International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 16 (2018) pp. 12531-12535

2 – Equitable Domination in Fuzzy Graphs

C. Gurubaran[1], A. Prasanna[2] and A. Mohamed Ismayil[3]

[1]*Department of Mathematics, National College, Tiruchirappalli, Tamilnadu, India.*
[2,3]*Department of Mathematics, Jamal Mohamed College, Tiruchirappalli, Tamilnadu, India.*

Abstract

In this paper, 2 – equitable dominating set and 2 – equitable domination number of a fuzzy graph is introduced. Some results on 2 – equitable dominating sets are proved upper and lower bounds of 2 – equitable domination number are obtained. The new parameters connected 2 – equitable dominating set, connected 2 – equitable domination number and exact 2 – equitable dominating set, exact 2 – equitable domination number of a fuzzy graph are discussed.

Keywords : strong neighbours, 2 – dominating set, 2 – domination number, 2 – equitable dominating set, 2 – equitable domination number.

AMS Mathematics Subject Classification (2010) : 03E72, 05C72

INTRODUCTION

Zadeh[8] introduced the concept of fuzzy sets in the year 1965. In 1975, fuzzy graph was introduced by Rosenfeld[4]. The notation of domination in fuzzy graphs was developed by A. Somasundaram and S. Somasundaram[6]. Nagoorgani and Chandrasekaran[3] discussed about domination in a fuzzy graph using strong arcs. The concept of degree equitable domination in graphs was introduced by Venkatasubramanian Swaminathan and Kuppusamy Markandan Dharmalingam[7]. Sivakumar.S., Soner.N.D., and Anwar Alwardi[5] introduced the concept 2-equitable dominaion in graphs. The concept of equitable domination in fuzzy graphs was introduced by Dharmalingam and Rani[2]. In this paper, the 2-equitable domination sets and its numbers are defined and discussed.

PRELIMINARIES

Definition 2.1

Let $G^* = (V,E)$ be a graph with vertex V and edge set $E \subseteq V \times V$. Let σ and μ be a fuzzy set of V and E respectively. Then $G = (\sigma,\mu)$ be a *fuzzy graph* if $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$ for all $(u,v) \in E$ and is denoted by $G = (\sigma,\mu)$.

Definition 2.2

Let $G = (\sigma,\mu)$ be a fuzzy graph then the *order* and *size* are defined as $p = \sum_{u\in V} \sigma(u)$ and $q = \sum_{(u,v)\in E} \mu(u,v)$

Definition 2.3

The *neighbourhood degree* of a vertex u is defined to be the sum of the weights of the vertices adjacent to u and is denoted by $d_N(u)$, the *minimum neighbourhood degree* of G is $\delta_N(G) = min\{d_N(u): u \in V\}$ and the *maximum neighbourhood degree* of G is $\Delta_N(G) = max\{d_N(u): u \in V\}$

Definition 2.4

An arc (u,v) in a fuzzy graph $G = (\sigma,\mu)$ is said to be strong if $\mu^{\infty}(u,v) = \mu(u,v)$ then u,v are called *strong neighbours*.

Definition 2.5

The *strong neighbourhood* of the vertex u is defined as $N_S(u) = \{v \in V \mid (u,v) is\ a\ strong\ arc\}$.

Definition 2.6

A vertex $u \in V$ dominates $v \in V$ if (u,v) is a strong arc. A subset D of V is called a *dominating set* of a fuzzy graph G if for every $v \in V - D$, there exists $u \in D$ such that u dominates v. The minimum scalar cardinality taken over all dominating set is called *domination number* and it is denoted by γ of a fuzzy graph G.

Definition 2.7

Let u and v be two vertices in a fuzzy graph G. A subset D of V is called an *equitable dominating set* if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|d(u) - d(v)| \leq 1$ and $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$. The minimum scalar cardinality of an *equitable dominating set* in a fuzzy graph is called equitable domination number and is denoted by $\gamma_e(G)$.

Definition 2.8

A subset D of V is called a 2 – dominating set of G if for every vertex $v \in V - D$ there exist atleast two strong neighbours in D. The 2 – domination number of a fuzzy graph G is the minimum cardinality of a set of all 2 – dominating set of G and is denoted by γ_2

2 – EQUITABLE DOMINATION IN FUZZY GRAPHS

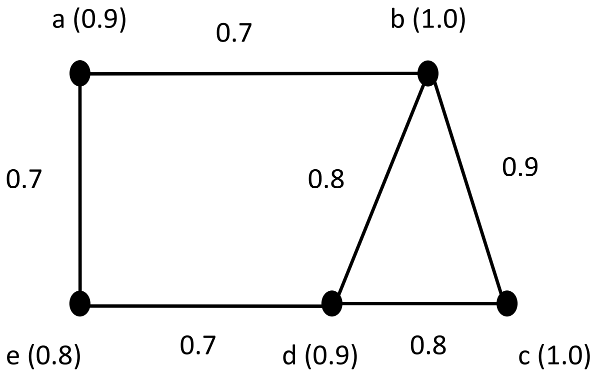
In this section, 2-equitable dominating set and 2-equitable domination numbers of a fuzzy graph are defined. The relation among domination number, equitable domination number and 2-equitable domination number are also obtained.

Definition 3.1

An equitable dominating set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is called 2 – equitable dominating set if for every vertex $v \in V - D$ there exist $v \in D$ or v is equitable dominated by atleast two vertices in D.

The minimum scalar cardinality of an 2 – equitable dominating set of G is called the 2 – equitable dominating number of G and is denoted by $\gamma_{2e}(G)$.

Example 3.2



Equitable Dominating set D of a fuzzy graph are $\{(b,e),(a,d),(e,c)\} \Rightarrow \gamma_e = 1.8$

2 – equitable dominating set $= \{e, d, b\}, \gamma_{2e} = 2.7$

Proposition 3.3

i) $\gamma(G) \leq \gamma_e(G) \leq \gamma_{2e}(G)$

ii) $\gamma_2(G) \leq \gamma_{2e}(G)$

Proof

From the definition, 2-equitable dominating set of a fuzzy graph G, it is clearly that for any fuzzy graph G any 2 – equitable dominating set D is also an equitable dominating set is also dominating set. Hence $\gamma(G) \leq \gamma_e(G) \leq \gamma_{2e}(G)$

Similarly, since every 2 – equitable dominating set is 2 – dominating set for any fuzzy graph G. Hence $\gamma_2(G) \leq \gamma_{2e}(G)$.

Definition 3.4

A 2 – equitable dominating set D is said to be minimal if no proper subset of D is 2 – equitable dominating set.

Proposition 3.5

For any fuzzy graph G with order p, then $\sum_{\substack{v_i, v_j \in G \\ v_i \neq v_j}} \min(\sigma(v_i), \sigma(v_j)) \leq \gamma_{2e}(G) \leq p$

Proof

Let D be a dominating set of a fuzzy graph G having atleast two vertices has minimum of V which is a sum of minimum value of a vertices$v_i, v_j \in D$, $\gamma_{2e}(G) \leq p$ it is obviously true.

Theorem 3.6

Let G be a fuzzy graph, $\gamma_{2e}(G) = p$ iff the fuzzy graph G has adjacent to less than two vertices.

Proof

Let G be a fuzzy graph then $\gamma_{2e}(G) = p$ then definition of fuzzy graph has all vertices in dominating set D. which shows that every vertex in G has adjacent to less than two vertices. Conversely, G be a fuzzy graph has adjacent to less than two vertices then every vertex in are in dominating set. Which is $\gamma_{2e}(G) = p$

Theorem 3.7

Let D is a minimal 2 – equitable dominating set then $V - S$ contains minimal 2 – equitable dominating set if every vertex of V in a fuzzy graph G adjacent to more than two vertices in V

Proof

Let D be a minimal 2 – equitable set of G suppose that $V - D$ is not an equitable dominating set, then there exist atleast one vertex $v \in D$ which is not equitable adjacent to any vertex in $V - D$. Therefore $V - D$ is equitable adjacent to atleast two vertices in D then $D - \{v\}$ is an 2 – equitable dominating set which is a contradiction. Hence every vertex in D must be equitable adjacent to atleast one vertex in $V - D$. Hence $V - D$ is equitable dominating set which contains minimal equitable dominating set.

Corollary 3.8

Every connected fuzzy graph has minimum 2 – equitable dominating set D then $V - D$ need not be 2 – equitable dominating set of G.

Proof

Let D be a 2 – equitable dominating set of G satisfies the condition also $|d(u) - d(v)| \leq 1$, suppose $v \in V$, then v be in every 2 – equitable dominating set of a fuzzy graph G, since it has only one neighbor vertex. This one neighbor also strong neighbor of v. Which shows that every vertex in $V - D$ does not has two strong neighbors for v. This implies that $V - D$ is not a 2 – equitable dominating set of G.

Theorem 3.9

Let G be a connected fuzzy graph has no non – equitable edge and H is spanning subgraph of G then $\gamma_{2e}(G) \leq \gamma_{2e}(H)$

Proof

Let G be a connected fuzzy graph and H is the spanning subgraph of H. consider D is minimum 2 – equitable dominating set of G, D also an 2 – equitable dominate all the vertices in $V(H) - D$ that is D is an 2 – equitable dominating set in H. Hence $\gamma_{2e}(G) \leq \gamma_{2e}(H)$.

Theorem 3.10

For cyclic fuzzy graph G with odd cycle then,

$$\gamma_{2e} = \begin{cases} \gamma_e + \min\sigma(v_i), & v_i \notin D \text{ if odd cycle} \\ \gamma_e, & \text{if even cycle} \end{cases}$$

Theorem 3.11

For any fuzzy graph G, $\gamma_e + \min\sigma(v_i) \leq \gamma_{2e}(G)$, for $v_i \notin D$.

Proof

Let D be 2 – equitable dominating set with minimum cardinality γ_{2e}. for any vertex $v_i \in D, D - \{v_i\}$ is equitable dominating set. Hence $\gamma_e + \min\sigma(v_i) \leq \gamma_{2e}(G)$.

Theorem 3.12

For any fuzzy graph G of order p, size q and maximum neighbourhood degree Δ_N, $\gamma_{2e}(G) \geq \frac{p+q}{2\Delta_N^2+1}$

Proof

Let D be the 2 – equitable dominating set of G. consider the vertex x of G dominates atmost itself. It has n number of edges and m number of vertices, each of these vertices dominating atmost n number of edges of G. i.e., vertex x of G incident on n number of edges, that vertex having maximum neighbourhood degree Δ_N of fuzzy graph G. Which shows that $\Delta_N + \Delta_N(\Delta_N - 1) = \Delta_N^2$. The 2 – equitable dominating set has another vertex with maximum degree of G. i.e. $2\Delta_N^2 + 1$.In the same way, an edge v_1, v_2 of G enables to dominate atmost itself $2(\Delta_N - 1) + 2 = 2\Delta_N$ and $2(\Delta_N - 1) + 2(\Delta_N - 1)^2 = 2\Delta_N^2 - 2\Delta_N$, which has maximum number of vertices adjacent to it. i.e $2\Delta_N^2 + 1$. The result is $\gamma_{2e}(G) \geq \frac{p+q}{2\Delta_N^2+1}$ is obvious.

CONNECTED 2–EQUITABLE DOMINATION IN FUZZY GRAPHS

Definition 4.1

Let $G = (\sigma, \mu)$ be a fuzzy graph. An 2 – equitable dominating set $D \subseteq V(G^*)$ if the subgraph of G induced by D is connected. The connected 2 – equitable domination of G with minimum cardinality is called connected 2 – equitable domination number of a fuzzy graph G and it is denoted by γ_{2ce}.

Observation 4.2

For any fuzzy tree T, with size p then $\gamma_{2ce} = p$

Proposition 4.3

For any fuzzy graph G, $\gamma(G) \leq \gamma_e(G) \leq \gamma_{2e}(G) \leq \gamma_{2ce}(G)$

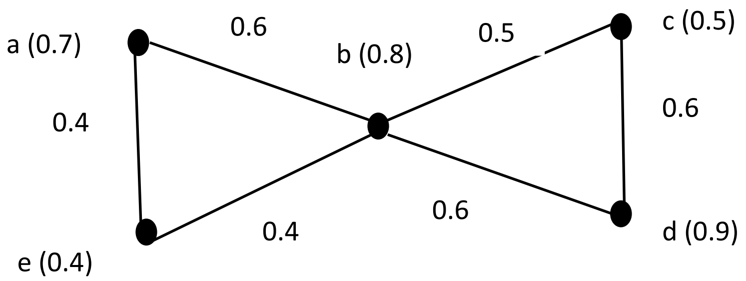
Theorem 4.4

Let G be a fuzzy graph without any equitable isolated vertices, then $\gamma_{2ce}(G) \leq \Delta_N + 1$

Proof

Let v be any vertex with $d(v) = \Delta_N(G)$ then obviously $N(v)$ is connected equitable dominating set and hence $\gamma_{2ce}(G) \leq \Delta_N + 1$.

Example 4.5



Connected 2 – equitable dominating set = {a, b, c, e}, $\gamma_{2ce} = 2.4$

Theorem 4.6

For any fuzzy graph, $\gamma_{2ce}(G) \geq \frac{2p}{\Delta_N+2}$

Proof

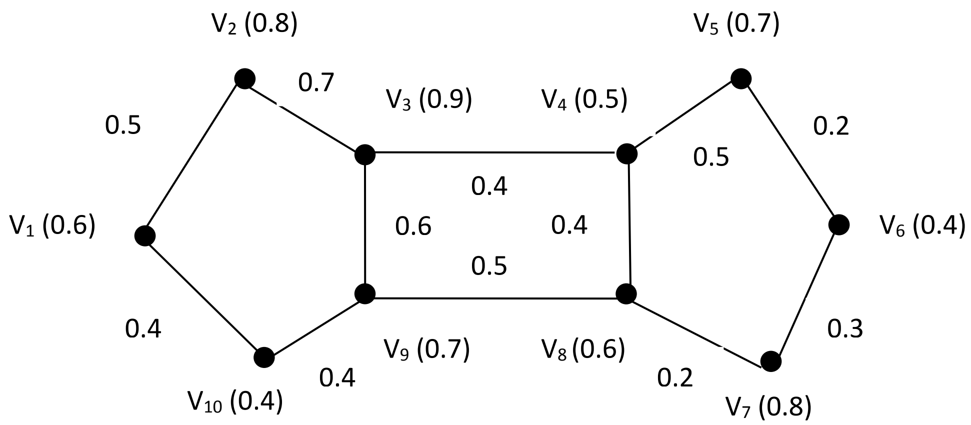
Let D be a minimum connected 2 – equitable dominating set and let k number of strong edges between dominating set D and $V - D$. Since the degree of each vertex in D is atmost Δ_N, which shows that $\sum \mu(u_i, u_j) \leq \Delta_N$. but each vertex in V – D is adjacent to atleast 2 vertices in D, $\sum \mu(u_i, u_j) \geq 2(p - \gamma_{2ce})$ combining these two inequalities produce $\gamma_{2ce}(G) \geq \frac{2p}{\Delta_N+2}$

EXACT 2 – EQUITABLE DOMINATION IN FUZZY GRAPHS

Definition 5.1

Let $G = (\sigma, \mu)$ be a fuzzy graph. A subset D of V be exact 2 – equitable dominating set of a fuzzy graph which has each of vertex in $V - D$ is dominated by exactly two vertices of D. The exact 2 – equitable domination of G with minimum cardinality is called exact 2 – equitable domination number of a fuzzy graph G and it is denoted by γ_{2ee}

Example 5.2



Equitable dominating set of a fuzzy graph $G = \{v_1, v_4, v_6, v_{10}\}$, $\gamma_e = 1.9$

Exact 2 – equitable dominating set = $\{v_2, v_4, v_6, v_8, v_{10}\}$, $\gamma_{2ee} = 2.7$

Theorem 5.3

If G be a fuzzy has an exact 2 – equitable dominating set then all such sets have the same size.

Proof

Let D_1, D_2 be the two exact 2 – equitable dominating sets if G. Let us write $C = D_1 \cap D_2$ and let X_0 and X_1 be the subsets of $D_1 - I$ such that every vertex of X_0 has no neighbours in I and every vertex of X_1 has one strong neighbor in I. which gives that $D_1 - I = X_0 \cup X_1$. In the same manner defined the subset Y_0 and Y_1 of $D_2 - I$. Claim that $|X_1| = |Y_1|$. Certainly, let x be any vertex of X_1 adjacent to a vertex $z \in I$. Since D_2 is an exact 2 – equitable dominating set z has a only one strong neighbor y in D_2. We have $y \in D_2 - I$ for otherwise z has two neighbours x, y in D_2. which is a contradiction. Thus $y \in Y_1$. The same argument holds for every vertex of Y_1 and so $|X_1| = |Y_1|$. Since D_2 is an exact 2 – equitable dominating set, every vertex of X_1 has exactly one neighbor in $Y_0 \cup Y_1$ and every vertex of X_0 has exactly two neighbours in $Y_0 \cup Y_1$. The same holds about the vertices of Y_0 and Y_1. This implies $|X_0| = |Y_0|$ and thus $|D_1| = |D_2|$.

Theorem 5.4

If D is an exact 2 – equitable dominating set of a fuzzy graph G, then $\gamma_{2ee} \leq \frac{2p}{\delta_N+1}$

Proof

Let D be an exact 2 – equitable dominating set of a fuzzy graph G and the minimum number of strong edges joining the vertices of D to the vertices of $V - D = 2(p - \gamma)$ since D is an exact 2 – equitable dominating set. D induces the matching of G since it has two strong neighbours. Then which implies that $\gamma(\delta_N + 1) \leq 2(p - \gamma)$. Hence $\gamma_{2ee} \leq \frac{2p}{\delta_N+1}$.

Proposition 5.5

Every exact 2 – equitable dominating is 2 – equitable dominating set, but converse need not be true.

REFERENCES

[1] Gurubaran. C., Prasanna. A., Mohamed Ismayil. A., Paired Equitable Domination in Fuzzy Graphs, International Journal of Mathematical Archive (IJMA), Vol 9, No 6, (2018), 43-47.

[2] Dharmalingam K.M., and Rani.M., Equitable domination in fuzzy graphs, International Journal of Pure and Applied Mathematics, Vol 94, No.5, (2014), 661-667.

[3] Nagoor Gani. A., and Chandrasekaran. T., Domination in fuzzy graph, Adv. Fuzzy Sets and System 1(1) (2006), 17-26.

[4] Rosenfeld.A., Fuzzy graphs, in: Zadeh.L.A., Fu.K.S., Tanaka. K., Shimura. M (Eds.), Fuzzy sets and Their Applications to cognitive and Decision Processes, Academic Press, NewYork, (1975), 77-95.

[5] Sivakumar.S., Soner.N.D., and Anwar Alwardi, 2-Equiable domination in graphs, IJMA., (2012), 1612-1616.

[6] Somasundaram. A and Somasundaram. S, Domination in fuzzy graphs – I, Pattern Recognition Letters 19 (1998), 787 – 791.

[7] Venkatasubramanian Swaminathan, Dharmalingam.K.M., Degree Equitable domination in graphs, Kragujevac J. Math., 35 (2011), 191-197.

[8] Zadeh.L.A., Fuzzy Sets, Information Control 8 (1965), 338-353.