Effect of internal heating on weakly non-linear stability analysis of Rayleigh-Bénard Convection in a vertically oscillating Micropolar fluid

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Abstract

In this paper, we study the combined effect of internal heating and time-periodic gravity modulation on thermal instability in a micropolar fluid layer, heated from below. The time-periodic gravity modulation, considered in this problem can be realized by vertically oscillating the fluid layer. A weak non-linear stability analysis has been performed to get expression for Nusselt number using Ginzburg–Landau equation derived from Lorenz equations. Effects of various parameters such as internal Rayleigh number, Prandtl number, amplitude and frequency of gravity modulation have been analysed on heat transport. It is found that the response of the convective system to the internal Rayleigh number is destabilizing. Further, it is found that the heat transport can be controlled by suitably adjusting the amplitude and frequency of gravity modulation.

INTRODUCTION

In recent years, the dynamics of micropolar fluids has been a popular area of research for engineers and scientists who works with hydrodynamic problems (Lukaszewicz[1], Eringen[2], Sheremet[3], Miroshnichenko[4], Dupuy[5], Gibanov[6], Boukrache[7], Benes[8]) . Several interesting aspects are revealed during the study of these problems which we cannot found in Newtonian fluids. These micropolar fluid, which is non-symmetric to stress tensor, consists of randomly oriented particles and each element has translation as well as rotational motions. Eringen[2] was the first who formulated general theory of the micropolar fluids. Siddheshwar and Pranesh[9,10], Pranesh et al[11-14] have carried out investigations in micropolar fluid under different situations.

In the research of fluid dynamics, much attention was drawn to controlling the convection by considering the effects of time periodic vertical oscillations or gravity modulation or g-jitter. The complex body forces which make these oscillating forces to arise occurs in different ways. This time periodic vertical oscillations have applications in experiments of space laboratory, crystal growth field, large-scale convection of atmosphere and so on. Nelson[15], Wadih[16,17] carried out experiments under microgravity conditions with materials processing or physics of fluids in an arbitrary space craft. Effect of g-jitter on the stability of a heated fluid layer was first studied by Gershuni[18] and Greshof[19]. Some studies related to time periodic vertical oscillations are Siddheshwar[19,20], Pranesh [12,13], Bhadauria [21], Lyubimova[22] and Maria et al[23].

In earth’s mantle, radioactive decay of elements produces heat which induces convection[24], in atmosphere, motion takes place due to heat absorption from sunlight [25] and the convection resulted from volumetric heat produced by nuclear fusion reaction is important in studying many astronomical events[26]. These are the geophysical, astrophysical and industrial processes which we come across convection induced by internal heat generation. Literature that reports internal heat generation is Bhattacharya [27], Takashima[28], Pranesh et al [13,14,29,30] and recently Maria et al[23].

Several works have been done to study the onset of convection and heat transfer in micropolar fluid with gravity modulation. But, for non-linear analysis of micropolar fluid with internal heat generation case, no study is being reported under gravity modulation, where heat transport, in terms of amplitude and frequency of modulation, is regulated in the system because of vibrations produced in the system. Objectives of this work are

(i) Deriving Ginzburg Landau equation from Lorenz equations (Siddheshwar[31]).

(ii) Quantifying the heat transport in the system by considering different values to the parameters.

MATHEMATICAL FORMULATION

Consider an infinite horizontal layer of Boussinesquian, micropolar fluid of depth d, where the fluid is heated from below with the internal heat generation existing within the fluid system. Let ΔT be the temperature difference between the lower and upper surfaces with the lower boundary at a higher temperature than the upper boundary. These boundaries maintained at constant temperature. A Cartesian system is taken with origin in the lower boundary and z-axis vertically upwards (see figure 1).
Basic State:
The basic state of the fluid is quiescent and is described by
\( q = q_0(0,0,0), \quad \dot{\omega} = \dot{\omega}_b(0,0,0) \), \( p = p_b(z), \quad \rho = \rho_b(z), \quad T = T_b(z) \). (7)

Substituting equation (7) into basic governing equations (1)-(6), we obtain the following quiescent state solutions:
\[
\frac{dp_b}{dz} + \rho_b g_0 (1 + \alpha \cos(\omega t)) \dot{k} = 0, \quad (8)
\]
\[
\chi \frac{d^2 T_b}{dz^2} = - Q(T - T_0), \quad (9)
\]
\[
\rho_b = \rho_0 [1 - \alpha (T_b - T_0)]. \quad (10)
\]
Solving equation (9) subject to the boundary conditions
\( T_b = T_0 + \Delta T \) at \( z = 0 \) and \( T_b = T_0 \) at \( z = d \),
we obtain \( T_b = T_0 + \Delta T \frac{\sin \left( \frac{Qd^2}{\chi} \left( 1 - \frac{z}{d} \right) \right)}{\sin \left( \frac{Qd^2}{\chi} \right)} \). (11)

Stability Analysis:
The stability of the basic state is analyzed by introducing the following perturbations
\( \ddot{q} = \ddot{q}_b + \dot{\omega} \), \( \dot{\omega} = \ddot{\omega}_b + \dot{\omega} \), \( p = p_b + p' \), \( \rho = \rho_b + \rho' \), \( T = T_b + T' \). (12)
where, the prime indicates that the quantities are infinitesimal perturbations.

Substituting equation (12) into the equations (1)-(6) and using the basic state solutions, we get linearized equations governing the infinitesimal perturbations in the form:
\[
\nabla q' = 0, \quad (13)
\]
\[
\rho_0 \left[ \frac{\partial \ddot{q}'}{\partial t} + (\ddot{q}' \nabla) \ddot{\omega} \right] = \frac{\beta}{\rho_0 C_v} \nabla \times \ddot{\omega} \nabla T + \chi \nabla^2 T + Q(T - T_0), \quad (14)
\]
\[
\rho_0 \left[ \frac{\partial \dot{\omega}'}{\partial t} + (\dot{q}' \nabla) \dot{\omega} \right] = (\lambda' + \eta') \nabla (\nabla \dot{\omega}) + \eta' \nabla^2 \dot{\omega}, \quad (15)
\]
\[
\frac{\partial \dot{T}'}{\partial t} + \frac{\partial T'}{\partial z} = \frac{\Delta T}{d} \frac{Qd^2}{\chi} \left( \frac{1 - z}{d} \right) \frac{Qd^2}{\chi} \left[ W + \frac{\beta}{\rho_0 C_v} \nabla \times \dot{\omega} \right], \quad (16)
\]
\[
\rho' = -\alpha p_0 T'. \quad (17)
\]
Substituting equation (17) in equation (14) and eliminating pressure by cross differentiation, we get

\[
\rho_0 \left[ \frac{\partial}{\partial t} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + u \left( \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial z^2} \right) + w \left( \frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 u}{\partial x^2} \right) \right] = -\rho_0 g_0 \left[ 1 + \epsilon \cos(\omega t) \right] \frac{\partial^2 T}{\partial x^2} + \left( 2 \zeta + \eta \right) \nabla^2 \left( \frac{\partial w}{\partial z} - \frac{\partial w}{\partial x} \right) + \zeta \left( \frac{\partial^2 \omega_y}{\partial z^2} - \frac{\partial^2 \omega_y}{\partial x^2} - 2 \omega_y \right)
\]

(18)

Writing y-component of the equation (15) we get

\[
\rho_0 \left[ \frac{\partial \omega_y}{\partial t} + \alpha_b \frac{\partial \omega_y}{\partial z} + \alpha_a \frac{\partial \omega_y}{\partial x} + \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial z} \right) \omega_y \right] = \eta \nabla^2 \omega_y + \zeta \left( \frac{\partial^2 \omega_y}{\partial z^2} - \frac{\partial^2 \omega_y}{\partial x^2} - 2 \omega_y \right)
\]

(19)

We consider only two dimensional disturbances and thus restrict ourselves to the xz-plane.

We now introduce the stream functions in the form

\[
u = \frac{\partial \psi}{\partial z}, w = -\frac{\partial \psi}{\partial x},
\]

which satisfies the continuity equation (13).

On using equations (20) in equations (16), (18) and (19), we get

\[
\rho_0 \left[ \frac{\partial}{\partial t} \left( \frac{\partial \psi}{\partial z} \right) \right] = -\rho_0 g_0 \left[ 1 + \epsilon \cos(\omega t) \right] \frac{\partial^2 T}{\partial x^2} + \left( 2 \zeta + \eta \right) \nabla^2 \psi - \zeta \nabla^2 \omega_y + \rho_0 \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \right)
\]

(21)

\[
\rho_0 \frac{\partial \omega_y}{\partial t} = \eta \nabla^2 \omega_y + \zeta \left( \frac{\partial^2 \omega_y}{\partial z^2} - \frac{\partial^2 \omega_y}{\partial x^2} - 2 \omega_y \right) + \rho_0 \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} \right)
\]

(22)

\[
\frac{\partial T}{\partial t} = -\frac{\partial T_0}{\partial z} \left( \frac{\partial \psi}{\partial x} + \frac{\beta}{\rho_0 c_v} \frac{\partial \omega_y}{\partial x} \right) + \chi \nabla^2 T + QT
\]

(23)

where, \( J \) stands for Jacobian. Let

\[
\left( x^*, y^*, z^* \right) = \left( \frac{x}{d}, \frac{y}{d}, \frac{z}{d} \right), \quad t^* = \frac{t}{\sqrt{\chi/d^2}} \quad v^* = \frac{\psi}{\chi/d},
\]

(24)

\[
T^* = \frac{T - T_0 \omega_y}{\Delta T}, \quad \Omega = \frac{\gamma}{d^2} \quad \lambda = \frac{\chi}{d^2}
\]

where, * denotes the non-dimensionalised quantity.

Substituting equation (24) into equations (21)-(23) we get the following dimensionless (on dropping the asterisks for simplicity);

\[
\frac{1}{\Pr} \frac{\partial}{\partial t} \left( \frac{\partial^2 \psi}{\partial z^2} \right) - J \left( \frac{\partial \psi}{\partial x} \right) = -R \left( 1 + \epsilon \cos(\omega t) \right) \frac{\partial T}{\partial x} + \left( 1 + N_1 \right) \nabla^2 \psi - N_2 \nabla^2 \omega_y
\]

(25)

\[
\frac{\partial \omega_y}{\partial t} = N_3 \nabla^2 \omega_y + N_5 \nabla^2 \psi - 2 N_4 \omega_y
\]

(26)

\[
\frac{\partial T}{\partial t} = -\frac{T_0}{\Delta T} \left( \frac{\partial \psi}{\partial x} + \frac{\beta}{\rho_0 c_v} \frac{\partial \omega_y}{\partial x} \right) + \chi \nabla^2 T + QT
\]

(27)

where,

\[
\Pr = \frac{\zeta + \eta}{\chi / \rho_0}
\]

(Prandtl Number),

\[
R = \frac{\rho_0 g_0 \Delta T d^3}{\chi (\zeta + \eta)}
\]

(Rayleigh Number),

\[
N_1 = \frac{\zeta}{\zeta + \eta}
\]

(Coupling Parameter),

\[
N_2 = \frac{I}{d^2}
\]

(Inertia Parameter),

\[
N_3 = \frac{\lambda + \eta}{\chi / \rho_0 \Delta T d^2}
\]

(Couple Stress Parameter),

\[
N_5 = \frac{\beta}{\rho_0 c_v d^2}
\]

(Micropolar Heat Conduction Parameter),

\[
Ri = \frac{Q d^2}{\chi}
\]

(Internal Rayleigh Number)

\[
\frac{\partial T_0}{\partial z} = \sqrt{Ri} \cos \left( \frac{\sqrt{Ri}}{Ri} \right)
\]

(Equations (25) to (27) are solved subject to free-free, isothermal and no spin boundary conditions, given by

\[
\psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial^2 \psi}{\partial x^2} = \omega_y = T = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = 1.
\]

Linear Stability Theory:

In this section, we discuss the linear stability analysis, which is of great utility in the local nonlinear stability analysis to be discussed further on. The linearized version of equations (25)-(27), after neglecting the Jacobian, is

\[
\frac{1}{\Pr} \frac{\partial}{\partial t} \left( \frac{\partial^2 \psi}{\partial z^2} \right) - J \left( \frac{\partial \psi}{\partial x} \right) = -R \left( 1 + \epsilon \cos(\omega t) \right) \frac{\partial T}{\partial x} - N_1 \nabla^2 \omega_y,
\]

(28)
where, \( f = \text{Real part of } (e^{-\text{i} \omega t}) \) and \( f' = (\text{-i} \Omega ) \text{ Real part of } (e^{-\text{i} \omega t}) \). In the dimensionless form, the velocity boundary conditions for solving equation (31) are obtainable from equations (28)-(30) in the form

\[
\psi = \frac{\partial^2 \psi}{\varepsilon z^2} + \frac{\partial^4 \psi}{\varepsilon^2 z^4} - \frac{\partial^6 \psi}{\varepsilon^3 z^6} - \frac{\partial^8 \psi}{\varepsilon^4 z^8} = 0 \text{ at } z = 0, 1.
\]  

(32)

**Finite Amplitude Convection:**

The finite amplitude analysis is carried out here via Fourier series representation of stream function \( \psi \), the spin \( \omega_x \), and the temperature distribution \( T \). Linear analysis gives only the condition for onset of convection, it is insufficient to explain the rate of heat transport, therefore non-linear analysis is carried out.

The first effect of non-linearity is to distort the temperature field through the interaction of \( \psi \) and \( T \). The distortion of temperature field will correspond to a change in the horizontal mean, i.e., a component of the form \( \sin(2\pi z) \) will be generated. Thus, a minimal double Fourier series which describes the finite amplitude convection is given by

\[
\psi(x, y, t) = A(t) \sin(\pi \alpha x) \sin(\pi y) \]

\[
\omega_x(x, y, t) = B(t) \sin(\pi \alpha x) \sin(\pi y) \]

\[
T(x, y, t) = E(t) \cos(\pi \alpha x) \sin(\pi y) + F(t) \sin(\pi x) \]

(33)

where, A, B, E and F are the amplitudes to be determined from the dynamics of the system. Since the spontaneous generation of large scale flow has been discounted, the functions \( \psi \) and \( \omega_x \) do not contain an \( x \)-independent term.

Substituting equation (33) into equations (25)-(27) and following standard procedure, we obtain the following non-linear non-autonomous system (generalized Lorenz model) of differential equations:

\[
A(t) = -\frac{R(1 + \varepsilon \cos(\Omega t)) \Pr \pi \alpha}{k^2} E(t) - (1 + N_1) \Pr k^2 A(t) - N_1 \Pr B(t),
\]

(34)

\[
E(t) = -\frac{\partial T_0}{\varepsilon z} N_5 \pi \alpha B(t) - \frac{\partial T_0}{\varepsilon z} \pi \alpha A(t) - (k^2 - Rl) E(t)
- \pi^2 \varepsilon N_5 B(t) F(t) - \pi^2 \varepsilon A(t) F(t),
\]

(35)

\[
F(t) = 4\pi^2 F(t) + Rl F(t) + \frac{\pi^2 \varepsilon N_5}{2} B(t) E(t)
+ \frac{\pi^2 \varepsilon}{2} A(t) E(t),
\]

(36)

where, over dot denotes time derivative with respect to \( t \). It is important to observe that the non-linearities in equations (34)-(37) stem from the convective terms in the energy equation (4) as in the classical Lorenz system.

**The Ginzburg-Landau equation from Lorenz model**

In this section, we derive the Ginzburg- Landau equation from Lorenz equations (34)-(37), from which we have

\[
A = -Y_1 \dot{B} - Y_2 B,
\]

(38)

\[
E = \frac{k^2}{R[1 + \varepsilon \cos(\Omega t)] \Pr \pi \alpha} - \frac{(1 + N_1) k^4}{R[1 + \varepsilon \cos(\Omega t)] \pi \alpha} A,
\]

(39)

\[
F = \frac{1}{\pi^2 \varepsilon N_5 B + \pi^2 \alpha A} \left[ -\dot{C} - \frac{\partial T_0}{\varepsilon z} N_5 \pi \alpha B - \frac{\partial T_0}{\varepsilon z} \pi \alpha A \right].
\]

(40)

Using equation (38) in equation (39) and then using the resulting equation along with equation (38) in (40), we get

\[
E = \frac{Y_1 \dot{B} + Y_1 \dot{B}}{[1 + \varepsilon \cos(\Omega t)]},
\]

(41)

\[
F = \frac{Y_1 \dot{B} + \left[ Y_1 + Y_1 \cos(\Omega t) \right] B + \left[ Y_1 + Y_1 \cos(\Omega t) \right] Y_1 B}{Y_1 \dot{B} + Y_1 B + \left[ Y_1 \sin(\Omega t) \right] \dot{B} + Y_1 \dot{B} + Y_1 \dot{B}},
\]

(42)

where

\[
Y_1 = \frac{N_2}{N_1 \Pr k^2},
\]

\[
Y_2 = \frac{N_1}{N_1} + \frac{2}{k^2},
\]

\[
Y_3 = \frac{Y_1 k^2}{R \Pr \pi \alpha},
\]

\[
Y_4 = \frac{Y_1 + (1 + N_1) \Pr k^2 Y_1}{R \Pr \pi \alpha},
\]

\[
Y_5 = \frac{1 + N_1 \Pr k^2 Y_1 - N_1 \Pr k^2}{R \Pr \pi \alpha}.
\]
\[ Y_6 = -\pi^2 \alpha Y_1, \]
\[ Y_7 = \pi^2 \alpha N_5 - \pi^2 \alpha Y_2, \]
\[ Y_8 = -\frac{Y_3}{[1 + \varepsilon \cos(\Omega t)]}, \]
\[ Y_9 = -\frac{Y_4 - \left(k^2 - R_i\right) Y_1}{[1 + \varepsilon \cos(\Omega t)]}, \]
\[ Y_{10} = -Y_1 \Omega \frac{[1 + \varepsilon \cos(\Omega t)]^2}{[1 + \varepsilon \cos(\Omega t)]^2}, \]
\[ Y_{11} = -\frac{Y_2 - \left(k^2 - R_i\right) Y_1}{[1 + \varepsilon \cos(\Omega t)]}, \]
\[ Y_{12} = -\frac{Y_6 \Omega}{[1 + \varepsilon \cos(\Omega t)]^2}, \]
\[ Y_{13} = \frac{\partial T_0}{\partial z} N_5 \pi \alpha + \frac{\partial T_0}{\partial z} \pi a Y_1, \]
\[ Y_{14} = -\frac{Y_4 \Omega}{[1 + \varepsilon \cos(\Omega t)]^2}, \]
\[ Y_{15} = \frac{-k^2 - R_i Y_2}{[1 + \varepsilon \cos(\Omega t)]}, \]
\[ Y_{16} = -\frac{\partial T_0}{\partial z} N_5 \pi \alpha + \frac{\partial T_0}{\partial z} \pi a Y_2, \]

Substituting equations (38), (41) and (42) in equation (37), we get a third order equation in B after neglecting the terms of the type:
\[ \frac{\partial^3 B}{\partial t^3} + \frac{\partial^3 B}{\partial z^3} + \frac{\partial^3 B}{\partial z^2} + \left(\frac{\partial B}{\partial t}\right)^2 + \frac{\partial B}{\partial t} \frac{\partial^3 B}{\partial z^2} + \frac{\partial^2 B}{\partial t^2} = \frac{\partial B}{\partial t} + \frac{\partial^2 B}{\partial t^2}. \]

Equation (43) is obviously the Ginzburg-Landau equation for non-linear convection in a micropolar fluid with vertical oscillation and heat source. By using equation (43) in equation (42), we get \( F \).

**Heat Transport**

The influence of gravity modulation with internal heat source on heat transport which is quantified in terms of Nusselt number (Nu) is defined as follows:

\[ Nu = \frac{Heat \text{ transport by (conduction+convection)}}{Heat \text{ transport by (conduction)}} \]

\[ = \left[ \frac{k}{2\pi} \int_0^z \left(\frac{2\pi}{k}\right)^2 \left(1 - z + T_z\right) dx \right]_{z=0} \]

where, subscript in the integrand denotes the derivative with respect to \( z \).

Substituting equation (33) in equation (44) and completing the integration, we get

\[ Nu = 1 - 2\pi F(t). \]

**RESULTS AND DISCUSSIONS**

In the study of thermal instability in a fluid layer, external regulation of convection is important. In this paper, time depended vertical oscillation (gravity modulation) is considered as an external force for convection. The non-linear analysis is carried out in order to study the effects of vertical oscillations and internal heating on heat transport in a micropolar fluid. The fourth order non-autonomous Lorenz model is obtained by using truncated Fourier series. Ginzburg-Landau equation is then derived from Lorenz model. The effect of Coupling Parameter \( N_1 \), Inertia Parameter \( N_2 \), Couple Stress Parameter \( N_3 \), Micropolar Heat Conduction Parameter \( N_4 \), Prandtl number \( Pr \), internal Rayleigh number \( R_i \), amplitude \( \varepsilon \) and frequency \( \Omega \) of the gravity modulation are analyzed and the results are depicted in figures (2)-(9).

From the figures, we observe that for small time, Nusselt number, Nu is less than one, which indicates that the heat transfer is due to conduction, as time increases, Nu also increases and becomes greater than one, which shows that convective region is in place. Further increase in time, the graphs remains oscillatory which means that early chaos is precipitated.

From figures (2)-(9), we observe that

(i) Increase in internal Rayleigh number increases the amount of heat transfer.

(ii) increase in \( N_1 \), decrease the rate of heat transfer and thus stabilizes the system. This
is on account of the presence of suspended particles, the critical Rayleigh number increases and hence the heat transport decreases.

(iii) increase in $N_2$, increases inertia of the fluid due to the suspended particles and then increases the amount of heat transfer.

(iv) increase in $N_3$ increases couple stress of the fluid and decreases gyrational velocities. So, increases the measure of heat transfer.

(v) When $N_5$ increases, the heat induced into the fluid, due to this microelements, also increases, thus reduces the heat transfer.

(vi) $Pr$ reduces the measure of heat transfer.

(vii) amplitude $\varepsilon$ of the gravity modulation increase the measure of heat transfer

(viii) frequency $\Omega$ of the modulation decrease the measure of heat transfer

CONCLUSION

(i) By adding suspended particles into Newtonian fluids, heat transport decreases and thus stabilizes the system.

(ii) $N_1$, $\Omega$ and $Pr$ decreases the rate of heat transfer.

(iii) $N_2, N_3, Pr, \varepsilon$ and $Ri$ increases the rate of heat transfer.

(iv) Heat transport can be controlled effectively by gravity modulation.

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Figures:

Figure 2: Plot of Nusselt number $Nu$ versus time $t$ for various values of internal Rayleigh number $Ri$ for $N_1 = 0.1,N_2 = 0.5,N_3 = 2,N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

Figure 3: Plot of Nusselt number $Nu$ versus time $t$ for various values of Coupling Parameter $N_1$ for $N_2 = 0.5,N_3 = 2,N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

Figure 4: Plot of Nusselt number $Nu$ versus time $t$ for various values of Inertia Parameter $N_3$ for $N_1 = 0.1,N_3 = 2,N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$. 

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Figure 5: Plot of Nusselt number $Nu$ versus time $t$ for various values of Couple Stress Parameter $N_3$ for $N_1 = 0.1, N_2 = 0.5, N_5 = 1, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

Figure 6: Plot of Nusselt number $Nu$ versus time $t$ for various values of Micropolar Heat Conduction Parameter $N_5$ for $N_1 = 0.1, N_3 = 2, N_2 = 0.5, Pr = 10, \varepsilon = 0.2, \Omega = 2, Ri = 2$.

Figure 7: Plot of Nusselt number $Nu$ versus time $t$ for various values of Prandtl number $Pr$ and internal Rayleigh number $Ri$ for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, \varepsilon = 0.2, \Omega = 2, Ri = 2, Pr = 10$.

Figure 8: Plot of Nusselt number $Nu$ versus time $t$ for various values of amplitude of modulation $\varepsilon$ for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, Pr = 10, \Omega = 2, Ri = 2$.

Figure 9: Plot of Nusselt number $Nu$ versus time $t$ for various values of frequency of modulation $\Omega$ for $N_1 = 0.1, N_2 = 0.5, N_3 = 2, N_5 = 1, \varepsilon = 0.2, Ri = 2, Pr = 10$. 