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SOME OPERATIONS ON MULTI-HYPERGRAPH GRAMMAR

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Abstract: This study presents the concept of hyperedge replacement grammar and introduces it as multi-hypergraph generating device. We define some operations over multi-hypergraph grammars such as sum, union and intersection. We prove the existence of isomorphism between the n^{th} component of the language of the given grammars and the n^{th} component of the language of the defined grammar. The results are illustrated with examples.

Keywords: multi-hypergraph grammar, Sum, union, intersection, regular multi-hypergraph language.

INTRODUCTION

Graphs are well known in Computer Science both as formal and an illustrative tool. The concept of graph grammar is an attractive way of formalizing the notion of a recursively defined set of graphs. Graph grammars were introduced by Pflatz. J.L, Rosenfeld. A, and Schneider. H.J initially for picture processing problems.

Node replacement and edge replacement are the two main approaches of graph rewriting. G. Rozenberg and J. Engelfriet discussed the basic notions of node replacement graph grammar and the properties of the class of generated graph languages such as closure properties, structural properties, and decidability properties. Hyperdege replacement is an elementary approach of graph and hypergraph rewriting. It was introduced by Feder and Pavlidis and has been extensively studied starting with special case of edge replacement by Bauderon, Courcelle, Engel friet, Hey kerlieh and Rozenberg. [1,2,4,7,8,9,10]. Courcelle presented the axiomatic definition of context-free rewriting [3]. Hyperedge replacement indeed satisfies these requirements and so hyperdege replacement is context-free in nature. Annegret Habel and Hansjorg Kreowsi formulated context-freeness lemma for both edge replacement and hyperedge replacement [1,2]. D. Caucal focused on providing some of the basic tools to reason out the deterministic graph grammar and on structural study of their generated graphs [5,6].

In this paper, we define some operations over the two hypergraph grammars such as sum, union and intersection. We consider that the rules of the grammar are deterministic and context free. We prove the existence of isomorphism between the n^{th} the component of the language of the given grammars and the n^{th} component of the language of the newly defined grammar. We illustrate the results with examples.

BASIC DEFINITIONS

We use the terminology and notation of D. Caucal [5] unless otherwise specified. This study concerns the following definition as key definition:

Definition 1. A multi-hypergraph grammar $R = (G_0, P)$ is an ordered pair where G_0 is the base graph and

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P is a finite set of rules. Each rule of P is of the form $fx_1x_2x_3\cdots x_{\varrho(f)} \rightarrow H$ or $H' \rightarrow H''$, where $fx_1x_2x_3\cdots x_{\varrho(f)}$ is a hyperarc joining pairwise distinct vertices $x_1, x_2, x_3\cdots, x_{\varrho(f)}$ and H', H'' are finite multi-hypergraphs. The multi-hypergraph H' is a multi-subset of $Im(P_i)$ where $Im(P_i)$ is the right hand side of the i^{th} rule P_i and H'' is the multi-subset of the terminal hyperarcs of H'. The labels of the left hand side of the rules which is of the form $fx_1x_2x_3\cdots x_{\varrho(f)} \rightarrow H$ are the non-terminals of R and is denoted by $N_R = \{f \in F \mid \exists P_j \in P, Dom(P_j) = fX\}$. The labels of rules of the grammar which is of the form $fx_1x_2x_3\cdots x_{\varrho(f)} \rightarrow H$, that are not non-terminals are the terminals of R and is denoted by $T_R = \{f \in F - N_R \mid \exists P_k \in P, Dom(P_k) = gX', fX \in Im(P_k)\}$. $F_R = N_R \cup T_R$ be the labels of R and $\varrho(R) = Max\{\varrho(A) \mid A \in N_R\}$ be the arity of R.

Note: The rule $H' \rightarrow H''$ is applied only when one prefers to terminate the process of derivation after finite number of steps so as to get a finite multi-hypergraph.

METHODS OF DERIVATION

In this section, we explain the process of derivation when the multi-hypergraph N is derived from M. A multi-hypergraph N can be derived from M in two ways i) by eliminating the non-terminal hyperarc X of M ii) by eliminating the sub-hypergraph H' of M.

A multi-hypergraph M derives N written as $M \underset{R,X}{\longrightarrow} N$ when we choose a non-terminal hyperarc X in M where $X = As_1s_2\ldots s_{\varrho(A)}$ and a rule $X' \rightarrow H$ in R where $X' = Ax_1x_2\ x_3x_4\ldots x_{\varrho(A)}$ in R such that N can be obtained by replacing X by H in M. Thus, $N = (M - X) + h(H)$ for some function h, mapping each x_i to s_i and other vertices of H injectively to vertices outside of M.

$$\begin{aligned} N(Y) &= M(Y) + (h(H))(Y) \text{ if } Y \neq X \\ N(X) &= (M(X) - 1) + (h(H))(X) \end{aligned}$$

The multi-hypergraph M derives N written as $M \underset{R,H'}{\longrightarrow} N$ when we choose a sub-hypergraph H' in M and a rule $K \rightarrow K'$ in R where K is isomorphic to H' such that N can be obtained by replacing H' by K' in M. Thus, $N = (M - H') + h(K')$ for some function h mapping each vertex of K into the vertices of H' such that $h(K) = H'$.

$$\begin{aligned} N(Y) &= M(Y) \text{ if } Y \neq X \\ N(X) &= (M(X) - 1) \end{aligned}$$

The derivation $\underset{R,X}{\longrightarrow}$ of a hyperarc X is extended in an obvious way to the derivation $\underset{R,E}{\longrightarrow}$ of any multi-subset E of non-terminal hyperarcs. The complete derivation $\Rightarrow$ is the derivation according to the multi-subset of all non-terminal hyperarcs.

Definition 2. A multi-hypergraph G is generated by a hypergraph grammar R from a multi-hypergraph H if G is isomorphic to a hypergraph in the following set called multi-hypergraph language.

$$R(H) = \left\{ \bigcup_{n\geq 0} \sqsubset H_n \sqsupset \mid H_0 = H \wedge \forall n \geq 0\ H_n \Rightarrow H_{n+1} \right\}$$

Definition 3. A regular multi- hypergraph is a multi-hypergraph generated by a deterministic hypergraph grammar from a finite multi-hypergraph.

OPERATIONS ON HYPERGRAPH GRAMMAR

In this section, we define some operations like union, sum and intersection over the two given grammars and it is proved that an isomorphism is preserved between the n^{th} component of the language of the given grammars and the language of the grammar computed by the defined operator.

Let $R_1 = (G_0, P_1)$ and $R_2 = (G'_0, P_2)$ be two multi-hypergraph grammars having G_0 and G'_0 as its initial multi-hypergraphs respectively with $N_{R_1} \cap N_{R_2} = \phi$.

Sum $(R_1 + R_2)$: The sum of R_1 and R_2 is given by $R = ((G_0 + G'_0), P)$ where $P = P_1 \cup P_2$.
The number of occurrences of X in $(G_0 + G'_0)$ is given by,

$$(G_0 + G'_0)(X) = G_0(X) + G'_0(X)$$
$$\text{(i.e.,)}(G_0 + G'_0)(X) = \begin{cases} G_0(X) & , \text{ if } X \in G_0 \text{ and } X \notin G'_0 \\ G'_0(X) & , \text{ if } X \notin G_0 \text{ and } X \in G'_0 \\ G_0(X) + G'_0(X), & \text{ if } X \in G_0 \text{ and } X \in G'_0 \end{cases}$$

Union $(R_1 \cup R_2)$: The union of R_1 and R_2 is given by $R = ((G_0 \cup G'_0), P)$ where $P = P_1 \cup P_2$. The number of occurrences of X in $(G_0 \cup G'_0)$ is given by,

$$(G_0 \cup G'_0)(X) = \vee(G_0(X), G'_0(X))$$

$$(\text{i.e.,})(G_0 \cup G_0')(X) = \begin{cases} G_0(X) & , \text{ if } X \in G_0 \text{ and } X \notin G_0' \\ G_0'(X) & , \text{ if } X \notin G_0 \text{ and } X \in G_0' \\ \vee(\,G_0(X), G_0'(X)\,), & \text{ if } X \in G_0 \text{ and } X \in G_0' \end{cases}$$

Intersection $(R_1 \cap R_2)$: The intersection of R_1 and R_2 is given by $R = (K_0, P)$ where $K_0 = (G_0 \cap G_0') \cup \{(\,X, G_0(X)\,) \in G_0 \mid X(1) \in N_{R_1}\} \cup \{(\,X, G_0'(X)\,) \in G_0' \mid X(1) \in N_{R_2}\}$ and $P = P_1 \cup P_2$. The number of occurrences of X in K_0 is given by,

$$K_0(X) = \wedge(\,G_0(X), G_0'(X)\,)$$

$$(\text{i.e.,})K_0(X) = \begin{cases} G_0(X) & , \text{ if } X \in G_0 \text{ and } X(1) \in N_{R_1} \\ G_0'(X) & , \text{ if } X \in G_0' \text{ and } X(1) \in N_{R_2} \\ \wedge(\,G_0(X), G_0'(X)\,), & \text{ if } X \in G_0 \text{ and } X \in G_0' \\ 0 & , \text{ otherwise} \end{cases}$$

Theorem 4. Let $R_1(G_0)$ and $R_2(G_0')$ be the multi-hypergraph languages of the grammar $R_1 = (G_0, P_1)$ and $R_2 = (G_0', P_2)$ respectively with $N_{R_1} \cap N_{R_2} \neq \emptyset$. If $(V(G_i) - V(G_0)) \cap V(G_j') = \emptyset \forall j \geq 0, i \geq 1$ and $(V(G_j') - V(G_0')) \cap V(G_i) = \emptyset \forall j \geq 1, i \geq 0$ then $\sqsubset K_n \sqsupset \sim \sqsubset G_n \sqsupset + \sqsubset G_n' \sqsupset$ $\forall n \geq 0$ where K_n be the multi-hypergraph generated in the n^{th} step of parallel derivation using the grammar $R = R_1 + R_2$.

Proof. Let n be the number of steps of derivation. We can prove the result $K_n \sim (G_n + G_n')$ by the method of induction on n. For $n = 0$, the result is obviously true because $(G_0 + G_0')$ becomes an initial multi-hypergraph of the grammar R and we let it as K_0.

By induction hypothesis, the result is true for all multi-hypergraphs that are generated in fewer than n steps. At the $(n-1)^{th}$ step of derivation, $K_{n-1} \sim (G_{n-1} + G_{n-1}')$. Let H_{n-1} be an isomorphism exists between K_{n-1} and $(G_{n-1} + G_{n-1}')$. Let us define $H_{n-1} : V(K_{n-1}) \to V(G_{n-1} + G_{n-1}')$ as

$$H_{n-1}(q_i^{(n-1)}) = p_i^{(n-1)} = \begin{cases} q_i^{(n-2)}, & \text{if } q_i^{(n-1)} = q_i^{(n-2)} \\ v_i^{(n-1)}, & \text{if } q_i^{(n-1)} = l_i^{(n-1)} \\ w_i^{(n-1)}, & \text{if } q_i^{(n-1)} = o_i^{(n-1)} \end{cases}$$

where

$$q_i^{(n-1)} = \begin{cases} q_i^{(n-2)}, & \text{if } q_i^{(n-1)} \in V(\sqsubset K_{n-2} \sqsupset) \\ l_i^{(n-1)}, & \text{if } q_i^{(n-1)} \in \bigcup\limits_{(i,j,k) \in T_{n-1}} V(\,h_{\{ijk\}}^{n-1}(H_j)\,) - V(\sqsubset K_{n-2} \sqsupset) \\ o_i^{(n-1)}, & \text{if } q_i^{(n-1)} \in \bigcup\limits_{(i,j,k) \in T_{n-1}'} V(\,h'^{n-1}_{\{ijk\}}(H_j')\,) - V(\sqsubset K_{n-2} \sqsupset) \end{cases}$$

Let G_n and G_n' be the multi-hypergraphs generated in the n^{th} step of parallel derivation using the grammar R_1 and R_2 respectively by replacing the k^{th} occurrence of their non-terminals $X_i^{(n-1)}$ in G_{n-1} of multiplicity $G_{n-1}(X_i^{(n-1)})$ and $X_i'^{(n-1)}$ in G_{n-1}' of multiplicity $G_{n-1}'(X_i'^{(n-1)})$ by $g_{\{ijk\}}^{(n)}(H_j)$ and $g'^{(n)}_{\{ijk\}}(H_j')$ respectively where H_j, H_j' are multi-hypergraphs of the rules $Y_j \to H_j$ in R_1 and $Y_j' \to H_j'$ in R_2 respectively. $g_{\{ijk\}}^{(n)}, g'^{(n)}_{\{ijk\}}$ are the transformations such that

$g^{(n)}_{\{ijk\}}(Y_j) = X^{(n-1)}_i$, $g'^{(n)}_{\{ijk\}}(Y'_j) = X'^{(n-1)}_i$. The multi-hypergraphs G_n and G'_n can be written as follows.

$$G_n = G_{n-1} - \Big\{ \bigcup_{i\in L_n} (X^{(n-1)}_i, G_{n-1}(X^{(n-1)}_i))\Big\} + \sum_{(i,j,k)\in M_n} g^{(n)}_{\{ijk\}}(H_j)$$

$$\text{where } L_n = \big\{i \mid X^{(n-1)}_i \in G_{n-1}, X^{(n-1)}_i(1) \in N_{R_1}\big\},$$

$$M_n = \big\{(i, j, k) \mid \exists Y_j \to H_j \in R_1,\ i \in L_n,\ g^{(n)}_{\{ijk\}}(Y_j) = X^{(n-1)}_i\big\}.$$

$$G'_n = G'_{n-1} - \Big\{ \bigcup_{i\in L'_n} (X'_i, G'_{n-1}(X'_i))\Big\} + \sum_{(i,j,k)\in M'_n} g'^{(n)}_{\{ijk\}}(H'_j)$$

$$\text{where } L'_n = \big\{i \mid X'^{(n-1)}_i \in G'_{n-1},\ X'_i(1) \in N_{R_2}\big\},$$

$$M'_n = \big\{(i, j, k) \mid \exists Y'_j \to H'_j \in R_2,\ i \in L'_n,\ g'^{(n)}_{\{ijk\}}(Y'_j) = X'^{(n-1)}_i\big\}.$$

Let K_n be a multi-hypergraph derived from K_0 using the grammar R in the n^{th} step of derivation. We classify the non-terminal hyperarcs of K_{n-1} into two groups according to the labels of them.

$$K_n = K_{n-1} - \Big\{ \bigcup_{i\in S_n} (Q^{(n-1)}_i, K_{n-1}(Q^{(n-1)}_i))\Big\} - \Big\{ \bigcup_{i\in S'_n} (Q_i'^{(n-1)}, K_{n-1}(Q_i'^{(n-1)}))\Big\}$$

$$+ \sum_{(i,j,k)\in T_n} h^{(n)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_n} h'^{(n)}_{\{ijk\}}(H'_j) \text{ where}$$

$$S_n = \big\{i \mid Q^{(n-1)}_i \in K_{n-1}, Q^{(n-1)}_i(1) \in N_{R_1} \cap N_R\big\},$$

$$T_n = \big\{(i, j, k) \mid \exists Y_j \to H_j \in R_1 \cap R,\ i \in S_n,\ h^{(n)}_{\{ijk\}}(Y_j) = Q^{(n-1)}_i\big\},$$

$$S'_n = \big\{i \mid Q'^{(n-1)}_i \in K_{n-1},\ Q'_i(1) \in N_{R_2} \cap N_R\big\},$$

$$T'_n = \big\{(i, j, k) \mid \exists Y'_j \to H'_j \in R_2 \cap R,\ i \in S'_n,\ h'^{(n)}_{\{ijk\}}(Y'_j) = Q'^{(n-1)}_i\big\}.$$

For every non-terminal hyperarc $Q^{(n-1)}_i$ in K_{n-1}, we can find a non-terminal hyperarc say $X^{(n-1)}_i$ in G_{n-1} with same multiplicity such that $H_{n-1}(Q^{(n-1)}_i) = X^{(n-1)}_i$. Both the non-terminal hyperarcs $Q^{(n-1)}_i$ in K_{n-1} and $X^{(n-1)}_i$ in G_{n-1} use the same rule $Y_j \to H_j$ in $R_1 \cap R$ under the transformation $h^{(n)}_{\{ijk\}}$ and $g^{(n)}_{\{ijk\}}$ respectively. Hence, $L_n = S_n$; $M_n = T_n$; $h^{(n)}_{\{ijk\}}(H_j) \sim g^{(n)}_{\{ijk\}}(H_j)$.

Let $G^{(n)}_{\{ijk\}}$ be an isomorphism exists between $h^{(n)}_{\{ijk\}}(H_j)$ and $g^{(n)}_{\{ijk\}}(H_j)$ and we can define $G^{(n)}_{\{ijk\}} : V(h^{(n)}_{\{ijk\}}(H_j)) \to V(g^{(n)}_{\{ijk\}}(H_j))$ as follows.

$$G^{(n)}_{\{ijk\}}(s^{(n)}_c) = t^{(n)}_c = \begin{cases} H_{n-1}(q^{(n-1)}_c), & \text{if } s^{(n)}_c = q^{(n-1)}_c \\ a^{(n)}_c, & \text{if } s^{(n)}_c = d^{(n)}_c \end{cases}$$

$$\text{where } s^{(n)}_c = \begin{cases} q^{(n-1)}_c, & \text{if } s^{(n)}_c \in \bigcup_{(i,j,k)\in T_n} V(h^{(n)}_{\{ijk\}}(H_j)) \bigcap V(\sqsubset K_{n-1} \sqsupset) \\ d^{(n)}_c, & \text{if } s^{(n)}_c \in \bigcup_{(i,j,k)\in T_n} V(h^{(n)}_{\{ijk\}}(H_j)) - V (\sqsubset K_{n-1} \sqsupset) \end{cases}$$

In the same way, the non-terminal multi-hyperarc $Q'^{(n-1)}_i$ in K_{n-1} and the non-terminal hyperarc $X'^{(n-1)}_i$ in G'_{n-1} use the same rule $Y'_j \to H'_j$ in $R \cap R_2$ under the transformation $h'^{(n)}_{\{ijk\}}$ and $g'^{(n)}_{\{ijk\}}$ respectively. Hence, $L'_n = S'_n$; $M'_n = T'_n$ and $h'^{(n)}_{\{ijk\}}(H'_j) \sim g'^{(n)}_{\{ijk\}}(H'_j)$. Let $G'^{(n)}_{\{ijk\}}$ be an isomorphism exists between $h'^{(n)}_{\{ijk\}}(H'_j)$ and $g'^{(n)}_{\{ijk\}}(H'_j)$. We define $G'^{(n)}_{\{ijk\}}$: $V(h'^{(n)}_{\{ijk\}}(H'_j)) \to V(g'^{(n)}_{\{ijk\}}(H'_j))$ as follows:

$$G'^{(n)}_{\{ijk\}}(s'^{(n)}_c) = t'^{(n)}_c = \begin{cases} H_{n-1}(q^{(n-1)}_c), & \text{if } s'^{(n)}_c = q^{(n-1)}_c \\ a'^{(n)}_c, & \text{if } s'^{(n)}_c = d'^{(n)}_c \end{cases}$$

$$\text{where} \quad s_c^{'(n)} = \begin{cases} q_c^{(n-1)}, & \text{if } s_c^{'(n)} \in \bigcup_{(i,j,k)\in T_n'} V(h_{\{ijk\}}^{'(n)}(H_j')) \bigcap V(\sqsubset K_{n-1} \sqsupset) \\ d_c^{'(n)}, & \text{if } s_c^{'(n)} \in \bigcup_{(i,j,k)\in T_n'} V(h_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset K_{n-1} \sqsupset) \end{cases}$$

For the n^{th} case, we have to prove $K_n \sim (G_n + G_n')$.

$$\begin{aligned} \mid V(G_n + G_n') \mid &= \mid V(G_n) \mid + \mid V(G_n') \mid - \mid V(G_n \cap G_n') \mid \\ &= \mid V(G_n) \mid + \mid V(G_n') \mid - \mid V(G_0 \cap G_0') \mid \\ &= \mid V(\sqsubset G_{n-1} \sqsupset) \mid + \sum_{(i,j,k)\in M_n} \mid V(g_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset G_{n-1} \sqsupset) \mid \\ &\quad + \mid V(\sqsubset G_{n-1}' \sqsupset) \mid + \sum_{(i,j,k)\in M_n'} \mid V(g_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset G_{n-1}' \sqsupset) \mid \\ &\quad - \mid V(G_0) \cap V(G_0') \mid \end{aligned}$$

$$\begin{aligned} \mid V(K_n) \mid &= \mid V(\sqsubset K_{n-1} \sqsupset) \mid + \sum_{(i,j,k)\in T_n} \mid V(h_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset K_{n-1} \sqsupset) \mid \\ &\quad + \sum_{(i,j,k)\in T_n'} \mid V(h_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset K_{n-1} \sqsupset) \mid \end{aligned}$$

$$\begin{aligned} \mid V(\sqsubset K_{n-1} \sqsupset) \mid &= \mid V(\sqsubset G_{n-1} + G_{n-1}' \sqsupset) \mid \\ &= \mid V(\sqsubset G_{n-1} \sqsupset) \mid + \mid V(\sqsubset G_{n-1}' \sqsupset) \mid - \mid V(\sqsubset G_0 \sqsupset) \cap V(\sqsubset G_0' \sqsupset) \mid \end{aligned}$$

Since $h_{\{ijk\}}^{(n)}(H_j) \sim g_{\{ijk\}}^{(n)}(H_j)$, $T_n = M_n$, and $Q_i^{(n)}(1) = X_i^{(n)}(1)$,

$$\begin{aligned} \mid V(h_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset K_{n-1} \sqsupset) \mid &= \mid V(g_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset G_{n-1} + G_{n-1}' \sqsupset) \mid \\ &= \mid V(g_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset G_{n-1} \sqsupset) \mid \end{aligned}$$

Since $h_{\{ijk\}}^{'(n)}(H_j') \sim g_{\{ijk\}}^{'(n)}(H_j')$, $T_n' = M_n'$, and $Q_i^{'(n)}(1) = X_i^{'(n)}(1)$,

$$|V(h_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset K_{n-1} \sqsupset)| = |V(g_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset G_{n-1} + G_{n-1}' \sqsupset)|$$

$$\begin{aligned} &= |V(g_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset G_{n-1}' \sqsupset)| \\ |V(K_n)| &= |V(\sqsubset G_{n-1} \sqsupset)| + |V(\sqsubset G_{n-1}' \sqsupset)| - |V(\sqsubset G_0 \sqsupset) \cap V(\sqsubset G_0' \sqsupset)| \\ &\quad + |V(g_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset G_{n-1} \sqsupset)| + |V(g_{\{ijk\}}^{'(n)}(H_j')) - V(\sqsubset G_{n-1}' \sqsupset)| \\ &= |V(G_n + G_n')| \end{aligned}$$

Let us define a transformation $H_n : V(K_n) \to V(G_n + G_n')$ as

$$H_n(q_c^{(n)}) = \begin{cases} H_{n-1}(q_c^{(n-1)}), & \text{if } q_c^{(n)} = q_c^{(n-1)} \\ a_c^{(n)}, & \text{if } q_c^{(n)} = d_c^{(n)} \\ a_c^{'(n)}, & \text{if } q_c^{(n)} = d_c^{'(n)} \end{cases}$$

$$\text{where} \quad q_c^{(n)} = \begin{cases} q_c^{(n-1)}, & \text{if } q_c^{(n)} \in V(\sqsubset K_{n-1} \sqsupset) \\ d_c^{(n)}, & \text{if } q_c^{(n)} \in \bigcup_{(i,j,k)\in T_n} V(h_{\{ijk\}}^{(n)}(H_j)) - V(\sqsubset K_{n-1} \sqsupset) \\ d_c^{'(n)}, & \text{if } q_c^{(n)} \in \bigcup_{(i,j,k)\in T_n'} V(h'^{(n)}_{\{ijk\}}(H_j')) - V(\sqsubset K_{n-1} \sqsupset) \end{cases}$$

Let us choose a multi-hyperarc $X = f q_1^{(n)} q_2^{(n)} q_3^{(n)} \cdots q_{\varrho(f)}^{(n)}$ in K_n.
Case 1: Suppose that X be in $(\sqsubset K_{n-1} \sqsupset)$ but not in $\bigcup_{(i,j,k)\in T_n} h_{\{ijk\}}^{(n)}(H_j)$ and $\bigcup_{(i,j,k)\in T'_n} h'^{(n)}_{\{ijk\}}(H'_j)$.

In this case, X be a terminal hyperarc and each $q_i^{(n)} = q_i^{(n-1)}$ $\forall i = 1, 2, 3, \cdots \varrho(f)$.

$$\begin{aligned}
H_n(X) &= H_n(f q_1^{(n)} q_2^{(n)} q_3^{(n)} \cdots q_{\varrho(f)}^{(n)}) \\
&= f H_n(q_1^{(n)}) H_n(q_2^{(n)}) \cdots H_n(q_{\varrho(f)}^{(n)}) \\
&= f H_n(q_1^{(n-1)}) H_n(q_2^{(n-1)}) \cdots H_n(q_{\varrho(f)}^{(n-1)}) \\
&= f H_{n-1}(q_1^{(n-1)}) H_{n-1}(q_2^{(n-1)}) \cdots H_{n-1}(q_{\varrho(f)}^{(n-1)}) \\
&= H_{n-1}(f q_1^{(n-1)} q_2^{(n-1)} q_3^{(n-1)} \cdots q_{\varrho(f)}^{(n-1)}) \\
H_n(X) &= H_{n-1}(X) = Y(say) \in (G_{n-1} + G'_{n-1})
\end{aligned}$$

Since X is a terminal hyperarc, $H_{n-1}(X)$ is also a terminal hyperarc and hence $H_n(X) \in (G_n + G'_n)$. Since no $h_{\{ijk\}}^{(n)}(H_j)$ and $h_{\{ijk\}}^{'(n)}(H'_j)$ contains X, $G_{\{ijk\}}^{(n)}(X)$ and $G_{\{ijk\}}^{'(n)}(X)$ can not be in $g_{\{ijk\}}^{(n)}(H_j)$ and $g_{\{ijk\}}^{'(n)}(H'_j)$ respectively.

$$\begin{aligned}
G_{\{ijk\}}^{(n)}(X) &= G_{\{ijk\}}^{(n)}(f q_1^{(n)} q_2^{(n)} q_3^{(n)} \cdots q_{\varrho(f)}^{(n)}) \\
&= f G_{\{ijk\}}^{(n)}(q_1^{(n)}) G_{\{ijk\}}^{(n)}(q_2^{(n)}) \cdots G_{\{ijk\}}^{(n)}(q_{\varrho(f)}^{(n)}) \\
&= f G_{\{ijk\}}^{(n)}(q_1^{(n-1)}) G_{\{ijk\}}^{(n)}(q_2^{(n-1)}) \cdots G_{\{ijk\}}^{(n)}(q_{\varrho(f)}^{(n-1)}) \\
&= f H_{n-1}(q_1^{(n-1)}) H_{n-1}(q_2^{(n-1)}) \cdots H_{n-1}(q_{\varrho(f)}^{(n-1)}) \\
&= H_{n-1}\big(f q_1^{(n-1)} q_2^{(n-1)} q_3^{(n-1)} \cdots q_{\varrho(f)}^{(n-1)}\big) \\
&= H_{n-1}(X) = Y
\end{aligned}$$

$$\text{Hence, } Y \notin \bigcup_{(i,j,k)\in M_n} g_{\{ijk\}}^{(n)}(H_j)$$

Similarly, $G_{\{ijk\}}^{'(n)}(X) = Y$ and hence $Y \notin \bigcup_{(i,j,k)\in M'_n} g_{\{ijk\}}^{'(n)}(H'_j)$.

$$\begin{aligned}
(G_n + G'_n)(Y) &= (\sqsubset G_{n-1} \sqsupset)(Y) + (\sqsubset G'_{n-1} \sqsupset)(Y) + \sum_{(i,j,k)\in T_n} (g_{\{ijk\}}^{(n)}(H_j))(Y) + \\
&\quad \sum_{(i,j,k)\in T'_n} (g_{\{ijk\}}^{'(n)}(H'_j))(Y) \\
&= (\sqsubset G_{n-1} \sqsupset)(Y) + (\sqsubset G'_{n-1} \sqsupset)(Y) \\
(G_n + G'_n)(Y) &= (\sqsubset G_{n-1} + G'_{n-1} \sqsupset)(Y). \\
K_n(X) &= (\sqsubset K_{n-1} \sqsupset)(X) \\
&= (\sqsubset G_{n-1} + G'_{n-1} \sqsupset)(H_{n-1}(X)) = (\sqsubset G_{n-1} + G'_{n-1} \sqsupset)(Y) \\
\text{Thus, } K_n(X) &= (G_n + G'_n)(Y).
\end{aligned}$$

Case 2: Suppose that X be in $\sqsubset K_{n-1} \sqsupset$ and $\bigcup_{(i,j,k)\in T_n} h_{\{ijk\}}^{(n)}(H_j)$ but not in $\bigcup_{(i,j,k)\in T'_n} h_{\{ijk\}}^{'(n)}(H'_j)$.

In this case also, X be a terminal hyperarc and each vertex $q_i^{(n)}$ of X is equal to $q_i^{(n-1)}$. Hence, $H_n(X) = H_{n-1}(X) = Y \in (G_n + G'_n)$.
Since $X \in \bigcup_{(i,j,k)\in T_n} h_{\{ijk\}}^{(n)}(H_j)$, $X \in h_{\{ijk\}}^{(n)}(H_j)$ for some $(i, j, k) \in T_n$.

$$\begin{aligned}
\text{Let } R_n &= \big\{(i, j, k) \in T_n \mid X \in h_{\{ijk\}}^{(n)}(H_j)\big\} \\
K_n(X) &= K_{n-1}(X) + \sum_{(i,j,k)\in R_n} (h_{\{ijk\}}^{(n)}(H_j))(X)
\end{aligned}$$

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Since $X \in \sqsubset K_{n-1} \sqsupset$, $K_{n-1} \overset{H_{n-1}}{\sim} (G_{n-1} + G'_{n-1})$ and $H_n(X) = H_{n-1}(X)$, $Y \in (G_{n-1} + G'_{n-1})$. Since $X \in \bigcup_{(i,j,k)\in R_n} h^{(n)}_{\{ijk\}}(H_j)$, $h^{(n)}_{\{ijk\}}(H_j) \overset{G^{(n)}_{\{ijk\}}}{\sim} g^{(n)}_{\{ijk\}}(H_j)$ and $H_n(X) = G^{(n)}_{\{ijk\}}(X)$, $Y \in \bigcup_{(i,j,k)\in R_n} g^{(n)}_{\{ijk\}}(H_j)$. Also, $(\, h^{(n)}_{\{ijk\}}(H_j)\,)(X) = (\, g^{(n)}_{\{ijk\}}(H_j)\,)(Y)$.

Since $X \notin \bigcup_{(i,j,k)\in R'_n} h^{'(n)}_{\{ijk\}}(H'_j)$, $h^{'(n)}_{\{ijk\}}(H'_j) \overset{G^{'(n)}_{\{ijk\}}}{\sim} g^{'(n)}_{\{ijk\}}(H'_j)$ and $H_n(X) = G^{'(n)}_{\{ijk\}}(X)$, $Y \notin \bigcup_{(i,j,k)\in R'_n} g^{'(n)}_{\{ijk\}}(H'_j)$. Thus, $Y \in (\, G_{n-1} + G'_{n-1}\,)$ and $\bigcup_{(i,j,k)\in R_n} g^{(n)}_{\{ijk\}}(H_j)$.

$$\begin{aligned}(G_n + G'_n)(Y) &= (G_{n-1} + G'_{n-1})(Y) + \sum_{(i,j,k)\in R_n} (\, g^{(n)}_{\{ijk\}}(H_j)\,)(Y)\\ &= K_{n-1}(X) + \sum_{(i,j,k)\in R_n} (\, h^{(n)}_{\{ijk\}}(H_j)\,)(X)\\ &= K_n(X).\end{aligned}$$

Case 3: Suppose that X be in $\sqsubset K_{n-1} \sqsupset$ and $\bigcup_{(i,j,k)\in T'_n} h^{'(n)}_{\{ijk\}}(H'_j)$ but not in $\bigcup_{(i,j,k)\in T_n} h^{(n)}_{\{ijk\}}(H_j)$,

In this case also, X be a terminal hyperarc and each vertex $q^{(n)}_i$ of X is equal to $q^{(n-1)}_i$. Hence, $H_n(X) = H_{n-1}(X) = Y \in (G_n + G'_n)$. Since $X \in \bigcup_{(i,j,k)\in T'_n} h^{'(n)}_{\{ijk\}}(H'_j)$, $X \in h^{'(n)}_{\{ijk\}}(H'_j)$ for some $(i,j,k) \in T'_n$.

$$\begin{aligned}\text{Let } R'_n &= \{(i,j,k) \in T'_n \mid X \in h^{'(n)}_{\{ijk\}}(H'_j)\}\\ K_n(X) &= K_{n-1}(X) + \sum_{(i,j,k)\in R'_n} (\, h'^{(n)}_{\{ijk\}}(H'_j)\,)(X)\end{aligned}$$

Since $X \in \sqsubset K_{n-1} \sqsupset$, $K_{n-1} \overset{H_{n-1}}{\sim} (G_{n-1} + G'_{n-1})$ and $H_n(X) = H_{n-1}(X), Y \in (G_{n-1} + G'_{n-1})$. Since $X \in \bigcup_{(i,j,k)\in R'_n} h^{'(n)}_{\{ijk\}}(H'_j)$, $h^{'(n)}_{\{ijk\}}(H'_j) \overset{G^{'(n)}_{\{ijk\}}}{\sim} g^{'(n)}_{\{ijk\}}(H'_j)$ and $H_n(X) = G^{'(n)}_{\{ijk\}}(X)$, $Y \in \bigcup_{(i,j,k)\in R'_n} g^{'(n)}_{\{ijk\}}(H'_j)$. Also, $(\, h^{'(n)}_{\{ijk\}}(H'_j)\,)(X) = (\, g^{'(n)}_{\{ijk\}}(H'_j)\,)(Y)$.

Since $X \notin \bigcup_{(i,j,k)\in R_n} h^{(n)}_{\{ijk\}}(H_j), h^{(n)}_{\{ijk\}}(H_j) \overset{G^{(n)}_{\{ijk\}}}{\sim} g^{(n)}_{\{ijk\}}(H_j)$ and $H_n(X) = G^{(n)}_{\{ijk\}}(X)$, $Y \notin \bigcup_{(i,j,k)\in R_n} g^{(n)}_{\{ijk\}}(H_j)$. Thus, $Y \in (\, G_{n-1} + G'_{n-1}\,)$ and $\bigcup_{(i,j,k)\in R'_n} g^{'(n)}_{\{ijk\}}(H'_j)$.

$$\begin{aligned}(G_n + G'_n)(Y) &= (G_{n-1} + G'_{n-1})(Y) + \sum_{(i,j,k)\in R'_n} (\, g^{'(n)}_{\{ijk\}}(H'_j)\,)(Y)\\ &= K_{n-1}(X) + \sum_{(i,j,k)\in R'_n} (\, h^{'(n)}_{\{ijk\}}(H'_j)\,)(X)\\ &= K_n(X).\end{aligned}$$

Case 4: Suppose that X be in $\bigcup_{(i,j,k)\in T_n} h^{(n)}_{\{ijk\}}(H_j)$ and $\bigcup_{(i,j,k)\in T'_n} h^{'(n)}_{\{ijk\}}(H'_j)$ but not in $\sqsubset K_{n-1} \sqsupset$,

Since $X \in \bigcup_{(i,j,k)\in T_n} h^{(n)}_{\{ijk\}}(H_j)$, $X \in h^{(n)}_{\{ijk\}}(H_j)$ for some $(i,j,k) \in T_n$.

$$\begin{aligned}\text{Let } A_n &= \left\{(i,j,k) \in T_n \mid X \in h^{(n)}_{\{ijk\}}(H_j)\right\}\\ \text{Let } B_n &= \left\{Q^{(n-1)}_i \in S_n \mid (i,j,k) \in A_n\right\}\end{aligned}$$

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Since $X \in \bigcup_{(i,j,k)\in T'_n} h^{'(n)}_{\{ijk\}}(H'_j)$, $X \in h^{'(n)}_{\{ijk\}}(H'_j)$ for some $(i, j, k) \in T'_n$.

$$\begin{aligned} \text{Let } A'_n &= \left\{(i, j, k) \in T'_n \mid X \in h^{'(n)}_{\{ijk\}}(H'_j)\right\} \\ \text{Let } B'_n &= \left\{Q^{'(n-1)}_i \in S'_n \mid (i, j, k) \in A'_n\right\} \end{aligned}$$

In this case, X can not be a non-terminal hyperarc and the vertices of X are the vertices that are common to both G_0 and G'_0. The i^{th} vertex of each non-terminal hyperarc in B'_n, the i^{th} vertex of each non-terminal hyperarc in B_n and the i^{th} vertex of X are identical. By induction hypothesis, $K_{n-1} \sim (G_{n-1} + G'_{n-1})$. Therefore, for every non-terminal hyperarc $Q^{(n-1)}_i$ in B_n, we can find a arc say $X^{(n-1)}_i$ in G_{n-1} such that $X^{(n-1)}_i = H_{n-1}(Q^{(n-1)}_i)$. In the same way, for every non-terminal hyperarc $Q^{'(n-1)}_i$ in B'_n we can find a hyperarc say $X^{'(n-1)}_i$ in G'_{n-1} such that $X^{'(n-1)}_i = H_{n-1}(Q^{'(n-1)}_i)$
Since $X^{(n-1)}_i$ and $X^{'(n-1)}_i$ are the corresponding arcs of $Q^{(n-1)}_i$ and $Q^{'(n-1)}_i$, the i^{th} vertex of each non-terminal hyperarc in $\mathcal{L}_n$ is identical with the i^{th} vertex of each non-terminal hyperarc in $\mathcal{L}'_n$ where

$$\begin{aligned} \mathcal{L}_n &= \left\{X^{(n-1)}_i \in L_n \mid H^{-1}_{n-1}(X^{(n-1)}_i) \in B_n\right\} \\ \mathcal{L}'_n &= \left\{X^{'(n-1)}_i \in L'_n \mid H^{-1}_{n-1}(X^{'(n-1)}_i) \in B'_n\right\} \end{aligned}$$

Since each $h^{(n)}_{\{ijk\}}(H_j)$ is isomorphic to $g^{(n)}_{\{ijk\}}(H_j)$ under the transformation $G^{(n)}_{ijk}$ and each $h^{'(n)}_{\{ijk\}}(H'_j)$ is isomorphic to $g^{'(n)}_{\{ijk\}}(H'_j)$ under the transformation $G'^{(n)}_{ijk}$, $G^{(n)}_{ijk}(X) \in \bigcup_{(i,j,k)\in A_n} g^{(n)}_{ijk}(H_j)$ and $G^{'(n)}_{ijk}(X) \in \bigcup_{(i,j,k)\in A'_n} g^{'(n)}_{ijk}(H'_j)$. Since the vertices of X are the vertices of $Q^{(n-1)}_i$ and the vertices of $Q^{(n-1)}_i$ are the vertices of $Q^{'(n-1)}_i$, $X = f q^{(n-1)}_1 q^{(n-1)}_2 \cdots q^{(n-1)}_{\varrho(f)}$ and so $G^{(n)}_{ijk}(X) = H_{n-1}(X) = Y = G^{'(n)}_{ijk}(X)$. Thus, $Y \in \bigcup_{(i,j,k)\in A_n} g^{(n)}_{\{ijk\}}(H_j)$ and $Y \in \bigcup_{(i,j,k)\in A'_n} g^{'(n)}_{\{ijk\}}(H'_j)$. Hence, $Y \in (G_n + G'_n)$

$$\begin{aligned} (G_n + G'_n)Y &= \sum_{(i,j,k)\in A_n} (g^{(n)}_{\{ijk\}}(H_j))(Y) + \sum_{(i,j,k)\in A'_n} (g^{'(n)}_{\{ijk\}}(H'_j))(Y) \\ H_n(X) &= H_n(f q^{(n-1)}_1 q^{(n-1)}_2 q^{(n-1)}_3 \cdots q^{(n-1)}_{\varrho(f)}) = Y \\ K_n(X) &= \sum_{(i,j,k)\in A_n} (h^{(n)}_{\{ijk\}}(H_j))(X) + \sum_{(i,j,k)\in A'_n} (h^{'(n)}_{\{ijk\}}(H'_j))(X) \\ K_n(X) &= \sum_{(i,j,k)\in A_n} (g^{(n)}_{\{ijk\}}(H_j))(Y) + \sum_{(i,j,k)\in A'_n} (g^{'(n)}_{\{ijk\}}(H'_j))(Y) \\ K_n(X) &= (G_n + G'_n)Y \end{aligned}$$

■

Example 5. Let us consider an initial graph G_0 and the production P_1 of the grammar $R_1 = (G_0, P_1)$ as given below:

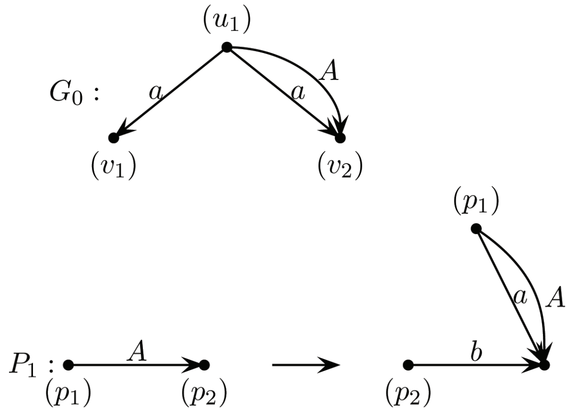

FIGURE 1. Grammar R_1

The first three steps of derivation according to the grammar R_1 are shown in the following figure 2.

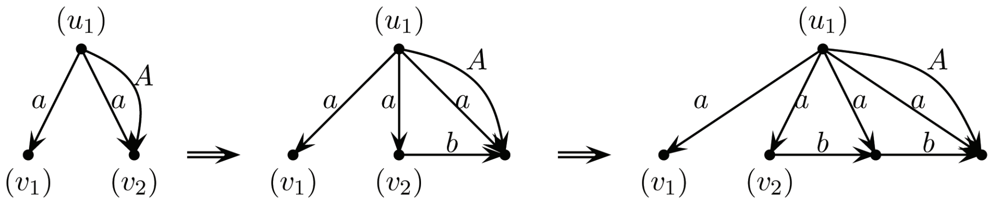

FIGURE 2. Steps of Derivation using R_1

Let us consider the grammar $R_2 = (G_0', P_2)$ as given in figure 3.

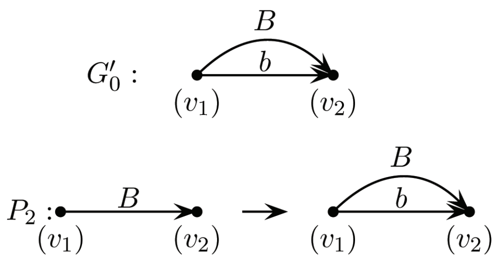

FIGURE 3. Grammar R_2

The first three steps of derivation from G_0' using the grammar R_2 are shown in the following figure 4.

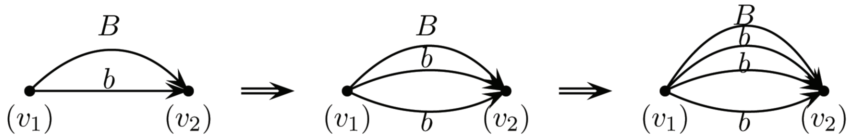

FIGURE 4. Steps of Derivation of R_2

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The sum ($R = R_1 + R_2$) of R_1 and R_2 is constructed in the next figure 5.

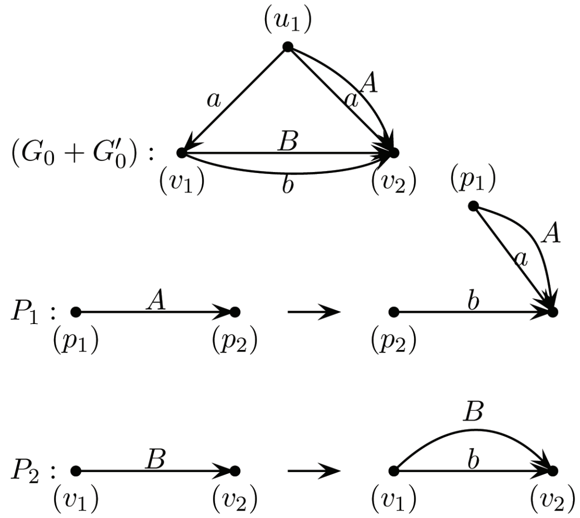

FIGURE 5. Sum of R_1 and R_2

The first three steps of derivation according to the grammar $R = (R_1 + R_2)$ are shown in figure 6. In the derivation given below, each K_n is the sum of $(G_n + G_n')$ and the graph language of the grammar $R = R_1 + R_2$ is the family of Fan Graphs $F_{1,m}$.

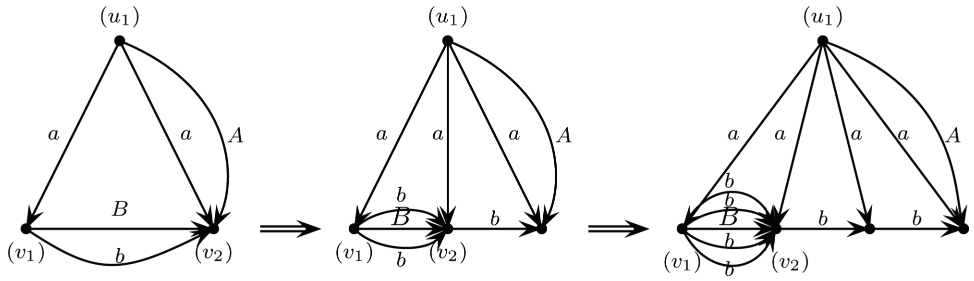

FIGURE 6. Steps of derivation using grammar R

Theorem 6. Let $R_1(G_0)$ and $R_2(G_0')$ be the multi-hypergraph languages of the grammars $R_1 = (G_0, P_1)$ and $R_2 = (G_0', P_2)$ respectively with $N_{R_1} \cap N_{R_2} = \emptyset$. If $(V(G_i) - V(G_0)) \cap V(G_j') = \emptyset\ \forall j \geq 0, i \geq 1$ and $(V(G_j') - V(G_0')) \cap V(G_i) = \emptyset \forall j \geq 1, i \geq 0$ then $\{ \bigcup_{n \geq 0} \sqsubset K_n \sqsupset \mid K_n \Rightarrow K_{n+1}\ \ \forall n \geq 0\}$ be the language of the grammar $R = R_1 \cup R_2$ with the following property.

$$\sqsubset K_0 \sqsupset \;=\; \sqsubset\big((G_0 + G_0') - \{(X, (G_0 + G_0')(X)) \in (G_0 + G_0') \mid X \in G_0 \cap G_0'\}\big)\sqsupset + (C_0 \cup C_0')$$

$$\sqsubset K_n \sqsupset \;\sim\; \sqsubset\big((G_n + G_n') - \{(X, (G_n + G_n')(X)) \in (G_n + G_n') \mid X \in G_0 \cap G_0'\}\big)\sqsupset + (C_0 \cup C_0') + \sum_{l=1}^{n} \sum_{(i,j,k) \in M_l} C_{ijk}^{(l)} + \sum_{m=1}^{n} \sum_{(i,j,k) \in M_m'} C_{ijk}^{'(m)}\ \ \forall n \geq 1 \text{ where}$$

$$C_0 = \{(X, G_0(X)) \in G_0 \mid X \in G_0 \cap G_0'\}$$
$$C_0' = \{(X, G_0'(X)) \in G_0' \mid X \in G_0 \cap G_0'\}$$
$$C_{ijk}^{(l)} = \big\{\big(X, (g_{ij(k)}^{(l)}(H_j))(X)\big) \in g_{ijk}^{(l)}(H_j) \mid X \in G_0 \cap G_0'\big\} \text{ and}$$
$$C_{ijk}^{'(m)} = \big\{\big(X, g_{ij(k)}^{'(m)}(H_j'))(X)\big) \in g_{ijk}^{'(m)}(H_j') \mid X \in G_0 \cap G_0'\big\}.$$

Proof. Let n be the number of steps of parallel derivation and K_0 be an initial multi-hypergraph of R. We can prove the result $K_n \sim K'_n$ by the method of induction on n by proceeding as same as in theorem 4 where $K_n = (G_0 \cup G'_0) + \sum_{l=1}^{n} \sum_{(i,j,k)\in T_l} h^{(l)}_{ijk}(H_j) + \sum_{m=1}^{n} \sum_{(i,j,k)\in T'_m} h'^{(m)}_{ijk}(H'_j)$ and $K'_n = (G_0 \cup G'_0) + \sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} g^{(l)}_{ijk}(H_j) + \sum_{m=1}^{n} \sum_{(i,j,k)\in M'_m} g'^{(m)}_{ijk}(H'_j)$.

For the case $n = 0$, let H_0 be an isomorphism exists between K_0 and K'_0.

$$\big((G_0 + G'_0) - \{(X, (G_0 + G'_0)(X)) \in (G_0 + G'_0) \mid X \in G_0 \cap G'_0\} \big) + (C_0 \cup C'_0)$$

$$\begin{aligned}
&= \big(G_0 - \{(X, G_0(X)) \in G_0 \mid X \in G_0 \cap G'_0\} \big) + (C_0 \cup C'_0) + \\
&\quad \big(G'_0 - \{(X', G'_0(X)) \in G'_0 \mid X' \in G'_0 \cap G_0\} \big) \\
&= \big\{(X, G_0(X)) \in G_0 \mid X \in G_0 \wedge X \notin G'_0\big\} \cup \big\{(X', G'_0(X')) \in G'_0 \mid \\
&\quad X' \in G'_0 \wedge X' \notin G_0\big\} \cup \big\{ (X, \vee(G_0(X), G'_0(X))) \mid X \in G_0 \cap G'_0\big\} \\
&= G_0 \cup G'_0 = K'_0
\end{aligned}$$

Thus, $K_0 \sim \big((G_0 + G'_0) - \{(X, (G_0 + G'_0)(X)) \in (G_0 + G'_0) \mid X \in G_0 \cap G'_0\} \big) +$
$(C_0 \cup C'_0)$.

For the case $n = 1$, let H_1 be an isomorphism exist between K_1 and K'_1.

$$\Big((G_1 + G'_1) - \big\{(X, (G_1 + G'_1)(X)) \in (G_1 + G'_1) \mid X \in G_0 \cap G'_0\big\} \Big)$$

$$\begin{aligned}
&= \Big(G_1 - \big\{(X, G_1(X)) \in G_1 \mid X \in G_0 \cap G'_0\big\} \Big) + \\
&\quad \Big(G'_1 - \big\{(X', G'_1(X')) \in G'_1 \mid X' \in G'_0 \cap G_0\big\} \Big) \\
&= \Big(\sqsubset G_0 \sqsupset - \big\{(X, G_0(X)) \in G_0 \mid X \in G_0 \cap G'_0\big\} \Big) + \\
&\quad \Big(\sqsubset G'_0 \sqsupset - \big\{(X', G'_0(X)) \in G'_0 \mid X' \in G'_0 \cap G_0\big\} \Big) + \\
&\quad \sum_{(i,j,k)\in M_1} \Big(g^{(1)}_{\{ijk\}}(H_j) - \big\{(X, g^{(1)}_{\{ijk\}}(X)) \in g^{(1)}_{\{ijk\}}(H_j) \mid X \in G_0 \cap G'_0\big\} \Big) + \\
&\quad \sum_{(i,j,k)\in M'_1} \Big(g'^{(1)}_{\{ijk\}}(H'_j) - \big\{ (X', g'^{(1)}_{\{ijk\}}(X')) \in g'^{(1)}_{\{ijk\}}(H_j) \mid X' \in G'_0 \cap G_0 \big\} \Big) \\
&= (\sqsubset G_0 \sqsupset - C_0) + (\sqsubset G'_0 \sqsupset - C'_0) + \sum_{(i,j,k)\in M_1} (g^{(1)}_{\{ijk\}}(H_j) - C^{(1)}_{ijk}) + \\
&\quad \sum_{(i,j,k)\in M'_1} (g'^{(1)}_{\{ijk\}}(H'_j) - C'^{(1)}_{ijk})
\end{aligned}$$

Now $\Big((G_1 + G'_1) - \big\{(X, (G_1 + G'_1)(X)) \in (G_1 + G'_1) \mid X \in G_0 \cap G'_0\big\} \Big) + (C_0 \cup C'_0)$
$\sum_{(i,j,k)\in M_1} C^{(1)}_{ijk} + \sum_{(i,j,k)\in M'_1} C'^{(1)}_{ijk}$

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$$= (\sqsubset G_0 \sqsupset - C_0) + (\sqsubset G'_0 \sqsupset - C'_0) + (C_0 \cup C'_0) + \sum_{(i,j,k)\in M_1} (g^{(1)}_{\{ijk\}}(H_j) - C^{(1)}_{ijk}) +$$
$$\sum_{(i,j,k)\in M'_1} (g'^{(1)}_{\{ijk\}}(H'_j) - C'^{(1)}_{ijk}) + \sum_{(i,j,k)\in M_1} C^{(1)}_{ijk} + \sum_{(i,j,k)\in M'_1} C'^{(1)}_{ijk}$$
$$= (\sqsubset G_0 \sqsupset \cup \sqsubset G'_0 \sqsupset) + \sum_{(i,j,k)\in M_1} g^{(1)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_1} g'^{(1)}_{\{ijk\}}(H'_j)$$
$$= K'_1$$
$$K_1 \sim \sqsubset\Big((G_1 + G'_1) - \big\{(X, (G_1 + G'_1)(X)) \in (G_1 + G'_1) \mid$$
$$X \in G_0 \cap G'_0\big\}\Big)\sqsupset + \sum_{(i,j,k)\in M_1} C^{(1)}_{ijk} + \sum_{(i,j,k)\in M'_1} C'^{(1)}_{ijk} + (C_0 \cup C'_0) \text{ (Since} K_1 \overset{H_1}{\sim} K'_1)$$

By induction we assume that the result is true for all multi-hypergraphs that are generated in fewer than n steps. (i.e.,) $\sqsubset K'_{n-1} \sqsupset = \sqsubset\Big((G_{n-1} + G'_{n-1}) - \big\{(X, (G_{n-1} + G'_{n-1})(X)) \in (G_{n-1} + G'_{n-1}) \mid$
$X \in G_0 \cap G'_0\big\}\Big)\sqsupset + \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} C^{(l)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} C'^{(m)}_{ijk} + (C_0 \cup C'_0)$
For the n^{th} case, let H_n be an isomorphism exists between K_n and K'_n.

$$K'_n = \sqsubset K'_{n-1} \sqsupset + \sum_{(i,j,k)\in M_n} g^{(l)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_n} g'^{(m)}_{\{ijk\}}(H'_j)$$
$$K'_n = \sqsubset(G_{n-1} + G'_{n-1}) - \big\{(X, (G_{n-1} + G'_{n-1})(X)) \in (G_{n-1} + G'_{n-1}) \mid$$
$$X \in G_0 \cap G'_0\big\}\sqsupset + \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} C^{(l)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} C'^{(m)}_{ijk} + (C_0 \cup C'_0)$$
$$\sum_{(i,j,k)\in M_n} g^{(n)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_n} g'^{(n)}_{\{ijk\}}(H'_j)$$
$$= \sqsubset\Big(G_{n-1} - \big\{(X, G_{n-1}(X)) \in G_{n-1} \mid X \in G_0 \cap G'_0\big\}\Big)\sqsupset +$$
$$\sqsubset\Big(G'_{n-1} - \big\{(X', G'_{n-1}(X')) \in G'_{n-1} \mid X' \in G'_0 \cap G_0\big\}\Big)\sqsupset +$$
$$\sum_{(i,j,k)\in M_n} g^{(n)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_n} g'^{(n)}_{\{ijk\}}(H'_j) + (C_0 \cup C'_0) +$$
$$\sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} C^{(l)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} C'^{(m)}_{ijk}$$
$$= \sqsubset\Big(G_{n-1} - \big\{(X, G_{n-1}(X)) \in G_{n-1} \mid X \in G_0 \cap G'_0\big\} \Big)\sqsupset +$$
$$\sqsubset\Big(G'_{n-1} - \big\{(X', G'_{n-1}(X')) \in G'_{n-1} \mid X' \in G'_0 \cap G_0\big\} \Big)\sqsupset +$$
$$\sum_{(i,j,k)\in M_n} \big(g^{(n)}_{\{ijk\}}(H_j) - C^{(n)}_{ijk}\big) + \sum_{(i,j,k)\in M'_n} \big(g'^{(n)}_{\{ijk\}}(H'_j) - C'^{(n)}_{ijk}\big) + (C_0 \cup C'_0) +$$
$$\sum_{(i,j,k)\in M_n} C^{(n)}_{ijk} + \sum_{(i,j,k)\in M'_n} C'^{(n)}_{ijk} + \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} C^{(l)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} C'^{(m)}_{ijk}$$

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$$= \Big(G_n - \big\{(X, G_n(X)) \in G_n \mid X \in G_0 \cap G_0'\big\} \Big) + \sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} C_{ijk}^{(l)} + (C_0 \cup C_0')$$

$$+ \Big(G_n' - \big\{(X', G_n'(X')) \in G_n' \mid X' \in G_0' \cap G_0\big\} \Big) + \sum_{m=1}^{n} \sum_{(i,j,k)\in M_m'} C'^{(m)}_{ijk}$$

$$= \Big((G_n + G_n') - \big\{(X, (G_n + G_n')(X)) \in (G_n + G_n') \mid X \in G_0 \cap G_0'\big\} \Big) +$$

$$\sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} C_{ijk}^{(l)} + \sum_{m=1}^{n} \sum_{(i,j,k)\in M_m'} C'^{(m)}_{ijk} + (C_0 \cup C_0')$$

Hence proved. ■

Example 7. Let us consider a hypergraph grammar (R_1, G_0) as in the following figure 7.

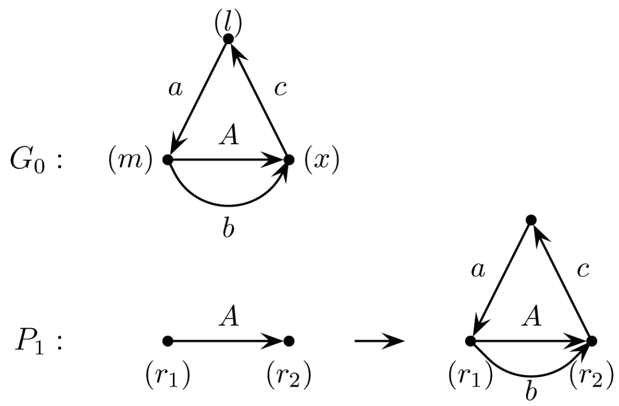

FIGURE 7. Hypergraph grammar R_1

The first three steps of parallel derivation from G_0 using R_1 are shown in the following graph 8.

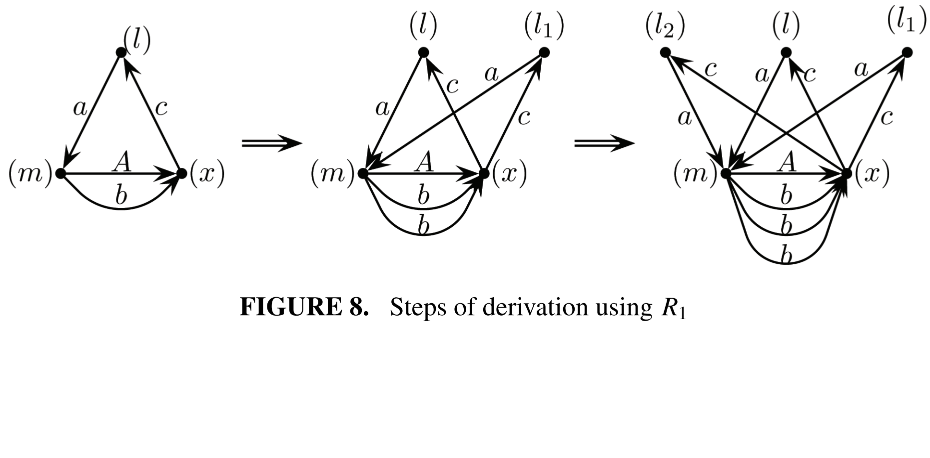

FIGURE 8. Steps of derivation using R_1

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Let us consider a hypergraph grammar (R_2, G'_0) as shown in the following figure 9:

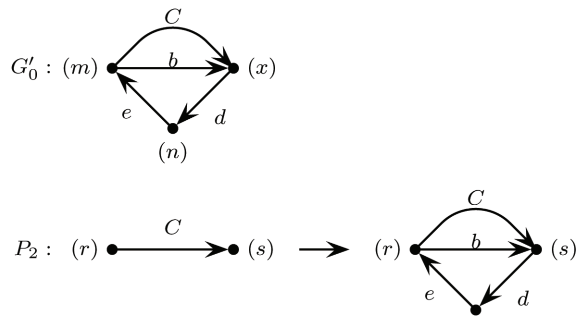

FIGURE 9. Hypergraph grammar R_2

The first three steps of derivation from G'_0 using the grammar R_2 are shown in figure 10.

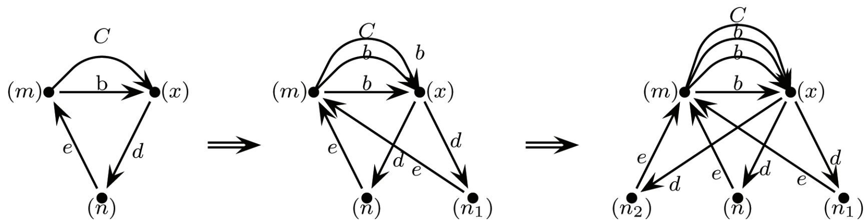

FIGURE 10. Steps of derivation from G'_0 using R_2

The graphs that are derived by the grammar $R = R_1 \cup R_2$ in the first three steps are shown in the following figure 11.

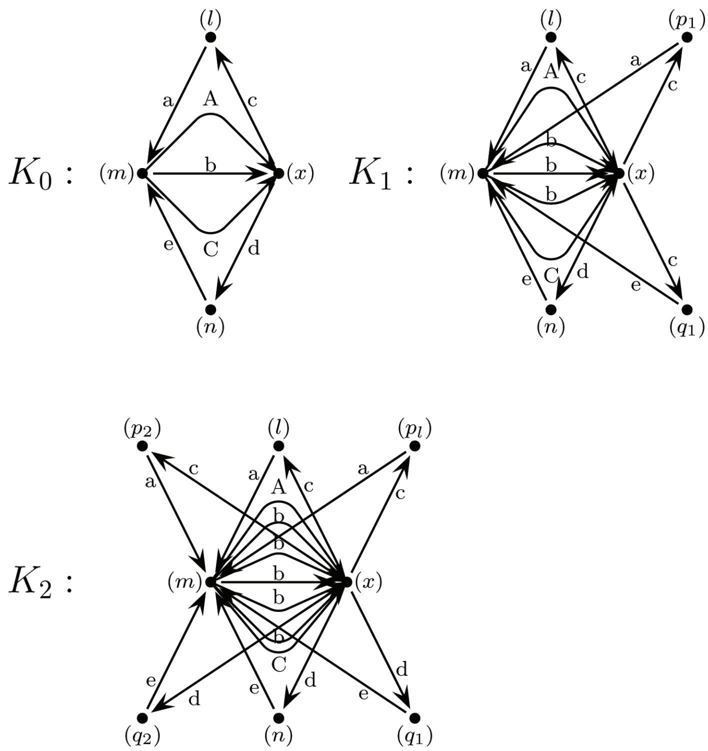

FIGURE 11. Derivation from K_0 using grammar R

Theorem 8. Let $R_1(G_0)$ and $R_2(G'_0)$ be the multi-hypergraph languages of the grammar $R_1 = (G_0, P_1)$ and $R_2 = (G'_0, P_2)$ respectively with $N_{R_1} \cap N_{R_2} = \emptyset$. If $(V(G_i) - V(G_0)) \cap V(G'_j) = \emptyset \ \forall j \geq 0, i \geq 1$ and $(V(G'_j) - V(G'_0)) \cap V(G_i) = \emptyset \forall j \geq 1, i \geq 0$ then $\big\{ \bigcup_{n\geq 0} \sqsubset K_n \sqsupset \mid K_n \Rightarrow K_{n+1} \ \ \forall n \geq 0 \big\}$ be the language of $R = R_1 \cap R_2$ with the following property.

$$\sqsubset K_0 \sqsupset = \sqsubset\Big((G_0 + G'_0) - \big\{(X, (G_0 + G'_0)(X)) \in (G_0 + G'_0) \mid X \in G_o \cup G'_0, X(1) \in T_R\big\}\Big)\sqsupset + (C_0 \cap C'_0) \text{ and}$$

$$\sqsubset K_n \sqsupset \sim \sqsubset\Big((G_n + G'_n) - \big\{(X, (G_n + G'_n)(X)) \in (G_n + G'_n) \mid X \in G_o \cup G'_0, X(1) \in T_R\big\}\Big)\sqsupset + (C_0 \cap C'_0) + \sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \sum_{m=1}^{n} \sum_{(i,j,k)\in M'_m} D'^{(m)}_{ijk} \ \forall n \geq 1 \text{ where}$$

$$C_0 = \big\{(X, G_0(X)) \in G_0 \mid X \in G_0 \cap G'_0\big\}$$
$$C'_0 = \big\{(X', G'_0(X)) \in G'_0 \mid X' \in G'_0 \cap G_0\big\}$$
$$D^{(l)}_{ijk} = \big\{(X, g^{(l)}_{ijk}(H_j)(X)) \in g^{(l)}_{ijk}(H_j) \mid X \in G_0, X(1) \in T_{R_1}\big\} \text{ and}$$
$$D'^{(m)}_{ijk} = \big\{(X', g'^{(m)}_{ijk}(H'_j)(X')) \in g'^{(m)}_{ijk}(H'_j) \mid X' \in G'_0, X'(1) \in T_{R_2}\big\}$$

Proof. Let n be the number of steps of parallel derivation and K_0 be an initial multi-hypergraph of R. We can prove that $K_n \sim K'_n$ by the method of induction on n by proceeding as same as in theorem 4 where

$K_n = (G_0 \cap G'_0) + \sum_{l=1}^{n} \sum_{(i,j,k)\in T_l} h^{(l)}_{ijk}(H_j) + \sum_{m=1}^{n} \sum_{(i,j,k)\in T'_m} h'^{(m)}_{ijk}(H'_j)$ and

$K'_n = (G_0 \cap G'_0) + \sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} g^{(l)}_{ijk}(H_j) + \sum_{m=1}^{n} \sum_{(i,j,k)\in M'_m} g'^{(m)}_{ijk}(H'_j)$.

For $n = 0$, H_0 be an isomorphism exist between K_0 and K'_0 and it will be as follows:

$$H_0(q^{(0)}_i) = \begin{cases} u^{(0)}_i, & \text{if } q^{(0)}_i = u^{(0)}_i \\ v^{(0)}_i, & \text{if } q^{(0)}_i = v^{(0)}_i \\ w^{(0)}_i, & \text{if } q^{(0)}_i = w^{(0)}_i \end{cases}$$

$$\text{where} \quad q^{(0)}_i = \begin{cases} u^{(0)}_i, & \text{if } q^{(0)}_i \in V(K_0) - V(G'_0) \\ v^{(0)}_i, & \text{if } q^{(0)}_i \in V(K_0) - V(G_0) \\ w^{(0)}_i, & \text{if } q^{(0)}_i \in V(K_0) \cap V(G_0) \cap V(G'_0) \end{cases}$$

$$\big((G_0 + G'_0) - \{(X, (G_0 + G'_0)(X)) \in (G_0 + G'_0) \mid X \in G_0 \cup G'_0, X(1) \in T_R\}\big) + (C_0 \cap C'_0)$$
$$= \big(G_0 - \{(X, G_0(X)) \in G_0 \mid X(1) \in T_{R_1}\}\big) + \big(G'_0 - \{(X', G'_0(X)) \in G'_0 \mid X(1) \in T_{R_2}\}\big) + (C_0 \cap C'_0)$$
$$= \big\{ (X, G_0(X)) \in G_0 \mid X(1) \in N_{R_1}\big\} \cup \big\{ (X', G'_0(X')) \in G'_0 \mid \in N_{R_2}\big\} \cup \big\{(X, \wedge(G_0(X), G'_0(X)) \mid X \in G_0 \cap G'_0\big\}$$
$$= K_0$$

Thus, the result is true for the case $n = 0$. For the case $n = 1$, let H_1 be an isomorphism exists between K_1 and K'_1 and it will be as follows:

$$H_1(q^{(1)}_c) = \begin{cases} H_0(q^{(0)}_c), & \text{if } q^{(1)}_c = q^{(0)}_c \\ a^{(1)}_c, & \text{if } q^{(1)}_c = d^{(1)}_c \\ a'^{(1)}_c, & \text{if } q^{(1)}_c = d'^{(1)}_c \end{cases}$$

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$$\text{where } q_c^{(1)} = \begin{cases} q_c^{(0)}, & \text{if } q_c^{(1)} \in V(\sqsubset K_0 \sqsupset) \\ d_c^{(1)}, & \text{if } q_c^{(1)} \in \bigcup\limits_{(i,j,k)\in T_1} V\big(h^{(1)}_{\{ijk\}}(H_j)\big) - V(\sqsubset K_0 \sqsupset) \\ d_c^{\prime(1)}, & \text{if } q_c^{(1)} \in \bigcup\limits_{(i,j,k)\in T_1'} V\big(h'^{(1)}_{\{ijk\}}(H_j')\big) - V(\sqsubset K_0 \sqsupset) \end{cases}$$

$$\Big((G_1+G_1') - \big\{(X,(G_1+G_1')(X)) \in (G_1+G_1') \mid X \in G_0 \cup G_0', X(1) \in T_R\big\}\Big)$$
$$= \Big(G_1 - \big\{(X, G_1(X)) \in G_1 \mid X \in G_0, X(1) \in T_{R_1}\big\}\Big) + \Big(G_1' - \big\{(X', G_1'(X')) \in G_1' \mid X' \in G_0', X'(1) \in T_{R_2}\big\}\Big)$$
$$= \sum_{(i,j,k)\in M_1} \big(g^{(1)}_{\{ijk\}}(H_j) - D^{(1)}_{ijk}\big) + \sum_{(i,j,k)\in M_1'} \big(g'^{(1)}_{\{ijk\}}(H_j') - D'^{(1)}_{ijk}\big)$$

Now $\Big((G_1 + G_1') - \big\{(X,(G_1+G_1')(X)) \in (G_1+G_1') \mid X \in G_0 \cup G_0', X(1) \in T_R\big\}\Big) + \sum\limits_{(i,j,k)\in M_1} D^{(1)}_{ijk} + \sum\limits_{(i,j,k)\in M_1'} D'^{(1)}_{ijk} + (C_0 \cap C_0')$

$$= \sum_{(i,j,k)\in M_1} \big(g^{(1)}_{\{ijk\}}(H_j) - D^{(1)}_{ijk}\big) + \sum_{(i,j,k)\in M_1'} \big(g'^{(1)}_{\{ijk\}}(H_j') - D'^{(1)}_{ijk}\big) + \sum_{(i,j,k)\in M_1} D^{(1)}_{ijk} + \sum_{(i,j,k)\in M_1'} D'^{(1)}_{ijk} + (C_0 \cap C_0')$$
$$= (C_0 \cap C_0') + \sum_{(i,j,k)\in M_1} \big(g^{(1)}_{\{ijk\}}(H_j)\big) + \sum_{(i,j,k)\in M_1'} \big(g'^{(1)}_{\{ijk\}}(H_j')\big)$$
$$= K_1'$$
$$K_1 \sim \Big((G_1+G_1') - \big\{(X,(G_1+G_1')(X)) \in (G_1+G_1') \mid X \in G_0 \cup G_0', X(1) \in T_R\big\}\Big) + \sum_{(i,j,k)\in M_1} D^{(1)}_{ijk} + \sum_{(i,j,k)\in M_1'} D'^{(1)}_{ijk} + (C_0 \cap C_0') \;(\text{Since} K_1 \overset{H_1}{\sim} K_1')$$

By induction, we assume that the result is true for all multi-hypergraphs that are generated in fewer than n steps.

$$K_{n-1}' \sim \Big((G_{n-1} + G_{n-1}') - \big\{(X,(G_{n-1}+G_{n-1}')(X)) \in (G_{n-1}+G_{n-1}') \mid X \in G_0 \cup G_0', X(1) \in T_R\big\}\Big) + \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M_m'} D'^{(m)}_{ijk} + (C_0 \cap C_0')$$

For the n^{th} case, let H_n be an isomorphism exists between K_n and K_n' and it will be as follows:

$$H_n(q_c^{(n)}) = \begin{cases} H_{n-1}(q_c^{(n-1)}), & \text{if } q_c^{(n)} = q_c^{(n-1)} \\ a_c^{(n)}, & \text{if } q_c^{(n)} = d_c^{(n)} \\ a'^{(n)}_c, & \text{if } q_c^{(n)} = d'^{(n)}_c \end{cases}$$

$$\text{where } q_c^{(n)} = \begin{cases} q_c^{(n-1)}, & \text{if } q_c^{(n)} \in V(\sqsubset K_{n-1} \sqsupset) \\ d_c^{(n)}, & \text{if } q_c^{(n)} \in \bigcup\limits_{(i,j,k)\in T_n} V\big(h^{(n)}_{\{ijk\}}(H_j)\big) - V(\sqsubset K_{n-1} \sqsupset) \\ d'^{(n)}_c, & \text{if } q_c^{(n)} \in \bigcup\limits_{(i,j,k)\in T_n'} V\big(h'^{(n)}_{\{ijk\}}(H_j')\big) - V(\sqsubset K_{n-1} \sqsupset) \end{cases}$$

$$
\begin{aligned}
K'_n &= \sqsubset K'_{n-1} \sqsupset + \sum_{(i,j,k)\in M_n} g^{(n)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_n} g'^{(n)}_{\{ijk\}}(H'_j) \\
&= \sqsubset\Big((G_{n-1} + G'_{n-1}) - \big\{(X, (G_{n-1} + G'_{n-1})(X)) \in (G_{n-1} + G'_{n-1}) \mid \\
&\quad X \in G_0 \cup G'_0, X(1) \in T_R\big\}\Big)\sqsupset + \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} D'^{(m)}_{ijk} + \\
&\quad (C_0 \cap C'_0) + \sum_{(i,j,k)\in M_n} g^{(n)}_{\{ijk\}}(H_j) + \sum_{(i,j,k)\in M'_n} g'^{(n)}_{\{ijk\}}(H'_j) \\
&= \sqsubset\Big(G_{n-1} - \big\{(X, G_{n-1}(X)) \in G_{n-1} \mid X \in G_0, X(1) \in T_{R_1}\big\}\Big)\sqsupset + \\
&\quad \sqsubset\Big(G'_{n-1} - \big\{(X', G'_{n-1}(X')) \in G'_{n-1} \mid X' \in G'_0, X'(1) \in T_{R_2}\big\}\Big)\sqsupset + \\
&\quad \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \sum_{(i,j,k)\in M_n} g^{(n)}_{\{ijk\}}(H_j) + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} D'^{(m)}_{ijk} + \\
&\quad \sum_{(i,j,k)\in M'_n} g'^{(n)}_{\{ijk\}}(H'_j) + +(C_0 \cap C'_0) \\
&= \sqsubset\Big(G_{n-1} - \big\{(X, G_{n-1}(X)) \in G_{n-1} \mid X \in G_0, X(1) \in T_{R_1}\big\}\Big)\sqsupset + \\
&\quad \sum_{(i,j,k)\in M_n} \big(g^{(n)}_{\{ijk\}}(H_j) - D^{(n)}_{ijk}\big) + \sum_{(i,j,k)\in M_n} D^{(n)}_{ijk} + \sum_{l=1}^{n-1} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \\
&\qquad \sqsubset\Big(G'_{n-1} - \big\{(X', G'_{n-1}(X')) \in G'_{n-1} \mid X' \in G'_0, X'(1) \in T_{R_2}\big\}\Big)\sqsupset + \\
&\quad \sum_{(i,j,k)\in M'_n} \big(g'^{(n)}_{\{ijk\}}(H'_j) - D'^{(n)}_{ijk}\big) + \sum_{(i,j,k)\in M'_n} D'^{(n)}_{ijk} + \sum_{m=1}^{n-1} \sum_{(i,j,k)\in M'_m} D'^{(m)}_{ijk} + \\
&\quad (C_0 \cap C'_0) \\
&= \Big(G_n - \big\{(X, G_n(X)) \in G_n \mid X \in G_0 \cup G'_0, X(1) \in T_{R_1}\big\}\Big) + \\
&\quad \Big(G'_n - \big\{(X', G'_n(X')) \in G'_n \mid X' \in G'_0 \cup G_0, X'(1) \in T_{R_2}\big\}\Big) + \\
&\quad \sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \sum_{m=1}^{n} \sum_{(i,j,k)\in M'_m} D'^{(m)}_{ijk} + (C_0 \cap C'_0) \\
&= (G_n + G'_n) - \big\{(X, (G_n + G'_n)(X)) \in (G_n + G'_n) \mid X \in G_0 \cup G'_0, \\
&\quad X(1) \in T_R\big\} + \sum_{l=1}^{n} \sum_{(i,j,k)\in M_l} D^{(l)}_{ijk} + \sum_{m=1}^{n} \sum_{(i,j,k)\in M'_m} D'^{(m)}_{ijk} + (C_0 \cap C'_0)
\end{aligned}
$$

Hence proved. ■

Example 9. Let us consider the grammars R_1 and R_2 given in the figures 7 and 9. The grammar $R = R_1 \cap R_2$ is constructed in figure 12.

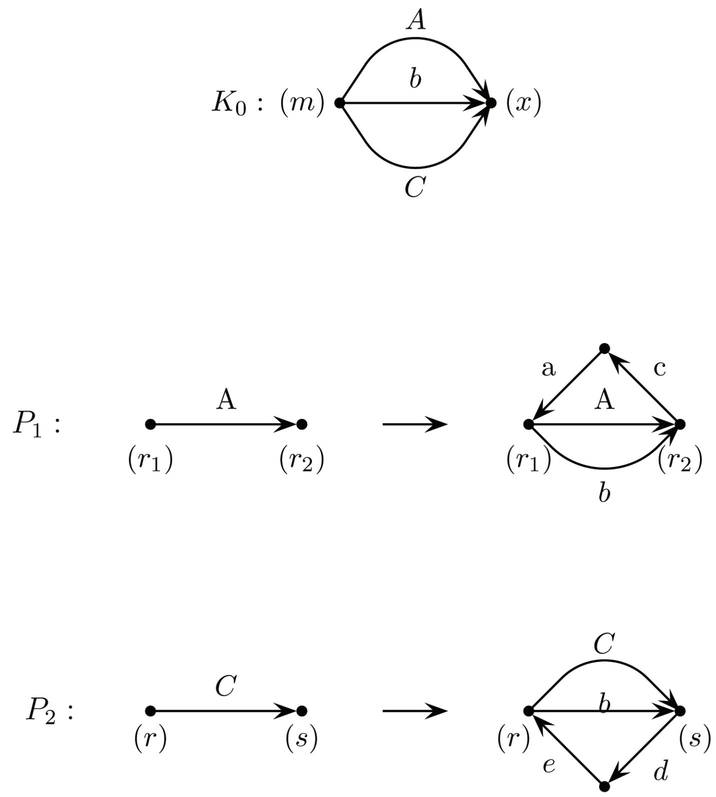

FIGURE 12. Hypergraph grammar R_2

The first three steps of parallel derivation from K_0 using the grammar R be given in the following figure. Each K_i satisfies the theorem statement.

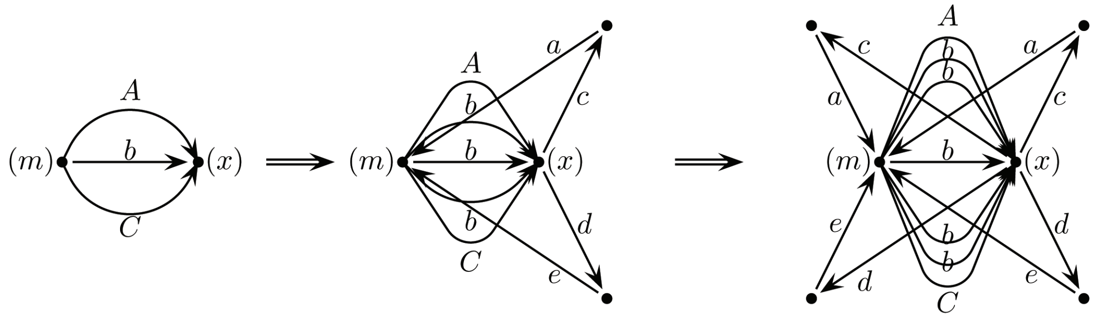

FIGURE 13. Steps of derivation using the grammar R of figure 12

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CONCLUSION

The study has defined operations over the grammars and proved some results on the existence of isomorphism between the languages of given grammars and the newly defined grammar with examples.

REFERENCES

[1] Annegret Habel and Hans - Jorg Kreowski, 1987, Characteristics of graph languages generated by edge replacement, Theoretical Computer Science, 51, 81–115.
[2] Annegret Habel and Hans - Jorg Kreowski, 1992, Hyperedge replacement: grammars and languages, Springer Science & Business Media, 643.
[3] Bruno Courcelle, 1987, An axiomatic definition of context-free rewriting and its application to nlc graph grammars, Theoretical Computer Science, 55(2-3), 141–181.
[4] Bruno Courcelle and Joost Engelfriet, 1995, A logical characterization of the sets of hypergraphs defined by hyperedge replacement systems, Mathematical Systems Theory, 28, 515–552.
[5] Didier Caucal, 2008, Deterministic Graph Grammars, Logic and automata, 2, 169–250.
[6] Didier Caucal, 1992, On the regular structure of prefix rewriting. Theoretical Computer Science, 106, 61–86.
[7] Jerome Feder, 1971, Plex languages, Information sciences, 3, 225–241.
[8] Micheal Bauderon and Bruno Courcelle, 1987, Graph expressions and graph rewriting. Mathematical Systems Theory, 20, 83–127.
[9] G. Rozenberg, 1997, Handbook of Graph Grammars and Computing by Graph Transformation, World Scientific, 1–3.
[10] Theodosios Pavlidis, 1972, Linear and context free graph grammars. Journal of the ACM, 19(1), 11–23.