

If we take $\chi_{\{2\}}$ and $\chi_{\{4\}}$ Then

$$[A \cap Ftricl(B)] \cup [Ftricl(A) \cap B] = \{\{\chi_{\{2\}}\} \cap \{\chi_{\{4\}}\}\} \cup \{\{\chi_{\{2\}}\} \cap \{\chi_{\{4\}}\}\} = \phi.$$

Hence $\chi_{\{2\}}$ and $\chi_{\{4\}}$ are Fuzzy tri separated sets.

Also $\chi_{\{1, 3\}}$ & $\chi_{\{1, 4\}}$ etc. are Fuzzy tri separated sets.

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Theorem 3.3: Suppose χ_λ and χ_δ are two fuzzy tri separated subsets of a fuzzy tri topological space (X,τ_1,τ_2,τ_3) and $\chi_{\lambda_1} \le \chi_\lambda$ and $\chi_{\delta_1} \le \chi_\delta$, then χ_{λ_1} and χ_{δ_1} are also fuzzy tri separated.

Proof: Since χ_λ and χ_δ are two fuzzy tri separated sets then $\chi_\lambda \wedge Ftricl(\chi_\delta) = \tilde{0}_X$ and $Ftricl(\chi_\lambda) \wedge \chi_\delta = \tilde{0}_X$ ……………..(1)

Also $\chi_{\lambda_1} \le \chi_\lambda \Rightarrow Ftricl\chi_{\lambda_1} \prec Ftricl\chi_\lambda$ and

$\chi_{\delta_1} \le \chi_\delta \Rightarrow Ftricl\chi_{\delta_1} \prec Ftricl\chi_\delta$ …………………(2)

By (1) and (2) that $\chi_{\lambda_1} \wedge Ftri\,cl(\chi_{\delta_1}) = \tilde{0}_X$ and $Ftri\,cl(\chi_{\lambda_1}) \wedge \chi_{\delta_1} = \tilde{0}_X$

Hence χ_{λ_1} and χ_{δ_1} are fuzzy tri separated.

Theorem 3.4: Two fuzzy tri closed (fuzzy tri open) subsets χ_λ, χ_δ of a fuzzy tri topological space (X,τ_1,τ_2,τ_3) are fuzzy separated iff they are disjoint.

Proof: Given that two fuzzy tri separated sets are disjoint, we have to prove that two disjoint fuzzy tri closed (fuzzy tri open) sets are fuzzy tri separated.

If χ_λ and χ_δ are both disjoint fuzzy tri closed, then:

$\chi_\lambda \wedge \chi_\delta = \chi_{\lambda_1}, Ftricl\chi_\lambda = \chi_\lambda$ and $Ftricl\chi_\delta = \chi_\delta$ …..[1]

So that $\chi_\lambda \wedge Ftricl(\chi_\delta) = \tilde{0}_X$ and $Ftricl(\chi_\lambda) \wedge \chi_\delta = \tilde{0}_X$

Therefore χ_λ and χ_δ are fuzzy tri separated.

If χ_λ and χ_δ are both disjoint and fuzzy tri open sets, then $\tilde{1}_X - \chi_\lambda$ and $\tilde{1}_X - \chi_\delta$ are both fuzzy tri closed so that:

$Ftricl(\tilde{1}_X - \chi_\lambda) = \tilde{1}_X - \chi_\delta$ and

$Ftricl(\tilde{1}_X - \chi_\delta) = (\tilde{1}_X - \chi_\delta)$

Also

$\chi_\lambda \wedge \chi_\delta = \tilde{0}_X \Rightarrow \chi_\lambda \le \tilde{1}_X - \chi_\delta and \chi_\delta \le \tilde{1}_X - \chi_\lambda$

$\Rightarrow Ftricl\chi_\lambda \prec Ftricl(\tilde{1}_X - \chi_\delta) = \tilde{1}_X - \chi_\delta$

and

$\Rightarrow Ftricl\chi_\delta \prec Ftricl(\tilde{1}_X - \chi_\lambda) = \tilde{1}_X - \chi_\lambda$

$Ftricl(\chi_\lambda) \wedge \chi_\delta = \tilde{0}_X$ and

$\chi_\lambda \wedge Ftricl(\chi_\delta) = \tilde{0}_X$

Hence χ_λ and χ_δ are fuzzy tri separated in fuzzy tri topological space.

FUZZY CONNECTED SETS AND FUZZY DISCONNECTED SETS IN FUZZY TRI TOPOLOGICAL SPACE

Definition 4.1: Suppose (X,τ_1,τ_2,τ_3) fuzzy tri topological space, $\chi_{\lambda_1} \subset X$ is called fuzzy tri disconnected iff it is the union of two non-empty fuzzy separated sets. That is, iff there exist two non-empty separated sets χ_{δ_1} and χ_{δ_2} such that $\chi_{\delta_1} \wedge Ftricl\chi_{\delta_2} = \tilde{0}_X$, $Ftricl\chi_{\delta_1} \wedge \chi_{\delta_2} = \tilde{0}_X$ and $\chi_{\delta_1} \vee \chi_{\delta_2} = \chi_{\lambda_1}$,

χ_{λ_1} is called fuzzy tri connected if and only if it is not fuzzy tri disconnected.

Example 4.2: Let $X = \{1,2,3,4\}$ be a non-empty fuzzy set.

Consider three fuzzy topologies on X

$\tau_1 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{2\}}, \chi_{\{1,2\}}\}$,

$\tau_2 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{1,3\}}, \chi_{\{1,3,4\}}\}$,

$\tau_3 = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{2,3,4\}}\}$,

Fuzzy open sets in fuzzy tri topological spaces are union of all three fuzzy topologies.

Fuzzy tri open sets of

$X = \{\tilde{1}_X, \tilde{0}_X, \chi_{\{1\}}, \chi_{\{2\}}, \chi_{\{3,4\}}, \chi_{\{1,2\}}, \chi_{\{1,3,4\}}, \chi_{\{2,3,4\}}\}$.

If we take $A = \chi_{\{1,3,4\}}$, $C = \chi_{\{1\}}$ and $D = \chi_{\{3,4\}}$ Then

$A = C \vee D$ And $\chi_{\{1\}} \wedge \chi_{\{3,4\}} = \tilde{0}_X$

$C \wedge Ftri\,cl(D) = \chi_{\{1\}} \wedge \chi_{\{3,4\}} = \tilde{0}_X$,

$F - tri\,cl(C) \wedge D = \chi_{\{1\}} \wedge \chi_{\{3,4\}} = \tilde{0}_X$.

Then $\chi_{\{1\}}$ and $\chi_{\{3,4\}}$ are fuzzy tri separated sets.

Hence the set $\chi_{\{1,3,4\}}$ is fuzzy tri disconnected.

But $\chi_{\{1\}}, \chi_{\{2\}}, \chi_{\{3,4\}}$ are fuzzy tri connected sets.

Remarks 4.3:

(i)The empty set in $Ftri\,O(X)$ is trivially fuzzy tri connected.

(ii) Every singleton set in $Ftri\,O(X)$ is fuzzy tri connected.

Definition 4.4: Two points $\chi_{\{1\}}$ and $\chi_{\{3,4\}}$ of a fuzzy tri topological space X are called fuzzy tri connected iff they are contained in a fuzzy connected subset of X.

Example 4.5: In last example the points $\chi_{\{1\}}$ and $\chi_{\{3,4\}}$ are fuzzy tri connected because they are contained in $\chi_{\{1,3,4\}}$ which is fuzzy tri connected subset of X.

Theorem 4.6: A fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ is fuzzy tri disconnected iff there exists a non-empty fuzzy proper subset of X which is both fuzzy tri open and fuzzy tri closed in X.

Proof: Suppose χ_λ be a non-empty fuzzy proper subset of X which is both fuzzy tri open and fuzzy tri closed in X. We have to show that X is fuzzy tri disconnected:

Let $\chi_\mu = \tilde{1}_X - \chi_\lambda$. Then χ_δ is non-empty since χ_λ is a fuzzy proper subset of X.Moreover,

$\chi_\mu \vee \chi_\lambda = \tilde{1}_X$ and $\chi_\mu \wedge \chi_\lambda = \tilde{0}_X$, since χ_λ is both fuzzy tri open and fuzzy tri closed, χ_μ is also both fuzzy tri open and fuzzy tri closed. Hence

$Ftri\,cl(\chi_\lambda) = \chi_\lambda$ and $Ftri\,cl(\chi_\mu) = \chi_\mu$ it follows that $Ftri\,cl(\chi_\lambda) \wedge \chi_\mu = \tilde{0}_X$ and

$\chi_\lambda \wedge Ftri\,cl(\chi_\mu) = \tilde{0}_X$. Thus X has been expressed as a union of two fuzzy tri separated sets and so X is fuzzy tri disconnected.

Conversely; Let X is fuzzy tri disconnected. Then there exist non empty subsets χ_λ and χ_μ of X such that

$Ftri\,cl(\chi_\lambda) \wedge \chi_\mu = \tilde{0}_X$ and

$\chi_\lambda \wedge Ftri\,cl(\chi_\mu) = \tilde{0}_X$ and $\chi_\lambda \cup \chi_\mu = \tilde{1}_X$.

Since $Ftri\,cl(\chi_\lambda) = \chi_\lambda$ and

$Ftri\,cl(\chi_\mu) = \chi_\mu \Rightarrow \chi_\mu \wedge \chi_\lambda = \tilde{0}_X$ Hence

$\chi_\lambda = \tilde{1}_X - \chi_\mu$ and χ_μ is non empty, χ_λ is a fuzzy proper subset of X. Now $\chi_\lambda \vee Ftri\,cl(\chi_\mu) = \tilde{1}_X$.

[Since $\chi_\lambda \vee \chi_\mu = \tilde{1}_X$ and

$\chi_\mu \prec Ftri\,cl(\chi_\mu) \Rightarrow \tilde{1}_X \prec \chi_\lambda \vee Ftri\,cl(\chi_\mu)$

but $\chi_\lambda \cup Ftri\,cl(\chi_\mu) \prec \tilde{1}_X$ always] Also

$\chi_\lambda \wedge Ftri\,cl(\chi_\mu) = \tilde{0}_X \Rightarrow \chi_\lambda = \tilde{1}_X - (Ftri\,cl(\chi_\mu))$

and similarly $\chi_\mu = \tilde{1}_X - (Ftri\,cl(\chi_\lambda))$.

Since $Ftri\,cl(\chi_\lambda)$ and $Ftri\,cl(\chi_\mu)$ are fuzzy tri closed sets, it follows that χ_λ and χ_μ are fuzzy tri open sets, and since $\chi_\lambda = \tilde{1}_X - \chi_\mu$, χ_λ is also fuzzy tri closed. Thus χ_λ is non empty fuzzy proper subset of X which is both fuzzy tri open and fuzzy tri closed.

In the same way we can show that χ_μ is also non empty fuzzy proper subset of X which is both fuzzy tri open and fuzzy tri closed.

Theorem 4.7: Let $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space and $\chi_\lambda \prec \tilde{1}_X$ .If χ_λ is fuzzy tri connected, then so is $Ftri\,cl(\chi_\lambda)$.

Proof: Let χ_λ be the fuzzy tri connected subset of a fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ .To prove that

$Ftricl(\chi_\lambda)$ is fuzzy tri connected. Suppose contrary, Then $Ftricl(\chi_\lambda)$ is fuzzy tri disconnected .Then there exist non empty tri fuzzy sets $\chi_{\lambda_1}, \chi_{\lambda_2} \prec \tilde{1}_X$ such that $(\tilde{1}_X - \chi_{\lambda_1}) \wedge \chi_{\lambda_2} = \tilde{0}_X$, $\chi_{\lambda_1} \wedge (\tilde{1}_X - \chi_{\lambda_2}) = \tilde{0}_X$.

$$Ftricl\,\chi_\lambda = \chi_{\lambda_1} \vee \chi_{\lambda_2}$$

$$\chi_{\lambda_1} \vee \chi_{\lambda_2} = Ftricl\,\chi_\lambda \succ \chi_\lambda,$$

$\Rightarrow \chi_\lambda \prec \chi_{\lambda_1} \vee \chi_{\lambda_2}$, χ_λ is fuzzy tri connected.

$\Rightarrow \chi_\lambda \prec \chi_{\lambda_1}$ or $\chi_\lambda \prec \chi_{\lambda_2}$

$$\chi_\lambda \prec \chi_{\lambda_1} \Rightarrow Ftricl\,\chi_\lambda \prec Ftricl\,\chi_{\lambda_1}$$

$$\Rightarrow \chi_\lambda \prec \chi_{\lambda_1} \cup \chi_{\lambda_2},$$

$$\Rightarrow Ftricl\,\chi_\lambda \wedge \chi_{\lambda_2} \prec Ftricl\,\chi_{\lambda_1} \wedge \chi_{\lambda_2} = \tilde{0}_X \quad .....(i)$$

$$Ftricl\,\chi_\lambda = \chi_{\lambda_1} \vee \chi_{\lambda_2}$$

$$\Rightarrow \chi_{\lambda_2} \prec Ftricl\,\chi_\lambda$$

$$\Rightarrow Ftricl\,\chi_\lambda \wedge \chi_{\lambda_2} = \chi_{\lambda_2}$$

$$\Rightarrow \chi_{\lambda_2} = \tilde{0}_X$$

For $Ftricl\,\chi_\lambda \wedge \chi_{\lambda_2} = \tilde{0}_X$ (from (i))

χ_H in X such that $Fbcl(\chi_\lambda) = \chi_G \vee \chi_H$.

Since $\chi_\lambda = (\chi_G \wedge \chi_\lambda) \vee (\chi_H \wedge \chi_\lambda)$ and $Fcl(\chi_G \wedge \chi_\lambda) \prec Fcl(\chi_G)$ and $Fcl(\chi_H \wedge \chi_\lambda) \prec Fcl(\chi_H)$ and $\chi_G \wedge \chi_H = \tilde{0}_X$ then $Fcl(\chi_G \wedge \chi_\lambda) \wedge \chi_H = \tilde{0}_X$. Hence $Fcl(\chi_G \wedge \chi_\lambda)) \wedge (\chi_H \wedge \chi_\lambda) = \tilde{0}_X$ .Similarly $(Fcl(\chi_H \wedge \chi_\lambda)) \wedge (\chi_G \wedge \chi_\lambda) = \tilde{0}_X$ .Therefore χ_λ is fuzzy connected a contradiction for $\chi_{\lambda_2} \neq \tilde{0}_X$ similarly $\chi_\lambda \prec \chi_{\lambda_2} \Rightarrow \chi_\lambda = \tilde{0}_X$ .Again we get a contradiction .Hence, If χ_λ is fuzzy tri connected, then so is $Ftricl(\chi_\lambda)$.

Corollary 4.8: A fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ is fuzzy tri connected if and only if the only non-empty fuzzy subset of X which is both fuzzy tri open and fuzzy tri closed in X is X itself.

Corollary 4.9: A fuzzy subset Y of a fuzzy tri topological space X is fuzzy disconnected if and only if Y is the union of two non-empty disjoint fuzzy sets both fuzzy tri open (fuzzy tri closed) in Y.

Theorem 4.10: $(X, \tau_1, \tau_2, \tau_3)$ be a fuzzy tri topological space .If χ_λ is a fuzzy connected set of X and $\chi_{\lambda_1}, \chi_{\lambda_2}$ are fuzzy tri separated sets of X with $\chi_\lambda \prec \chi_{\lambda_1} \vee \chi_{\lambda_2}$, then either $\chi_\lambda \prec \chi_{\lambda_1}$ or $\chi_\lambda \prec \chi_{\lambda_2}$.

Proof: Let $\chi_\lambda \prec \chi_{\lambda_1} \vee \chi_{\lambda_2}$ .Since $\chi_\lambda = (\chi_\lambda \wedge \chi_{\lambda_1}) \vee (\chi_\lambda \wedge \chi_{\lambda_2})$ then $(\chi_\lambda \wedge \chi_{\lambda_1}) \wedge Ftricl(\chi_\lambda \wedge \chi_{\lambda_2}) \prec \chi_{\lambda_1} \wedge cl(\chi_{\lambda_2}) = \tilde{0}_X$. By similar reasoning .we have $(\chi_\lambda \wedge \chi_{\lambda_1}) \wedge Ftricl(\chi_\lambda \wedge \chi_{\lambda_2}) \prec \chi_{\lambda_1} \wedge Ftricl(\chi_{\lambda_2}) = \tilde{0}_X$ .Suppose that $(\chi_\lambda \wedge \chi_{\lambda_1})$ and $\chi_\lambda \wedge \chi_{\lambda_2}$ are nonempty. Then χ_λ is not fuzzy tri connected. This is a contradiction. Thus either $\chi_\lambda \wedge \chi_{\lambda_1} = \tilde{0}_X$ or $\chi_\lambda \wedge \chi_{\lambda_2} = \tilde{0}_X$ .This implies that $\chi_\lambda \prec \chi_{\lambda_1}$ or $\chi_\lambda \prec \chi_{\lambda_2}$.

Theorem 4.11: Let $(X, \tau_1, \tau_2, \tau_3)$ and $(Y, \tau'_1, \tau'_2, \tau'_3)$ are two fuzzy tri topological spaces. Let $f : I^X \to I^Y$ be a fuzzy tri continuous function. If χ_λ is fuzzy tri connected in X, then $f(\chi_\lambda)$ is fuzzy tri connected in Y.

Proof : Suppose that $f(\chi_\lambda)$ is fuzzy tri disconnected in Y .There exist two fuzzy tri separated sets χ_{λ_1} and χ_{λ_2} of Y such that $f(\chi_\lambda) = \chi_{\lambda_1} \cup \chi_{\lambda_2}$. Set

$\chi_{\delta_1} = \chi_\lambda \wedge f^{-1}(\chi_{\lambda_1})$ and

$\chi_{\delta_2} = \chi_\lambda \wedge f^{-1}(\chi_{\lambda_2})$ Since

$f(\chi_\lambda) \wedge \chi_{\lambda_1} \neq \tilde{0}_X$ then $\chi_\lambda \wedge f^{-1}(\chi_{\lambda_1}) \neq \tilde{0}_X$ and so $\chi_{\delta_1} \neq \tilde{0}_X$. Similarly $\chi_{\delta_2} \neq \tilde{0}_X$. Since

$\chi_{\lambda_1} \wedge \chi_{\lambda_2} = \tilde{0}_X$,

$\chi_{\delta_1} \wedge \chi_{\delta_2} = \chi_\lambda \wedge f^{-1}(\chi_{\lambda_1} \wedge \chi_{\lambda_2}) = \tilde{0}_X$ and so

$\chi_{\delta_1} \wedge \chi_{\delta_2} = \tilde{0}_X$ .Since f is continuous then by Lemma 4.7, $Ftri\,cl(f^{-1}(\chi_{\lambda_2})) \prec f^{-1}(\mathrm{F}tri\,cl(\chi_{\lambda_2}))$ and

$\chi_{\delta_2} \prec f^{-1}(\chi_{\lambda_2})$ then

$Ftri\,cl(\chi_{\delta_2}) \prec f^{-1}(\mathrm{F}tri\,cl(\chi_{\lambda_2}))$ .Since

$\chi_{\lambda_1} \wedge Ftri\,cl(\chi_{\lambda_2}) = \tilde{0}_X$, then

$\chi_{\delta_2} \wedge f^{-1}(\mathrm{F}tri\ cl(\chi_{\lambda_2})) \prec f^{-1}(\chi_{\lambda_1}) \wedge f^{-1}(\mathrm{F}tri\ cl(\chi_{\lambda_2})) = \tilde{0}_X$

and then $\chi_{\delta_1} \wedge Ftri\ cl(\chi_{\delta_2}) = \tilde{0}_X$. Thus χ_{δ_1} and χ_{δ_2} are fuzzy tri separated.

Theorem 4.12: If χ_{λ_1} is a fuzzy tri connected set of a fuzzy tri topological space $(X, \tau_1, \tau_2, \tau_3)$ and

$\chi_{\lambda_1} \prec \chi_{\lambda_2} \prec Ftri\,cl(\chi_{\lambda_1})$ then χ_{λ_2} fuzzy tri connected.

Proof: Suppose χ_{λ_2} is not fuzzy tri connected. There exist fuzzy separated sets χ_{δ_1} and χ_{δ_2} of X such that

$\chi_{\lambda_2} = \chi_{\delta_1} \vee \chi_{\delta_2}$ .This implies that χ_{δ_1} and χ_{δ_2} are nonempty and

$Ftri\,cl(\chi_{\delta_1}) \wedge \chi_{\delta_2} = \chi_{\delta_1} \wedge Fcl(\chi_{\delta_2}) = \tilde{0}_X$. By Theorem 4.8, we have either $\chi_{\lambda_1} \prec \chi_{\delta_1}$ or $\chi_{\lambda_1} \prec \chi_{\delta_2}$.

Suppose that $\chi_{\lambda_1} \prec \chi_{\delta_1}$. Then

$Ftricl(\chi_{\lambda_1}) \prec Ftricl(\chi_{\delta_1})$ and

$\chi_{\delta_2} \wedge Ftricl\,\chi_{\lambda_1} = \tilde{0}_X$ .This implies that

$\chi_{\delta_2} \prec \chi_{\lambda_2} \prec Ftricl\,\chi_{\lambda_1}$ and

$\chi_{\delta_2} = Ftricl(\chi_{\lambda_1}) \wedge \chi_{\delta_2} = \tilde{0}_X$. Thus, χ_{δ_2} is an fuzzy tri empty set for if χ_{δ_2} is fuzzy tri nonempty, this is a contradiction. Suppose that $\chi_{\lambda_1} \prec \chi_{\delta_2}$. By similar way, it follows that χ_{δ_1} is empty. This is a contradiction. Hence, χ_{λ_2} is fuzzy tri connected.

Theorem 4.13: If χ_{λ_1} and χ_{λ_2} are fuzzy tri connected sets which are fuzzy tri separated, then $\chi_{\lambda_1} \vee \chi_{\lambda_2}$ is fuzzy tri connected.

Proof: Suppose $\chi_{\lambda_1} \vee \chi_{\lambda_2}$ is tri disconnected and suppose $\chi_{\delta_1} \vee \chi_{\delta_2}$ is fuzzy tri disconnected sets of $\chi_{\lambda_1} \vee \chi_{\lambda_2}$ .Since χ_{λ_1} is a fuzzy tri connected subsets of $\chi_{\lambda_1} \vee \chi_{\lambda_2}$ either $\chi_{\lambda_1} \prec \chi_{\delta_1}$ or $\chi_{\lambda_1} \prec \chi_{\delta_2}$ .Similarly either $\chi_{\lambda_2} \prec \chi_{\delta_1}$ or $\chi_{\lambda_2} \prec \chi_{\delta_2}$.

If $\chi_{\lambda_1} \prec \chi_{\delta_1}$ and $\chi_{\lambda_2} \prec \chi_{\delta_2}$ then

$(\chi_{\lambda_1} \vee \chi_{\lambda_2}) \wedge \chi_{\delta_2} = \chi_{\delta_2}$ and

$(\chi_{\lambda_1} \vee \chi_{\lambda_2}) \wedge \chi_{\delta_1} = \chi_{\delta_1}$ are fuzzy tri separated but this contradicts the hypothesis. Hence either

$\chi_{\lambda_1} \vee \chi_{\lambda_2} \prec \chi_{\delta_1}$ or $\chi_{\lambda_1} \vee \chi_{\lambda_2} \prec \chi_{\lambda_2}$ and so $\chi_{\delta_1} \vee \chi_{\delta_2}$ is not a fuzzy tri disconnected subset of $\chi_{\lambda_1} \vee \chi_{\lambda_2}$ .In other words $\chi_{\lambda_1} \vee \chi_{\lambda_2}$ is fuzzy tri connected.

CONCLUSION:

In this paper the idea of tri connectedness and tri separation were introduced and studied in fuzzy tri topological spaces.

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REFERENCES

[1] Birsan T.,(1968) , Surles espaces bitopological connexes, An sti. Univ. Al. cuza Iasi sect. I. t. XIV fase 2, 293-96.

[2] Change C.L., Fuzzy topological spaces, J. Math. Anal. Appl. 24,182-190(1968).

[3] Dvalishi B. (2004) .connectedness of a fine topology and localization in bitopological spaces, gergian. Math. J.,11(4) , pp 713 -732.

[4] Hameed N.F. & Mohammed Yahya Abid (2011).Certain types of separation axioms in tri topological spaces. Iraqi journal of science. Vol 52, (2),212-217.

[5] Kandil A., Tantwauy O., El. Sheikh S.A. & Hosny M., Connectedness in Ideal bitopological ordered space, Gen. Math. Notes, Vol 24, No.2, pp 37-52. ICSRS Publication(2014).

[6] Kandil A., Nouh A.A. and El-sheikh S.A., On fuzzy bitopological spaces, Fuzzy sets and system , vol.74, issue 3, pp:353-363

[7] Kelly J.C. ,(1963), Bitopological spaces, Proc.LondonMath.Soc.,3 PP. 17-89 .

[8] Kovar, M.,(2000).On 3-Topological version of Thet-Reularity. Internat. J. Matj, Sci. ,23(6),393-398.

[9] Palaniammal S**.** **,**(2011).Study of Quad topological spaces, Ph.D Thesis.

[10] Pervin. W.J.(1967), Connectedness in bitopological spaces, Indag Math.,29, 369-372.

[11] Reilly I.L.. (1971), On pairwise connected bitopological spaces, Kyungpook Math. J., 11(1) 25-28.

[12] Sethupathy Raja K.S. and Lakshmivarahan ,Connectedness in fuzzy topological space, Kybernetika, 13(3), (1977).

[13] Swart J., Total disconnectedness in bitopological spaces and product bitopological spaces, Indag.Math.33, 135-145(1971).