Influence of Hall current in the MHD Oscillatory flow of Nanofluid: Application to the Blood flow

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Abstract: In this study, we analyse the influence of hall current in the MHD oscillatory flow of blood carrying gold nanoparticle in a porous space with radiation is investigated. We consider blood as base fluid which is Non-Newtonian and gold (Au) as nanoparicle. Therefore the nanofluid is called blood-gold nanofluid. The governing boundary layer equations are solved analytically. Numerical solutions of these equations are obtained by using the software MATHEMATICA. The influence of various parameters on the flow field, heat transfer characteristics, skin friction and Nusselt number are discussed and presented through graphs using ORIGIN software and tables. It is found that the velocity of nanofluid decreases for a given increase of nanoparticle volume fraction. Further, the rate of heat transfer increases with increasing nanoparticle volume fraction at the upper wall.

Keywords: MHD, oscillatory flow, hall current, gold nanoparticle.

INTRODUCTION

Studies pertaining to the non-Newtonian fluids are important because of its applications in biological sciences and industry such as flow of blood, food mixing and cyme movement in the intestine, paint, polymer solutions, flow of nuclear fuel slurries and flow liquid metal and alloys. Misra et al. (2011) analysed the Hydromagnetic flow and heat transfer of a second-grade viscoelastic fluid in a channel with oscillatory stretching wall. Nirmala et al. (2018) presented MHD transport phenomena of oscillatory channel of blood flow with hall current.

Nanofluid is a mixture of nano-sized particles suspended in a base fluid, is used to enhance the rate of heat transfer through its higher thermal conductivity compared to the base fluid. Nanofluids play vital role to significantly influences the heat transfer rates in many areas such as industrial cooling applications, nuclear reactors, transportation industry, micro-electromechanical systems, heat exchangers, chemical catalytic reactors, fiber and granular insulation, packed beds, petroleum reservoirs and nuclear waste repositories and biomedical applications. Choi (1995) introduced the concept of nanofluid by immersing the nano meter sized particles into base fluids and he found the enhanced thermal conductivity in the base fluid due to the mixture of nanoparticles. Buongiorno (2006) has given clear description on convective heat transfer in nanofluids.

Das et al. (2007) written a book entitled “nanofluids science and technology”, where they discussed the applications of nanofluids along with importance of convectional

Timofeeva and Dileep Singh (2009) studied the effect of nano particle shape by considering Alumina based nanofluid. MHD effects on unsteady dusty viscous flow by considering volume fraction of dust particles was analyzed by Ibrahim Saidu et al. (2010). Makinde and Aziz (2011) analyzed boundary layer flow of a nanofluid by considering convective boundary conditions. Flow and heat transfer behavior over a nonlinearly stretching surface was studied by Cortell (2011). Pavithra and Gireesha (2013) discussed unsteady flow and heat transfer behavior of the dust particles suspended flow over a stretching sheet. Hayat et al. (2014) presented a nanofluids over an exponentially stretching sheet in a porous medium with convective boundary conditions.


From the literature survey and to the best of authors’ knowledge, the study pertaining to oscillatory flow of blood carrying gold nanoparticles with hall current has not been explored yet. Such a consideration is of great value in biomedical and engineering research. Gold nanoparticles with small size are very important in biomedical science and they can be used to activate or inhibit the growth of blood vessels. Hatami et al. (2014) discussed computer simulation of MHD blood conveying gold nanoparticles as a third grade Non-Newtonian nanofluid in a hollow porous vessel. Srinivas Reddy et al. (2017) presented MHD flow and heat transfer characteristics of Williamson nanofluid over a stretching sheet with variable thickness and variable thermal conductivity. Hence the main objective of the present investigation is to study the oscillatory flow of nanofluid in a porous space in the presence of thermal radiation with hall current. Analytical solutions are obtained for flow variables. The effect of various parameters on velocity, temperature, skin friction coefficient and heat transfer rate are investigated.

**MATHEMATICAL FORMULATION**

Consider the oscillatory flow of nanofluid with hall current effect between two parallel walls in a porous medium. As shown in figure 1, take Cartesian coordinate system \((x^*, y^*)\) where \(Ox^*\) lies along the lower wall, while \(Oy^*\) is normal to it. The lower and upper wall are maintained at uniform temperature \(T_0\) and \(T_1\), \((T_1 > T_0)\) respectively. A uniform magnetic field is applied parallel to \(Oy^*\) axis. Since the walls are infinite, all physical quantities excepting pressure may be taken as a function of \(y\) and \(t\).

**FIGURE 1. Mathematical Model**

Under these assumptions, the governing equations of continuity, momentum and heat transfer are as follows:

\[
\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{1}
\]

\[
\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho_{nf}} \frac{\partial p^*}{\partial x^*} + \frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u^*}{\partial y^*^2} - \frac{\mu_{nf}}{\rho_{nf}} \frac{u^*}{k} - \frac{\sigma B_0^2 x^*}{\rho(1 + m^2)} u^* \tag{2}
\]
\[
\frac{\partial T^*}{\partial t^*} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T^*}{\partial y^*^2} - \frac{1}{(\rho c_p)_{nf}} \frac{\partial q}{\partial y^*}
\]  

(3)

The corresponding boundary conditions are

\[
u^* = 0, \quad T^* = T_0, \quad \text{at} \quad y^* = 0 \quad \text{(4)}
\]

\[
u^* = 0, \quad T^* = T_0 + (T_1 - T_0)e^{\omega^*t^*}, \quad \text{at} \quad y^* = h \quad \text{(5)}
\]

where \(\nu^*\) is velocity component in \(x^*\) direction, \(\rho_{nf}\) is density of nanofluid, \(p^*\) is the pressure, \(\mu_{nf}\) is dynamic viscosity of the nanofluid, \(k^*\) is permeability of porous medium, \((\rho c_p)_{nf}\) is effective heat capacitance of nanofluid, \(k_{nf}\) thermal conductivity of nanofluid, \(q_r\) is radiative heat flux, \(T^*\) is the temperature of the nanofluid and \(h\) is the distance between the channels, \(\sigma\) is the electrical conductivity, \(m\) is the hall current parameter \((m = \tau_e \omega_e)\) where \(\tau_e\) is the electron collision time and \(\omega_e\) is the electron frequency.

The physical properties of nanofluid such as \(\mu_{nf}, \rho_{nf}, (\rho c_p)_{nf}\) and \(k_{nf}\) are given in Hatami et al. (2014)

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{1.5}}
\]

(6)

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi \rho_s
\]

(7)

\[
(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s
\]

(8)

\[
k_{nf} = \frac{k_s + (n - 1)k_f - \phi(n - 1)(k_f - k_s)}{k_s + (n - 1)k_f - \phi(k_f - k_s)}k_f
\]

(9)

where \(\rho_f\) is the density of the base fluid, \(\rho_s\) is density of the nanoparticle, \(\mu_f\) is viscosity of the base fluid, \(\phi\) is the nanoparticles volume fraction, \((\rho c_p)_f, (\rho c_p)_s\) are the heat capacitance of the base fluid and nanoparticles respectively, \(k_f, k_s\) are thermal conductivities of the base fluid and nanoparticle respectively, \(n\) is the nanoparticle shape and \(n = 3\) gives spherical shape, \(n = 6\) gives the cylindrical shape as given in Akbar et al. (2016) and the subscripts \(f\) and \(s\) denote fluid and solid properties respectively. Following Cogley et al. (1968), it is assumed that the fluid is optically thin with a relatively low density and the relative heat flux is given by

\[
\frac{\partial q}{\partial y^*} = 4\alpha^2(T_0 - T^*)
\]

(10)

where \(\alpha\) is the mean radiation absorption coefficient. The oscillatory flow is assumed to be induced by pressure gradient of the form

\[
- \frac{1}{\rho_f} \frac{\partial p^*}{\partial x^*} = Be^{i\omega^*t^*}
\]

(11)

The following dimensionless variables and parameters are introduced:

\[
y = \frac{y^*}{h}, \quad x = \frac{x^*}{h}, \quad u = \frac{u^*}{B}, \quad v = \frac{v^*}{B}
\]

\[
p = \frac{p^*}{B\rho_f h}, \quad \theta = \frac{T^* - T_0}{T_f - T_0}, \quad Da = \frac{k}{h^2}
\]

\[
Pr = \frac{(\rho c_p)_f}{k_f}, \quad t = t^*\omega_e, \quad M = \frac{\omega_e^2 B_0^2}{\mu_f}
\]

\[
m = \tau_e \omega_e, \quad R = \frac{\omega_e h^2}{\nu_f}, \quad N^2 = \frac{4\alpha^2 h^2}{k_f}
\]

(12)

Substitute equations (10) and (12) in equations (1)–(3) and (11) we have

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(13)

\[
\frac{\partial u}{\partial t} = \frac{-1}{\nu_f} \frac{\partial p}{\partial x} + \frac{A_2}{A_1} \frac{\partial^2 u}{\partial y^2} - \left( \frac{A_2}{A_1 R Da} + \frac{M}{R(1 + m^2)} \right) u
\]

(14)

\[
\frac{\partial \theta}{\partial t} = \frac{A_4}{A_3 R Pr} \frac{\partial^2 \theta}{\partial y^2} + \frac{N^2}{A_3 R Pr} \theta
\]

(15)

\[
- \frac{\partial p}{\partial x} = e^{i\omega^*t^*}
\]

(16)

The corresponding boundary conditions becomes,

\[
u = 0, \quad \theta = 0, \quad \text{at} \quad y = 0
\]

(17)

\[
u = 0, \quad \theta = e^{i\omega^*t^*}, \quad \text{at} \quad y = 1
\]

(18)

where,

\[
A_1 = \frac{\rho_{nf}}{\rho_f}, \quad A_2 = \frac{\mu_{nf}}{\mu_f}
\]

\[
A_3 = \frac{(\rho c_p)_f}{(\rho c_p)_s}, \quad A_4 = \frac{k_{nf}}{k_f}
\]

**SOLUTION OF THE PROBLEM**

Since the flow is induced by the pressure gradient given by equation (16), the velocity and temperature can be expressed as

\[
u(y, t) = u_f(y)e^{i\omega^*t^*}
\]

(19)
\[ \theta(y, t) = \theta_f(y)e^{it} \]  

(20)

Substituting equations (16), (19) and (20) in equations (14) and (15), we have

\[ A_6 u_f^2 - (A_7 + i) u_f = -A_5 \]  

(21)

\[ A_8 \theta_f^2 + (A_9 - i) \theta_f = 0 \]  

(22)

and boundary conditions (17) and (18) becomes

\[ u_f = 0, \ \theta_f = 0, \ \text{at} \ y = 0 \]  

(23)

\[ u_f = 0, \ \theta_f = 1, \ \text{at} \ y = 1 \]  

(24)

where,

\[ A_5 = \frac{1}{A_1}, A_6 = \frac{A_2}{A_1 R} \]

\[ A_7 = \left( \frac{A_2}{A_1 R Da} + \frac{M}{R(1 + m^2)} \right) \]

\[ A_8 = \frac{A_4}{A_3 R Pr}, A_9 = \frac{N^2}{A_3 R Pr} \]

Solving equations (21), (22) by using boundary conditions (23), (24) and substituting in the equations (19) and (20), we have the velocity and temperature given as follows:

\[ u(y, t) = \left( \frac{A_{11}(e^{-A_{10}} - 1)}{e^{A_{10}} - e^{-A_{10}}} e^{A_{10} y} + \frac{A_{11}(1 - e^{A_{10}})}{e^{A_{10}} - e^{-A_{10}}} e^{-A_{10} y} + A_{11} \right) e^{it} \]  

(25)

\[ \theta(y, t) = \left( \frac{e^{A_{12} y} - e^{-A_{12} y}}{e^{A_{12}} - e^{-A_{12}}} \right) e^{it} \]  

(26)

where,

\[ A_{10} = \sqrt{A_7 + i A_6} \]

\[ A_{11} = \frac{A_5}{A_7 + i}, A_{12} = \sqrt{\frac{i - A_9}{A_8}} \]

Further, the skin-friction coefficient \( c_f \) and Nusselt number \( Nu \) at the upper wall \( y = 1 \) are given by

\[ c_f = \frac{\mu_n f}{\mu_f} \left( \frac{\partial u}{\partial y} \right)_{y=1} \]  

(27)

\[ Nu = -\frac{k_n f}{k_f} \left( \frac{\partial \theta}{\partial y} \right)_{y=1} \]  

(28)

**RESULTS AND DISCUSSION**

For numerical results, we consider the dimensionless parameter values as, \( \phi = 0.1, n = 3, m = R = M = N = 1, Da = 0.5, Pr = 0.71, \pi = \frac{\pi}{4} \). These values are kept as common in the entire study except the varied values as displayed in respective figures and labels. The thermophysical properties of the base fluid (blood) and the nanoparticles (gold) are given in table 1. Figure 2–5 shows the velocity profile and Figure 6–10 shows the temperature profile for various parameters namely the frequency parameter \( R \), Magnetic parameter \( M \), hall current parameter \( m \), nanoparticle volume fraction \( \phi \), radiation parameter \( \gamma \).
FIGURE 4. Variation of axial velocity against $y$ for different values of Hall current

FIGURE 5. Variation of axial velocity against $y$ for different values of $\phi$

FIGURE 6. Variation of temperature against $y$ for different values of frequency parameter $R$. Also the skin friction $c_f$ and the rate of heat transfer $Nu$ are discussed in table 2.

Figure 2 shows the effect of frequency parameter $(R)$ on velocity profile of the flow. From the figure, the increase in the frequency parameter follows the increase in the velocity. When $R = 1$, the velocity increases slowly upto $y = 0.5$ and then slowly decreases. But when $R = 5.0$ and $R = 10.0$ the velocity increases rapidly upto $y = 0.5$ and then decreases. Figure 3 shows the effect of magnetic parameter $(M)$ on velocity profile. From the figure the increase in the magnetic parameter decline in the fluid velocity. This happens due to the fact that in the presence of magnetic field, there arises a resistive force called Lorenz force, which has a tendency to slow down the motion of the fluid in the boundary layer. Figure 4 displays the effect of hall current parameter $(m)$ on the velocity profile. The hall current parameter increases with increasing velocity profile. When the nanoparticle volume fraction $\phi$ increases the velocity decreases. Note that the fluid without nanoparticle ($\phi = 0$) has greater velocity than nanoparticle ($\phi > 0$) as shown in the figure 5.

Figure 6 shows the effect of frequency parameter on temperature profile. The increased value of frequency parameter $(R)$ with temperature profile shows a disturbance rather than a chance at different channel heights. At any frequency parameter $R$, the dependence of $y$ with temperature is linear. The effect of heat radiation parameter $N$ on temperature profile is presented in figure 7. It is noticed that when the heat radiation parameter increases, the temperature also increases. For $N = 1, 2$, the temperature increases gradually along $y$ and reaches 1. But, for $N = 3$, the temperature increases highly and attains maximum at $y = 0.5$ and decreases. Figure 8 presents the effect of Prandtl number $Pr$ with temperature profile and when $Pr$ increases, the temperature decreases, very small at the middle of the wall and converges at the end. In figure 9, when nanoparticle volume fraction $\phi$ increases, the temperature decreases, but is marginal. Figure 10 predicts the effect of nanoparticle shape with temperature. The temperature profile for spherical nanoparticles is little higher than the temperature profile for cylindrical nanoparticles, and the temperature profile increases linearly with $y$ irrespective of the shape of the nanoparticles.

Table 2 depicts the influence of skin friction coefficient and Nusselt number (upper wall $y = 1$) for the above said parameters. It is evident from the table that increase of hall
## Table 1. Some properties of Non-Newtonian fluid (blood) and Nanoparticle (Au)

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (Kg/m$^3$)</th>
<th>$c_p$ (J/KgK)</th>
<th>$k$ (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blood</td>
<td>1050</td>
<td>3617</td>
<td>0.52</td>
</tr>
<tr>
<td>Gold (Au)</td>
<td>19300</td>
<td>129</td>
<td>318</td>
</tr>
</tbody>
</table>

## Table 2. Numerical values of the Skin-friction coefficient and Nusselt number

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$m$</th>
<th>$Pr$</th>
<th>$R$</th>
<th>$M$</th>
<th>$N$</th>
<th>$C_f$</th>
<th>$Nu$(sphere)</th>
<th>$Nu$(cylinder)</th>
</tr>
</thead>
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<tr>
<td>0.1</td>
<td>1.0</td>
<td>0.71</td>
<td>1.0</td>
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<td>-0.314497</td>
<td>1.02196</td>
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<tr>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<td>2.15616</td>
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<tr>
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## FIGURE 7. Variation of temperature against $y$ for different values of Radiation parameter

Parameter and frequency parameters, decreases the skin friction coefficient, while increase of magnetic parameter increases skin friction. With the increase of nanoparticle volume fraction, Prandtl number, frequency parameter and radiation parameter, the nusselt number for spherical and cylindrical nanoparticles increase.

## CONCLUSIONS

In this study, the MHD oscillatory flow of blood carrying gold nanoparticle has been investigated. The analytical solutions are obtained for flow variables. The considered
problem is important in biomedical field due to its applications in drug delivery system. The gold nanoparticles are efficient in drug-carrying and drug-delivery vehicles because they can encapsulate large quantities of therapeutic molecules (Hatami et al. (2014)). The findings of the numerical results are summarized as follows:

1. The velocity of the nanofluid increases with an increase of frequency parameter and hall current parameter while decreases for a given increase of magnetic parameter and nanoparticle volume fraction.
2. The temperature of the nanofluid increases with increase of radiation parameter, while decreases with an increase of frequency parameter, Prandtl number, nanoparticle volume fraction.
3. The skin friction coefficient \((c_f)\) decreases as the hall current parameter and frequency parameter are increased while increase of magnetic parameter increases the skin friction.
4. When the rate of heat transfer \((Nu)\) for upper wall increases, the nanoparticle volume fraction, Prandtl number, frequency parameter and radiation parameter are increased.

REFERENCES


