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Optimum inventory control for imperfect quality item with maximum life-time under Quadratic demand and Preservation Technology Investment

Nita H. Shah[1,*], Urmila Chaudhari[2] and Mrudul Y. Jani[3]

[1]*Department of Mathematics, Gujarat University, Ahmedabad – 380009, Gujarat, India.*

[2]*Government Polytechnic Dahod, Dahod-389151, Gujarat, India.*

[3] *Department of Applied Sciences, Faculty of Engg & Technology, Parul University, Vadodara-391760, Gujarat, India.*

*(*Corresponding Author: Nita H. Shah)*

Abstract

The model is applicable for dry-fruits and food industry. India is considered to be the country of festivals and varieties of food. During the festival, the demand of dry-fruits extremely increases for preparation of sweets. This article focuses for dry-fruits. To maintain the quality of dry fruits, preservation technology investment is incorporated. The demand follows quadratic nature during season. The objective is to minimize the total cost of the inventory system with respect to screening time cycle time and investment for preservation technology. Here, to collect defective items from the imperfect quality items, we use concept of learning curve process. The model is supported with numerical examples and also established scenario of the model. Sensitivity analysis is done to assume decision-making insights.

Keywords: Inventory control, imperfect items, learning process, maximum fixed life-time, preservation technology investment.

INTRODUCTION

The classical EOQ has been an extensively developed model for inventory control purposes due to its modest and instinctively pleasing mathematical formulation. Salameh and Jaber (2000) established a mathematical model that permits some loose quality items or imperfect quality items requirements. The researchers assumed that each lot is screened 100 percent by learning process and that can be sold at lower price. Huang (2004) established for flawed items in a (JIT) manufacturing environment, a model to determine an optimal layered vendor–buyer inventory strategy. Maddah and Jaber (2008) studied a new model that remedies a flaw in the one given by Salameh and Jaber (2000) using renewal theory. Jaber *et al* (2008) extended it by assuming that the percentage imperfect per lot diminishes according to a learning curve. They inspected empirical data from the self-propelled industry for several learning curve models and the S-shaped logistic learning curve (Carlson (1973); Jordan (1958)) was found to fit well. Jaggi and Mittal (2011) examined when the items are of imperfect quality, the effect of deterioration on a retailer's EOQ. In that paper, defective items is assumed to be kept in the same warehouse until the end of the screening process. Jaggi *et al* (2011) and Sana (2012) presented inventory models for imperfect quality items under the condition of credit limit in payments. Haidar *et al* (2014) extended the work of Jaggi and Mittal (2011) to allow for shortages. Moreover, Alamri *et al* (2016) developed an inventory control model for imperfect quality items.

In classical inventory problems, it is assumed that products have an infinite shelf life, while the most of items lose their initial values over time and for some of them this occurs faster than usual which is called deterioration. (Soni & Patel, 2013). Ghare and Schrader (1963) determined deteriorating item's inventory model. The criticise articles by Raafat (1991), Shah and Shah (2000), Goyal and Giri (2001), Bakker *et al.* (2012), on deteriorating items for inventory system throw light on the part of deterioration. Chung and Cardenas-Barrón (2013) established supply chain inventory modelling algorithm for stock-dependent demand which comprising of three players for deteriorating items. Furthermore, Shah and Barrón (2015) developed when a distributor offers order-linked credit period or cash discount, byer's decision for credit policies and ordering for deteriorating items.

On the other hand to reduce deterioration, use preservation technology, Hsu *et al*. (2010) determined a model under preservation technology investment, an inventory model to minimize the deterioration rate of inventory for constant demand. Hsieh and Dye (2013) evaluated when demand is changing with time, a production inventory model including the effect of preservation technology investment. Recently, Shah, *et al*. (2016a) established an integrated inventory model for time dependent deteriorating item under time and price sensitive demand with preservation technology. Moreover, Shah *et al.* (2016b) developed supply chain inventory model under selling price and trade credit dependent quadratic demand for time dependent deteriorating item with preservation technology.

In the earlier research, constant demand was considered in many research articles however demand rarely remains constant over infinite planning horizon. In our study, demand is depended on time and quadratic in nature which is more feasible for the study seasonal product for example food industry, electronics items and fashion goods. Moreover, most of the products lose their utility over time. So, we consider time dependent deterioration rate and to reduce deterioration preservation technology investment is calculated. Furthermore, study of screening process is very interesting concept of inventory modelling. In our study we consider many aspects of business to calculate cost function. Therefore, this paper focus an inventory control model for imperfect quality items to minimize total cost. The model is applicable for dry-fruits. To maintain the quality of dry fruits, preservation technology investment is incorporated. The demand follows quadratic nature during season. In the model, each lot is subject to a 100 per cent screening where items that are not achieving to certain quality standards are stored in a different warehouse. Therefore, different holding costs for the perfect and imperfect items are considered in the mathematical model. Items deteriorate while they are in storage, with screening and deterioration rates being chance functions of time. The percentage of imperfect items per lot decreases according to a learning curve. After a 100 per cent screening, defective quality items may be sold at a low price as a single batch at the end of the screening process. Under above assumptions, the objective is to minimize the cost of inventory system with respect to the screening time, cycle time, and investment of preservation technology.

The rest of the paper is structured as follow. Section 2 is about the notations and the assumptions that are used. Section 3 is about formulation of the mathematical model of the proposed inventory control problem. Section 4 validates the derived inventory model with numerical example and its sensitivity analysis. This section also provides some managerial insights. Finally, Section 5 provides conclusion and future research directions.

NOTATION AND ASSUMPTIONS

The proposed inventory problem is based on the following notation and assumptions.

Notation

Inventory system's parameters:	
j	Cycle index
a	Total Scale demand of the product , $a>0$
b	Linear rate of change of demand of the product, $0\le b<1$
c	Quadratic rate of change of demand of the product, $0\le c<1$
$R(t)$	Time dependent quadratic demand rate; $R(t)=a\cdot(1+bt-ct^2)$, where $a>0$ is scale demand, $0\le b,c<1$ are rates of change of demand, respectively.
A	Ordering cost per order incurred by the inventory system ($/order)
C	purchase cost per unit item (in $)
t_{1j}	First phase duration (i.e. screening time)
$\theta(t)$	Deterioration rate; $0\le\theta(t)\le 1$
m	Fixed life-time of the product (in years)
$x(t)$	Screening function
P_j	Percentage of defectives per lot reduces according to a learning curve
u	Preservation technology investment per unit time (in $)(decision variable)
$f(u)$	$=1-\frac{1}{1+\mu u}$; proportion of reduced deterioration item (in year), $\mu>0$
$I_{g1j}(t)$	Good Inventory level of the inventory system for the item during first phase $0\le t\le t_{1j}$ (units)
$I_{g2j}(t)$	Good Inventory level of the inventory system for the item during second phase $t_{1j}\le t\le T_j$ (units)
$I_{dj}(t)$	Defective Inventory level of the inventory system for item at $0\le t\le t_{1j}$ (units)
T_j	Cycle time (in years) of the inventory system (decision variable)
Q_j	Order quantity at time $t=0$
h_g	Holding cost rate for inventory system for good item per unit per annum
h_d	Holding cost rate for inventory system for defective item per unit per annum, $h_g>h_d$
HC	Holding cost of the inventory system for item ($/unit / unit time)
$TC_j(t_{1j},T_j,u)$	Total cost of the inventory system for j^th^ cycle per unit time ($/unit / unit time)

Relations between parameters:

- $T_j\le m$
- $0\le\theta(t)<1$

The problem is expressed as follows:

$$\min\ TC_j(t_1,T,u)$$

Subject to constraints

$$T_j\le m$$

Assumptions:

1. The inventory system involves single instantaneous deteriorating item.
2. The demand, screening and deterioration rates are arbitrary functions of time denoted by $R(t)$, $x(t)$ and $\theta(t)$ respectively. The percentage of defectives per lot reduces according to a learning curve denoted by P_j, where j is cycle index. Here, we consider screening function as $x(t)=\alpha+\beta t$ and learning

curve is defined as $P_j = \dfrac{\tau}{\eta + e^{\gamma j}}$. Where, $\alpha, \tau, \eta > 0$ and $\beta, \gamma \geq 0$.

3. The demand rate, $R(t) = a \cdot (1 + bt - ct^2)$ (say) is function of time, $a > 0$ is scale demand, $0 \leq b < 1$ denotes the linear rate of change of demand with respect to time, $0 \leq c < 1$ denotes the quadratic rate of change of demand.
4. Time horizon is infinite.
5. Shortages are not allowed. i.e. $(1 - P_j)\, x(t) \geq R(t) \;\; , \forall t \geq 0$.
6. Lead time is zero or negligible.
7. The instantaneous rate of deterioration is $\theta(t) = \dfrac{1}{1 + m - t}$, $0 \leq t \leq T \leq m.$ for any finite value of m, we have $\theta(t) < 1$. If $m \to \infty$ then $\theta(t) \to 0$ i.e. the item is non- deteriorating.
8. The proportion of reduced deterioration rate, $f(u)$ is assumed to be a continuous increasing and concave function of investment u on preservation technology, i.e. $f'(u) > 0$ and $f''(u) > 0$. WLOG, assume $f(0) = 0$.

MATHEMATICAL MODEL

In this model, two phases are analyzed. The phases are screening phase $\left[0, t_{1j}\right]$ and non-screening phase $\left[t_{1j}, T_j\right]$. At the beginning of each cycle $j\,(j = 1, 2, 3, ...)$, Q_j units are received in the inventory system, which fulfil actual demand and deterioration during the screening phase and the non-screening phase. Each lot is subjected to a 100 % screening process at a rate of $x(t)$ that starts at the beginning of the cycle and terminates by time T_j, by which point in time Q_j units have been screened and y_j units have been depleted, which is the combined with demand and deterioration. During this phase, items not conforming to certain quality standards are stored in a different warehouse.

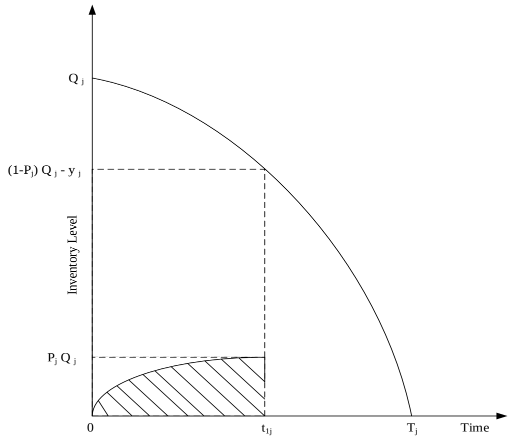

Figure 1: Inventory variation of the model for one cycle

The variation in the inventory level during the screening and non-screening phases (see Fig. 1 (as in Alamri 2016)) and inventory level variation for the defective items (shaded area in Fig. 1) is given by equations respectively as follows.

$$\frac{dI_{g1j}(t)}{dt} = -\theta(t) \cdot (1 - f(u))\, I_{g1j}(t) - R(t) - x(t) P_j \quad , 0 \leq t \leq t_{1j}$$

With boundary condition $I_{g1j}(0) = Q_j$, where $Q_j = \int_0^{T_j} x(u)\, du$.

and

$$\frac{dI_{g2j}(t)}{dt} = -\theta(t) \cdot (1 - f(u))\, I_{g2j}(t) - R(t) \quad , t_{1j} \leq t \leq T_j$$

With the boundary condition $I_{g2j}(T_j) = 0$.

In the different warehouse, the inventory level of defective item is given by the following equation $\dfrac{dI_{dj}(t)}{dt} = x(t) P_j \; , 0 \leq t \leq t_{1j}$ with the boundary condition $I_{dj}(0) = 0$.

Solving above differential equations, we get inventory level at any instant of time in the different phase, $I_{g1j}(\mathrm{t})$, $I_{g11j}(\mathrm{t})$, $I_{g12j}(\mathrm{t})$ and $I_{g2j}(\mathrm{t})$ **(see Appendix-1 to Appendix-5).**

Now, the total cost component per unit time of the inventory system is comprises of

- Ordering of the inventory cost per unit : $OC = A$
- Purchase cost of the item per unit : $PC = CQ_j$
- Screening cost of the item per unit : $SC = dQ_j$
- Holding cost per unit:

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$$HC = h_g \left[\int_0^{t_{1j}} I_{g1j}(t)\, dt + \int_{t_{1j}}^{T_j} I_{g2j}(t)\, dt \right] + h_d \int_0^{t_{1j}} I_{dj}(t)\, dt$$

- Investment for Preservation Technology : $PTI = u \cdot T_j$

The total cost of the inventory system for the item per unit time is

$$TC_j\left(t_{1j}, T_j, u\right) = \frac{1}{T_j}\left(\begin{array}{l} OC + PC + SC \\ +HC + PTI \end{array} \right)$$

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

Numerical Example

Example: During Indian festivals (generally festival is going on during whole year) there is demand of dry fruits. So, at that time demand rates are $a = 50,000$ kgs., $b = 1\%$, $c = 10\%$. The ordering cost is $A = \$100$ /100 kg. Purchase cost of the dry-fruits is $C = \$10$ / kg Moreover, holding cost rates for good inventory of dry fruits and defective dry fruits are $h_g = \$0.8$/kg/time unit, $h_d = \$0.3$/kg/time unit and has maximal life-time is $m = 0.5$ year. Rates for learning process are $\tau = 70.076$, $\gamma = 0.89$, $\eta = 819$ and constants of screening rate for dry fruits are $\alpha = 1520$, $\beta = 1000$ and screening cost rates is $d = \$\ 0.5$. Now, to reduce deterioration rate of the dry-fruits, rate of investment for preservation technology is $\mu = 5$. Here, we analyze only for one cycle consequently $j = 1$. The values of the decision variables are screening time is $t_{11} = 0.26$ year, cycle time of replenishment is $T_1 = 0.49$ year and investment for preservation technology is $u = \$\ 7.54$/kg. This results **inventory system's minimum cost as $ 8961.59**.Also, the convexity of the cost function obtained in Fig.2-4.

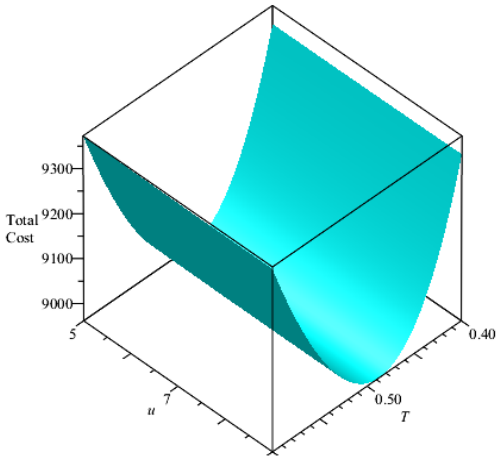

Figure 2: Convexity behaviour of the cost function for $t_{11} = 0.26$ year

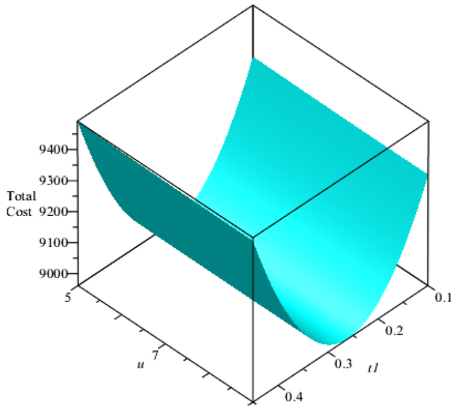

Figure 3: Convexity behaviour of the cost function for $T_1 = 0.49$ year

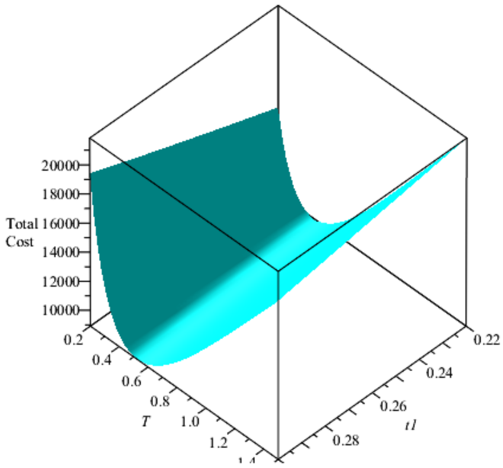

Figure 4: Convexity behaviour of the cost function for $u = \$7.54$

Table 1: Learning process

j (No. of Cycle)	P_j	Q_j	No. of Defective item P_jQ_j
1	0.0853	428.4	36.548
2	0.08494	428.4	36.393
3	0.08408	428.4	36.022
4	0.08204	428.4	35.150
5	0.07746	428.5	33.193
6	0.06820	428.6	29.230
7	0.05281	428.7	22.646
8	0.03409	428.9	14.624
9	0.018297	429.1	7.852
10	0.008597	429.2	3.690

It is clear that from the Table1and Fig 5 - 6 using learning curve process we can separate the defective items from the inventory system during the cycle. With increase in cycles percentage of defective items decreased which results in less number of defective items received in the inventory system.

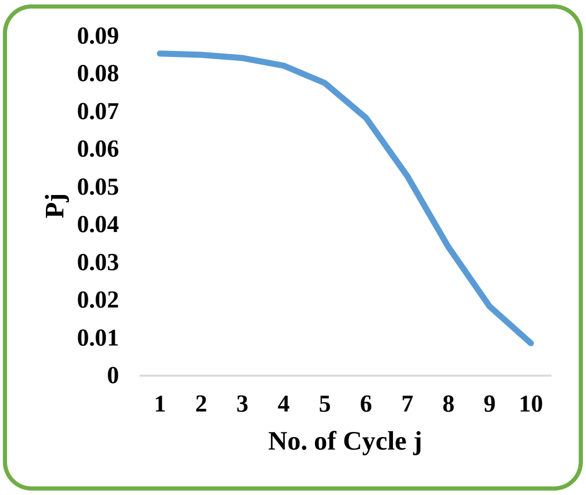

Figure 5: Fraction of defective items in cycles

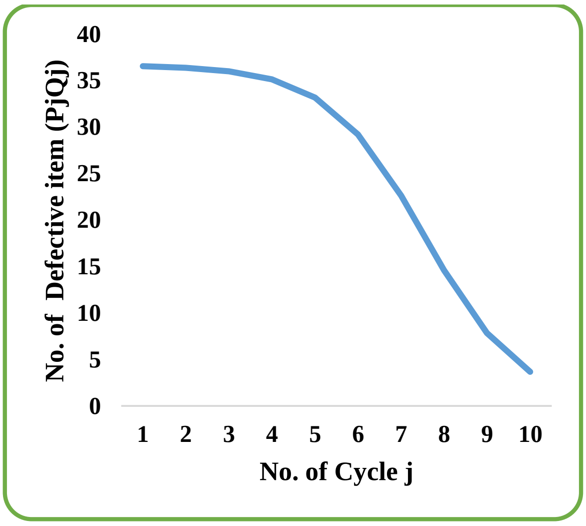

Figure 6: Number of defective items during cycles

Sensitivity Analysis for the Inventory Parameters

In Table 2, the sensitivity analysis of example is carried out by varying one variable at a time as-20%, -10%, 10% and 20%.

Table 2: Sensitivity analysis

Parameter	Change %	Values	t_{11} (in year)	T_1 (in year)	u (in $)	Total Cost TC (in $)
a	-20%	40000	0.3601	**0.6557 (Infeasible)**	9.40	9152.48
	-10%	45000	0.3017	**0.5584 (Infeasible)**	8.32	9040.33
	0%	50000	0.2597	0.4872	7.55	8961.59
	10%	55000	0.2279	0.4325	6.95	8905.26
	20%	60000	0.2029	0.3891	6.48	8864.73

Parameter	Change %	Values	t_{11} (in year)	T_1 (in year)	u (in $)	Total Cost TC (in $)
b	-20%	0.008	0.2597	0.4875	7.55	8960.32
	-10%	0.009	0.2597	0.4873	7.55	8960.96
	0%	0.01	0.2597	0.4872	7.55	8961.59
	10%	0.011	0.2596	0.4871	7.55	8962.23
	20%	0.012	0.2596	0.4869	7.55	8962.86
c	-20%	0.08	0.2580	0.4865	7.54	8968.27
	-10%	0.09	0.2574	0.4868	7.55	8964.94
	0%	0.1	0.2573	0.4872	7.55	8961.59
	10%	0.11	0.2572	0.4875	7.55	8958.23
	20%	0.12	0.2571	0.4879	7.55	8954.85
α	-20%	1216	0.2016	0.3840	5.81	7242.89
	-10%	1368	0.2305	0.4353	6.67	8098.09
	0%	1520	0.2597	0.4872	7.55	8961.59
	10%	1672	0.2890	**0.5398 (Infeasible)**	8.46	9831.52
	20%	1824	0.3182	**0.5926 (Infeasible)**	9.38	10694.99
β	-20%	800	0.2452	0.4688	7.23	8819.11
	-10%	900	0.2522	0.4777	7.38	8888.97
	0%	1000	0.2597	0.4871	7.55	8961.59
	10%	1100	0.2677	0.4974	7.73	9037.19
	20%	1200	0.2764	**0.5083 (Infeasible)**	7.92	9116.00
τ	-20%	56.0608	0.2598	0.4873	7.55	8962.54
	-10%	63.0684	0.2597	0.4873	7.55	8962.07
	0%	70.0760	0.2597	0.4872	7.55	8961.59
	10%	77.0836	0.2596	0.4871	7.55	8961.12
	20%	84.0912	0.2596	0.4871	7.55	8960.65
γ	-20%	0.712	0.2597	0.4872	7.55	8961.59
	-10%	0.801	0.2597	0.4872	7.55	8961.59
	0%	0.890	0.2597	0.4872	7.55	8961.59
	10%	0.979	0.2597	0.4872	7.55	8961.59
	20%	1.068	0.2597	0.4872	7.55	8961.60
h_d	-20%	0.24	0.2596	0.4871	7.55	8961.02
	-10%	0.27	0.2596	0.4872	7.55	8961.31
	0%	0.30	0.2597	0.4872	7.55	8961.59
	10%	0.33	0.2597	0.4873	7.55	8961.88
	20%	0.36	0.2597	0.4873	7.55	8962.16
h_g	-20%	0.64	0.3524	**0.6460 (Infeasible)**	9.30	9101.82
	-10%	0.72	0.2991	**0.5551 (Infeasible)**	8.29	9019.63
	0%	0.80	0.2597	0.4872	7.55	8961.59
	10%	0.88	0.2293	0.4343	6.97	8920.33

Parameter	Change %	Values	t_{11} (in year)	T_1 (in year)	u (in $)	Total Cost TC (in $)
	20%	0.96	0.2052	0.3920	6.50	8891.18
C	-20%	8	0.1961	0.3785	5.69	7242.98
	-10%	9	0.2271	0.4317	6.59	8092.08
	0%	10	0.2597	0.4871	7.55	8961.59
	10%	11	0.2939	**0.5454 (Infeasible)**	8.58	9850.49
	20%	12	0.3302	**0.6066 (Infeasible)**	9.69	10758.26
A	-20%	80	0.2613	0.4878	7.56	8920.57
	-10%	90	0.2605	0.4875	7.55	8941.07
	0%	100	0.2597	0.4872	7.55	8961.59
	10%	110	0.2589	0.4869	7.54	8982.12
	20%	120	0.2581	0.4867	7.54	9002.67
d	-20%	0.40	0.2563	0.4815	7.45	8873.76
	-10%	0.45	0.2580	0.4844	7.50	8917.65
	0%	0.50	0.2597	0.4872	7.55	8961.59
	10%	0.55	0.2613	0.4901	7.60	9005.58
	20%	0.60	0.2630	0.4929	7.65	9049.62
η	-20%	655.2	0.2595	0.4871	7.55	8960.42
	-10%	737.1	0.2596	0.4871	7.55	8961.07
	0%	819.0	0.2597	0.4872	7.55	8961.59
	10%	900.9	0.2597	0.4873	7.55	8962.02
	20%	982.8	0.2598	0.4873	7.55	8962.38
m	-20%	0.4	0.2597	0.4872	7.88	8962.26
	-10%	0.45	0.2597	0.4872	7.71	8961.92
	0%	0.5	0.2597	0.4872	7.55	8961.59
	10%	0.55	0.2597	0.4872	7.40	8961.29
	20%	0.6	0.2596	0.4872	7.25	8961.00
μ	-20%	4	0.2598	0.4872	8.41	8963.37
	-10%	4.5	0.2597	0.4872	7.94	8962.41
	0%	5	0.2597	0.4872	7.55	8961.59
	10%	5.5	0.2596	0.4872	7.21	8960.89
	20%	6	0.2596	0.4872	6.91	8960.28

N.B.: The solution is declared to be infeasible because life time is less than the cycle time.

In order to observe the sensitivity of the model parameters on the optimal solution, we consider the data as given in numerical Example. Optimal solutions for different values of a, b, c, α, β, τ, γ, h_g, h_d, A, C, d, η, m and μ are presented in Table 2. The following observation could be made from Table 2.

1. In Table 2. , constants of Screening rate and purchase cost increases screening time rapidly whereas scale demand and holding cost rate for good item decreases screening time rapidly. However, linear rate of screening and screening cost increases screening time slowly wherever quadratic rate of change of demand of the product, ordering cost and rate of investment of preservation technology decreases screening time slowly. In addition, change in linear rate of change of demand, learning process rate γ, η, τ, maximum life-time of the product and holding cost rate for defective item screening time remain constant.
2. In Table 2. , linear rate of screening, constants of Screening rate and purchase cost increases cycle time rapidly although scale demand and holding cost rate for good item decreases cycle

time rapidly. However, quadratic rate of change of demand of the product and screening cost increases screening time slowly wherever ordering cost and linear rate of change of demand decreases cycle time slowly. In addition, change in, rate of investment of preservation technology, learning process rate γ, η, τ, maximum life-time of the product and holding cost rate for defective item cycle time remain constant.

3. In Table 2. , constants of screening rate and purchase cost increases investment of preservation technology rapidly whereas scale demand, rate of investment of preservation technology and holding cost rate for good item decreases investment of preservation technology rapidly. However, linear rate of screening and screening cost increases investment of preservation technology slowly wherever maximum life-time of the product decreases investment of preservation technology slowly. In addition, change in linear rate of change of demand, learning process rate γ, η, τ, quadratic rate of change of demand of the product, ordering cost and holding cost rate for defective item investment of preservation technology remain constant.
4. In Table 2. , constants of screening rate and purchase cost increases total cost of the inventory system rapidly whereas scale demand decreases total cost of the inventory system rapidly. However, linear rate of screening, ordering cost and screening cost increases total cost of the inventory system slowly wherever quadratic rate of change of demand of the product and holding cost rate for good item decreases total cost of the inventory system slowly. In addition, change in linear rate of change of demand, learning process rate γ, η, τ, maximum life-time of the product, holding cost rate for defective item and rate of investment of preservation technology total cost of the inventory system remain constant.

CONCLUSION

In this paper, we consider inventory control model for instantaneous deteriorating imperfect quality item under screening time, replenishment time and preservation technology investment with quadratic demand. In this article, a general inventory control model for items with imperfect quality was presented. Each lot is subjected to a 100 per cent screening and items not conforming to certain quality standards are stored in a separate facility with different holding costs of the perfect and imperfect items being considered. The obtained numerical results reflect the learning process effects incorporated in the proposed model. Due to time varying deteriorating item retailer invest money on preservation technology to reduce deterioration. The total cost of the inventory system with respect to the screening time, replenishment time and investment of preservation technology is minimized. For numerical examples, inventory control system reaches the minimum cost and carry-out sensitivity analysis with respect to inventory parameters. Current research have numerous possible extension like, model can be further generalized by taken more items at a time. One can also analyzed multi layered supply chain. Research can be extended for finite or infinite planning horizons.

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REFERENCES

- Alamri, A., A., Irina H. and Aris, A. S., (2016). Efficient inventory control for imperfect quality items. *European Journal of Operational Research,* 254 (1), 92-104.
- Bakker, M., Riezebos, J. and Teunter, R., (2012). Review of inventory system with deterioration since 2001. *European Journal of Operation Research*, 221 (2), 275-284.
- Carlson, J. G. (1973). Cubic learning curve: Precession tool for labour estimating. *Manufacturing Engineering and Management, 71* (5), 22–25.
- Chung, K. J. and Cárdenas-Barrón, L.E., (2013). The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade-credit in the supply chain management. *Applied Mathematical Modelling,* 37 (7), 4653-4660.
- Ghare, P. M. and Scharender, G. H., (1963). A model for exponentially decaying inventory system. *Journal of Industrial Engineering,* 14 (5), 238-243.
- Goyal, S. K. and Giri, B. C., (2001). Recent trends in modeling of deteriorating inventory. *European Journal of Operational Research,* 134 (1), 1-16.
- Hsieh, T. P., and Dye, C. Y. (2013). A production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. *Journal of Computational and Applied Mathematics*, 239 (1), 2536.
- Hsu, P. H., Wee, H. M. and Teng, H. M. (2010). Preservation technology investment for deteriorating inventory *.International Journal of Production Economics* 124 (2), 388-394.
- Huang, C. K. (2004). An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration. *International Journal of Production Economics, 91* (1), 91–98.
- Jaber, M., Goyal, S., and Imran, M. (2008). Economic production quantity model for items with imperfect quality subject to learning effects. *International Journal of Production Economics, 115* (1), 143–150.
- Jaggi, C. K., and Mittal, M. (2011). Economic order quantity model for deteriorating items with imperfect quality. *Revista Investigación Operacional, 32* (2), 107–113.
- Jaggi, C. K., Goel, S. K., and Mittal, M. (2011). Economic order quantity model for deteriorating items with imperfect quality and permissible delay on payment. *International Journal of Industrial Engineering Computations,* 2(1), 237–248.
- Jordan, R. B. (1958). Learning how to use the learning curve. *NAA Bulletin, 39* (5), 27–39.
- Maddah, B., and Jaber, M. (2008). Economic order quantity for items with imperfect quality. Revisited. *International Journal of Production Economics, 112* (2), 808–815.

- Moussawi-Haidar, L., Salameh, M., and Nasr, W., (2014). Effect of deterioration on the instantaneous replenishment model with imperfect quality items. *Applied Mathematical Modelling, 38* (24), 5956–5966.
- Salameh, M. K., and Jaber, M. Y. (2000). Economic production quantity model for items with imperfect quality. *International Journal of Production Economics, 64* (1), 59–64.
- Sana, S. S. (2012). An economic order quantity model for nonconforming quality products. *Service Science, 4* (4), 331–348.
- Shah Nita. H., Cárdenas-Barrón, L.E., (2015). Retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount, *Applied Mathematics and Computation*, 259 (1), 569-578.
- Shah, Nita H. and Shah, Y. K., (2000). Literature survey on inventory models for deteriorating items. *Economic Annals,* 44 (145), 221-237.
- Shah, Nita. H., Chaudhari, U. B., Jani M.Y., (2016a). An integrated production-inventory model with preservation technology investment for time-varying deteriorating item under time and price sensitive demand. *International Journal of Inventory Research*, 3(1), 81-98.
- Shah, Nita. H., Chaudhari, U. B., Jani M.Y., (2016b). Optimal policies for time-varying deteriorating item with preservation technology under selling price and trade credit dependent quadratic demand in a supply chain. *Int. J. Appl. Comput. Math,* DOI 10.1007/s40819-016-0141-3.
- Soni, H. N., & Patel, K. A. (2012). Optimal pricing and inventory policies for non-instantaneous deteriorating items with permissible delay in payment: fuzzy expected value model. *International Journal of Industrial Engineering Computations*, 3(1), 281–300.

Appendix-1

$$I_{g1j}(t) = \left(I_{g11j}(t) + I_{g12j}(t)\right)\left(1+m-t\right)^{\frac{1}{1+\mu u}} \quad ,0 \le t \le t_{1j}\,.$$

Appendix-2

$$I_{g11j}\left(t\right) = -S_1\left(\left((1+m)\begin{pmatrix} 7ab\eta\mu - 6ac\eta\mu u + 7\beta m\mu\tau u + 3e^{\gamma j}ab\mu^3u^3 \\ -2e^{\gamma j}ac\mu^3u^3 + 3ab\eta\mu^3u^3 - 2ac\eta\mu^3u^3 \\ +3\beta m\mu^3u^3\tau + 8e^{\gamma j}ab\mu^2u^2 - 6e^{\gamma j}ac\mu^2u^2 \\ +8ab\eta\mu^2u^2 - 6ac\eta\mu^2u^2 + 8\beta m\mu^2u^2\tau \\ +7e^{\gamma j}ab\mu u - 6e^{\gamma j}ac\mu u - 2ac\eta m^2\mu^3u^3 \\ -12ac\eta m\mu^2u^2 - 6e^{\gamma j}acm^2\mu u - 6ac\eta m^2\mu u \\ +7e^{\gamma j}abm\mu u - 12e^{\gamma j}acm\mu u + 7ab\eta m\mu u \\ -12ac\eta m\mu u - 2e^{\gamma j}acm^2\mu^3u^3 + 3e^{\gamma j}abm\mu^3u^3 \\ -4e^{\gamma j}acm\mu^3u^3 + 3ab\eta m\mu^3u^3 - 4ac\eta m\mu^3u^3 \\ -6e^{\gamma j}acm^2\mu^2u^2 - 6ac\eta m^2 + 8e^{\gamma j}abm\mu^2u^2 \\ -12e^{\gamma j}acm^2\mu^2u^2 + 8ab\eta m\mu^2u^2 + 9a\eta\mu u \\ +9\alpha\mu\tau u + 7\beta\mu\tau u + 6\alpha\tau\mu^3u^3 + 3\beta\tau\mu^3u^3 \\ +13e^{\gamma j}a\mu^2u^2 + 13a\eta\mu^2u^2 + 13\alpha\mu^2u^2\tau + 8\beta\mu^2u^2\tau \\ +9e^{\gamma j}a\mu u + 6e^{\gamma j}a\mu^3u^3 + 6a\eta\mu^3u^3 + 2e^{\gamma j}ab \\ -2e^{\gamma j}ac + 2ab\eta - 2ac\eta + 2\beta m\tau - 2e^{\gamma j}acm^2 \\ -2ac\eta m^2 + 2e^{\gamma j}abm - 4e^{\gamma j}acm + 2ab\eta m \\ -4ac\eta m + 2e^{\gamma j}a + 2a\eta + 2\alpha\tau + 2\beta\tau \end{pmatrix} \\ -Q_j\left(6\eta\mu^3u^3 - 7\eta\mu^2u^2 - 2\eta\mu u - 6e^{\gamma j}\mu^3u^3 - 7e^{\gamma j}\mu^2u^2 - 2e^{\gamma j}\mu u\right)\right)(1+m)^{\frac{-1}{\mu u+1}}\right)$$

where, $S_1 = \dfrac{1}{\mu u\left(6e^{\gamma j}\mu^2u^2 + 6\eta\mu^2u^2 + 7\eta\mu u + 2e^{\gamma j} + 2\eta\right)}$

Appendix-3

$$I_{g12j}\left(t\right)=S_2\left(\left(\left(1+m-t\right)^{1-\frac{1}{1+\mu u}}\left(1+\mu u\right)\left(\begin{array}{l}7a\eta\mu u+7\alpha\mu\tau u+7e^{\gamma j}a\mu u+3\beta t\tau\mu^2u^2\\+3ab\eta\mu^2u^2-2ac\eta\mu^2u^2+3\beta m\mu^2u^2\tau\\+3e^{\gamma j}ab\mu^2u^2-2e^{\gamma j}ac\mu^2u^2-4ac\eta\mu u\\-4ace^{\gamma j}\mu u-2e^{\gamma j}ac+6a\eta\mu^2u^2\\+6\alpha\mu^2u^2\tau+3\beta\mu^2u^2\tau-2ac\eta m^2+6ae^{\gamma j}\mu^2u^2\\-2ace^{\gamma j}m^2-4acm\eta-4e^{\gamma j}acm+2abe^{\gamma j}\\+2ab\eta+2abm\eta+5\beta\mu\tau u+2e^{\gamma j}abm-2ac\eta\\+2\beta m\tau+5ab\eta\mu u+5\beta m\mu\tau u+5e^{\gamma j}ab\mu u\\+2\tau\beta+2a\eta+2\alpha\eta+2e^{\gamma j}a+2\beta\mu t\tau u\\-2ac\eta m\mu^2u^2t-2e^{\gamma j}acm\mu^2u^2t-2ac\eta\mu umt\\-2ace^{\gamma j}\mu umt-2ac\eta\mu^2u^2t^2+3ab\eta\mu^2u^2t\\-2ac\eta\mu^2u^2t-2e^{\gamma j}ac\mu^2u^2t^2+3abe^{\gamma j}\mu^2u^2t\\-2ace^{\gamma j}\mu^2u^2t+3ab\eta m\mu^2u^2-4ac\eta m\mu^2u^2\\-4ac\eta m^2\mu u-2e^{\gamma j}acm^2\mu^2u^2+3abme^{\gamma j}\mu^2u^2\\-4ace^{\gamma j}m\mu^2u^2-4ace^{\gamma j}m^2\mu u-2ac\eta m^2\mu^2u^2\\-8ac\eta m\mu u-8ace^{\gamma j}m\mu u-2ac\eta\mu ut\\+5abe^{\gamma j}m\mu u-2ace^{\gamma j}\mu ut+5ab\eta m\mu u\\-ac\eta\mu t^2u+2ab\eta\mu tu-e^{\gamma j}ac\mu t^2u+2e^{\gamma j}ab\mu tu\end{array}\right)\right)\right)$$

Where, $S_2=\dfrac{1}{\mu u\left(2\mu u+1\right)\left(3\mu u+2\right)\left(\eta+e^{\gamma j}\right)}$

Appendix-4

$$I_{g2j}\left(t\right)=\frac{1+\mu u}{(6\mu^2u^2+7\mu u+2)\mu u}\left(\begin{array}{l}\left(\left(1+m-t\right)^{1-\frac{1}{\mu u+1}}a\left(\begin{array}{l}-2cm^2\mu^2u^2-2cm\mu^2tu^2\\-2c\mu^2t^2u^2+3bm\mu^2u^2+3b\mu^2tu^2\\-4cm\mu^2u^2-2c\mu^2tu^2+3b\mu^2u^2\\-4cm^2\mu u-2cm\mu tu-2c\mu^2u^2\\-c\mu t^2u+5bm\mu u+2b\mu tu\\-8cm\mu u-2c\mu tu+6\mu^2u^2\\+5b\mu u-2cm^2-4c\mu u+2bm\\-4cm+7\mu u+2b-2c+2\end{array}\right)\right)\\-\left(\left(1+m-T_j\right)^{1-\frac{1}{\mu u+1}}a\left(\begin{array}{l}-2cm^2\mu^2u^2-2cm\mu^2T_ju^2\\-2c\mu^2{T_j}^2u^2+3bm\mu^2u^2\\+3b\mu^2T_ju^2-4cm\mu^2u^2\\-2c\mu^2T_ju^2+3b\mu^2u^2\\-4cm^2\mu u-2cm\mu tu-2c\mu^2u^2\\-c\mu {T_j}^2u+5bm\mu u+2b\mu T_ju\\-8cm\mu u-2c\mu T_ju+6\mu^2u^2\\+5b\mu u-2cm^2-4c\mu u+2bm\\-4cm+7\mu u+2b-2c+2\end{array}\right)\right)\end{array}\right)$$

$$,t_{1j}\le t\le T_j$$

Appendix-5

and $I_{dj}(t) = \dfrac{\tau\left(\frac{1}{2}t^2\beta + \alpha t\right)}{\eta + e^{\gamma j}} \qquad , 0 \le t \le t_{1j}$.