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Heat transfer in a Second order Fluid over a Continuous Stretching Surface subject to Transverse Magnetic Field

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Abstract

This paper presents numerical analysis of flow and heat transfer of MHD viscoelastic fluid over a stretching sheet subject to suction. The governing partial differential equations are converted to ordinary differential equations by a similarity transformation. The effects of viscous dissipation and work done by deformation are considered in the energy equation. The variations of flow and heat transfer characteristics with various physical parameters such as viscoelasticity (k_1), Prandtl number (Pr), magnetic parameter (Mn), viscous dissipation (Ec) and suction parameter(R) are presented through graphs and tables.

INTRODUCTION

Boundary-layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering processes. Such processes include heat-treated materials travelling between a feed roll and a wind-up roll or materials manufactured by extrusion and many others. Since the pioneering work of Sakiadis [1], various aspects of the problem have been investigated by many authors [2-5].

Danberg and Fansler [6] studied the solution for the boundary layer flow past a wall that is stretched with a speed proportional to the distance along the wall. Later many authors [7-9] extended the stretching sheet problem to viscoelastic fluid with various parameters.

Abel and Veena [10] investigated a viscoelastic fluid flow and heat transfer in a porous medium over a stretching sheet and observed that the dimensionless surface temperature profiles increases with an increase in viscoelastic parameter k_1. However, later, Abel et al. [11] studied the effect of heat transfer on MHD viscoelastic fluid over a stretching surface and an important finding was that the effect of visco-elasticity is to decrease the dimensionless surface temperature profiles in that flow. Furthermore, Char [12] studied MHD flow of a viscoelastic fluid over a stretching sheet; however, only the thermal diffusion is considered in the energy equation. Vajravelu and Rollins [13] obtained analytical solution for heat transfer characteristics in viscoelastic second order fluid over a stretching sheet with frictional heating and internal heat generation. Later, Sarma and Rao [14] extended the work of [13], studied the effect of work due deformation in the energy equation. Vajravelu and Roper [15] and Cortell [16] analyzed the effects of work due to deformation in viscoelastic second grade fluid over a stretching sheet.

Motivated by the above studies, in present paper studies the flow and heat transfer of an incompressible MHD viscoelastic fluid (Walter's liquid B model) past stretching sheet, by taking viscous dissipation and work due deformation terms in the energy equation. Nonlinear differential equations are solved using Quasilinearization technique. Thermal boundary condition is taken as Prescribed Surface Temperature. Results are in good agreement with available literature. This paper highlights the effect of visco-elasticity and work due to deformation on heat transfer characteristics of the fluid.

MATHEMATICAL FORMULATION

Momentum boundary layer equation

Following the postulates of gradually fading memory, Coleman and Noll [17] derived the constitutive equation of second-order fluid flow in the form

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \qquad (1)$$

where T is the Cauchy stress tensor, $-pI$ is the spherical stress due to constraint of incompressibility, μ is the dynamics viscosity, α_1, α_2 are the material constants and A_1 and A_2 are the first two Rivlin–Ericken tensors [18] defined as

$$A_1 = (grad\ v) + (grad\ v)^T \qquad (2)$$

$$A_2 = \frac{dA_1}{dt} + A_1(grad\ v) + (grad\ v)^T A_1 \qquad (3)$$

Here, v denotes the velocity field and d/dt is the material time derivative. If the fluid of second grade modeled by (1) is to be compatible with thermodynamics and is to satisfy the Clausius-Duhem inequality for all motions and the assumption that the specific Helmholtz free energy of the fluid is a minimum when it is locally at rest, Dunn and Fosdick [19] found that the material moduli must satisfy

$$\mu \geq 0, \alpha_1 \geq 0, \alpha_1 + \alpha_2 = 0 \qquad (4)$$

But later on Fosdick and Rajagopal [20] have reported, by using the data reduction from experiments, that in the case of a second order fluid the material constants μ, α_1, α_2 should satisfy the relation

$$\mu \geq 0, \alpha_1 \leq 0, \alpha_1 + \alpha_2 \neq 0 \qquad (5)$$

They also reported that that the fluids modeled by (1) with the relationship (5) exhibit some anomalous behavior. A critical review on this controversial issue can be found in the work of Dunn and Rajagopal [21]. We must mention that second-order fluid, obeying model equation (1) with $\alpha_1 < \alpha_2, \alpha_1 < 0$ although exhibits some undesirable instability characteristics, the second order approximations are valid at low shear rate. Now in literature the fluid satisfying the model equation (1) with $\alpha_1 < 0$ is termed as second-order fluid and with $\alpha_1 > 0$ is termed as second grade fluid.

We consider a laminar steady flow of an incompressible viscoelastic (Walters' liquid B model) fluid over a wall coinciding with the plane y = 0, the flow being confined to y > 0.Two equal and opposite forces are applied along the x-axis, so that a sheet is stretched with a velocity proportional to the distance from the origin. The resulting motion of the quiescent fluid is thus caused solely by the moving surface. The flow satisfies the rheological equation of state derived by Beard and Walters [22].

The governing boundary layer equations for momentum, in the usual form, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - k_0\left\{\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right\} - \frac{\sigma B_0^2 u}{\rho} \qquad (7)$$

$$where \;\; \upsilon = \frac{\mu}{\rho}, k_0 > 0$$

where u and v are the velocity components along the x and y directions respectively, υ are the kinematic viscosity $k_0 = -\alpha_1 / \rho$ is the co-efficient of elasticity, and ρ is the density, σ_0 is the electric conductivity, B_0 is the uniform magnetic field along the y-axis. Hence, in the case of second order fluid flow k_0 takes positive value as α_1 takes negative value and other quantities have their usual meanings. In deriving (7) it is assumed that the normal stress is of the same order of magnitude as that of the shear stress, in addition to usual boundary layer approximations.

The boundary conditions for the velocity field are:

$$u = u_w = bx, \;\; v = -v_0 \quad at \; y = 0, b > 0$$
$$u \to 0, \frac{\partial u}{\partial x} \to 0 \; as \; y \to \infty \qquad (8)$$

The condition $\partial u / \partial y \to 0 \;\; as \;\; y \to \infty$ is the augmented condition since the flow is in an unbounded domain, which has been discussed by Rajagopal etc., [23-25]. In this case, the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.

Defining new variables:

$$u = bxf_\eta(\eta), v = -\sqrt{b\upsilon}f(\eta), \eta = \sqrt{b/\upsilon}\,y \qquad (9)$$

where $f_\eta(\eta)$ denotes differentiation with respect to η. Clearly u and v defined above satisfy the continuity equation (6) and equation (7) is transformed as

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1\{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\} - Mn\, f_\eta \qquad (10)$$

where $k_1 = k_0 b / \upsilon$

The boundary conditions (8) become

$$f(0) = R, f_\eta(0) = 1 \qquad (11a)$$

$$f_\eta(\eta) \to 0, f_{\eta\eta}(\eta) \to 0 \;\; as\, \eta \to \infty \qquad (11b)$$

Here k_1 is the viscoelastic parameter, Mn= $\sigma B_0^2 / \rho b$ is the magnetic parameter and R= ($v_0 / \sqrt{b/\upsilon}$) is the suction parameter.

Heat transfer Analysis:

By using boundary layer approximations, and taken into account both viscous dissipation and work due to deformation, the equation of energy for temperature T is given by

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \rho k_0\frac{\partial u}{\partial y}\left[\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)\right]$$
(1 2)

where k is the thermal diffusivity and C_p the specific heat of a fluid at constant pressure.

The thermal boundary conditions depend on the type of heating process under consideration. Here it is considered as Prescribed

Surface Temperature (PST case).

Prescribed Surface Temperature (PST case):

For this circumstance, the boundary conditions are

$$T = T_w [= T_\infty + A\left(\frac{x}{l}\right)^2 \quad \text{at y=0} \tag{13a}$$

$$T \to T_\infty \qquad as \quad y \to \infty \tag{13b}$$

where l is the characteristic length. Define non-dimensional temperature as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{14}$$

Using (9) and (14), equation (12) reduces to

$$\theta'' + \Pr f\theta' - 2\Pr f'\theta = -\Pr Ec\left[(f'')^2 - k_1 f''(f'f'' - ff''')\right] \tag{15}$$

with boundary conditions

$$\theta(0) = 1, \theta(\infty) \to 0 \tag{16}$$

NUMERICAL SOLUTION OF THE PROBLEM

The flow equation (10) coupled with energy equation (15) constitute a set of highly nonlinear differential equations in each thermal boundary condition, so Obtaining closed form solution for this set is cumbersome and time consuming. Hence Quasilinearization method, given by Bellman & Kalaba [26] is used to solve this system.

For convenience equations (10) and (15) are rearranged as

$$f^{iv} = \frac{1}{k_1 f}\left[(f')^2 - ff'' - f''' + Mnf' + 2k_1 f'f''' - k_1 (f'')^2\right] \tag{17}$$

$$\theta'' = -\Pr f\theta' + 2\Pr f'\theta - \Pr Ec\left[(f'')^2 - k_1 f''(f'f'' - ff''')\right] \tag{18}$$

with boundary conditions:

$$f = R,\ f' = 1,\ \theta = 1 \qquad at \quad \eta = 0 \tag{19}$$

$$f' \to 0,\ f'' \to 0,\ \theta \to 0 \quad as\ \eta \to \infty \tag{20}$$

In order to implement the Quasilinearization method, the equations (20) and (21) are converted to a system of first order differential equations by substituting

$$(f, f', f'', f''', \theta, \theta') = (x_1, x_2, x_3, x_4, x_5, x_6)$$

Then equations (17) and (18) give

$$\frac{dx_1}{d\eta} = x_2$$

$$\frac{dx_2}{d\eta} = x_3$$

$$\frac{dx_3}{d\eta} = x_4$$

$$\frac{dx_4}{d\eta} = \frac{1}{k_1 x_1}\left\{x_2^2 - x_1 x_3 - x_4 + Mnx_2 + 2k_1 x_2 x_4 - k_1 x_3^2\right\} \tag{21}$$

$$\frac{dx_5}{d\eta} = x_6$$

$$\frac{dx_6}{d\eta} = -\Pr x_1 x_6 + 2\Pr x_2 x_5 - \Pr Ec\left[x_3^2 - k_1 x_3 (x_2 x_3 - x_1 x_4)\right]$$

Using Quasilinearization technique, the system (21) can be linearized as

$$\frac{dx_1^{r+1}}{d\eta} = x_2^{r+1}$$

$$\frac{dx_2^{r+1}}{d\eta} = x^{r+1}$$

$$\frac{dx_3^{r+1}}{d\eta} = x_4^{r+1}$$

$$\begin{aligned}\frac{dx_4^{r+1}}{d\eta} = &\left(\frac{-1}{k_1 (x_1^r)^2}\left((x_2^r)^2 - x_4^r + Mnx_2^r + 2k_1 x_2^r x_4^r - k_1 (x_3^r)^2\right)\right)x_1^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r}\left(2x_2^r + 2k_1 x_4^r + Mn\right)\right)x_2^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r}\left(-x_1^r - 2k_1 x_3^r\right)\right)x_3^{r+1} \\ &+ \left(\frac{1}{k_1 x_1^r}\left(-1 + 2k_1 x_2^r\right)\right)x_4^{r+1} + \left(\frac{-x_4^r + Mnx_2^r}{k_1 x_1^r}\right)\end{aligned} \tag{22}$$

$$\frac{dx_5^{r+1}}{d\eta} = x_6^{r+1}$$

$$\frac{dx_6^{r+1}}{d\eta} = \left(-\Pr x_6^r - \Pr Eck_1 x_3^r x_4^r\right)x_1^{r+1} + (2\Pr x_5^r + \Pr Eck_1\left(x_3^r\right)^2)x_2^{r+1}$$
$$+\left(-2Ec\Pr x_3^r + \Pr Eck_1\left(2x_2^r x_3^r - x_1^r x_4^r\right)\right)x_3^{r+1}$$
$$-\left(\Pr Eck_1 x_1^r x_3^r\right)x_4^{r+1} + (2\Pr x_2^r)x_5^{r+1} + (-\Pr x_1^r)x_6^{r+1}$$
$$+\left(Ec\Pr\left(x_3^r\right)^2 + \Pr x_1^r x_6^r - 2\Pr x_2^r x_5^r - 2\Pr Eck_1 x_3^r\left(x_2^r x_3^r - x_1^r x_4^r\right)\right)$$

The above system of equations (22) is linear in $x_i^{r+1}(i=1,2,...6$ and general solution can be obtained by using the principle of superposition.

The boundary conditions given by (19) and (20) reduce to

$$x_1^{r+1}(\eta)=0,\ x_2^{r+1}(\eta)=1,\ x_5^{r+1}(\eta)=1 \qquad at\ \eta=0$$

$$x_2^{r+1}(\eta)\to 0, x_3^{r+1}(\eta)\to 0, x_5^{r+1}(\eta)\to 0 \qquad as\ \eta\to\infty \tag{23}$$

The initial values are chosen as follows:

For the homogeneous solution:

$$x_i^{h_1}(\eta)=[0\ 0\ 1\ 0\ 0\ 0]$$
$$x_i^{h_2}(\eta)=[0\ 0\ 0\ 1\ 0\ 0] \tag{24}$$
$$x_i^{h_3}(\eta)=[0\ 0\ 0\ 0\ 0\ 1]$$

For particular solution:

$$x_i^{p}(\eta)=[R\ 1\ 0\ 0\ 1\ 0] \tag{25}$$

The general solution of system of equations is given by

$$x_i^{r+1}(\eta)=c_1 x_i^{h_1}(\eta)+c_2 x_i^{h_2}(\eta)+c_3 x_i^{h_3}(\eta)+x_i^{p}(\eta) \tag{26}$$

where C_1, C_2, C_3 are the unknown constants and are determined by considering the boundary conditions as $\eta \to \infty$. This solution $(x_i^{r+1}, i=1,2,...6)$ is then compared with solution at the previous step $x_i^r, i=1,2,...6$ and next iteration is performed if the convergence has not been achieved or greater accuracy is desired.

RESULTS AND DISCUSSION:

Fig 1 depicts the effect of magnetic field parameter (Mn) on the horizontal velocity profile (f_η (η)). Horizontal velocity profile decreases with increase in Magnetic field parameter, since increase of Magnetic field parameter signifies the increase of Lorentz force, which opposes the horizontal flow in the reverse direction.

Fig 2(a) and Fig 2(b) depict the effect of viscoelastic parameter k_1 on longitudinal and transverse velocity components. It can be seen, for a fixed value of η, both f'(η) and f(η) decrease with increasing values of viscoelastic parameter k_1. This can be explained by the fact that, as the viscoelastic parameter k_1 increases, the boundary layer adheres strongly to the surface, which in turn retards the flow in longitudinal and transverse directions.

Fig 3 shows the effect of Magnetic field parameter on temperature distribution. Temperature profile increases with increase in Magnetic field. Since increase of magnetic field increases the thermal boundary layer thickness. The increasing frictional drag due to Lorentz force is responsible for increasing the thermal boundary layer thickness.

Fig 4 reveals the effect of Prandtl number (Pr) on non-dimensional temperature θ(η) profiles are shown. Temperature θ (η) decreases with increase in the Prandtl number Pr. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing values Prandtl number Pr.

Fig 5 depicts the effect of suction parameter(R) on the heat transfer θ (η). Temperature profiles decreases with increasing values of suction parameter(R). Due to suction parameter(R) there will be loss of fluid in the boundary layer region, hence there will be less scope for heat transfer from the sheet to the fluid. This causes the declination in the heat transfer for increasing values of suction parameter.

The effect of work due to deformation term in the energy equation can be analyzed from Table 1 and Table 2. It can be seen from both the tables that, at a given point, temperature $\theta(\eta)$ decreases with increase in viscoelastic parameter k_1. From Table 2, it is observed that when work due to deformation is taken into account, for given k_1, temperature $\theta(\eta)$ decreases, which is in contrast to the second grade fluids [16]. And values of $|-\theta'(0)|$ in Table 2 are larger than in Table1. Physically it means that heat transfer rate is more from the sheet, which results in decrease in temperature $\theta(\eta)$.

CONCLUSIONS:

From our numerical results, it can be concluded that:

i. Horizontal velocity profile decreases with increase in viscoelastic parameter (k_1) and it also decreases with increase in magnetic field parameter (Mn).

ii. Temperature profiles increases with increase in magnetic field parameter (Mn).

iii. Thermal boundary layer thickness decreases with increase in Prandtl number (Pr).

iv. Temperature profiles decreases with increasing values of suction parameter (R).

v. Work done deformation term in energy equation reduces the temperature profiles, this is in contrast to the second grade fluids.

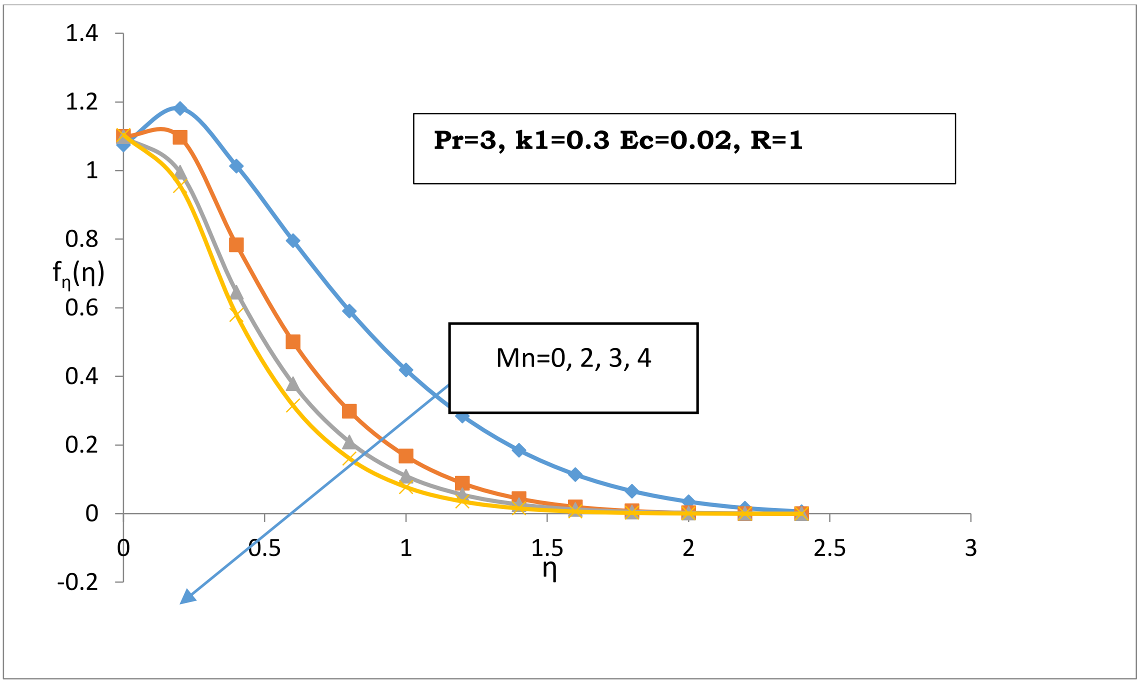

Figure1. Plot of velocity (f_η (η)) vs η for different values of Magnetic parameter (Mn)

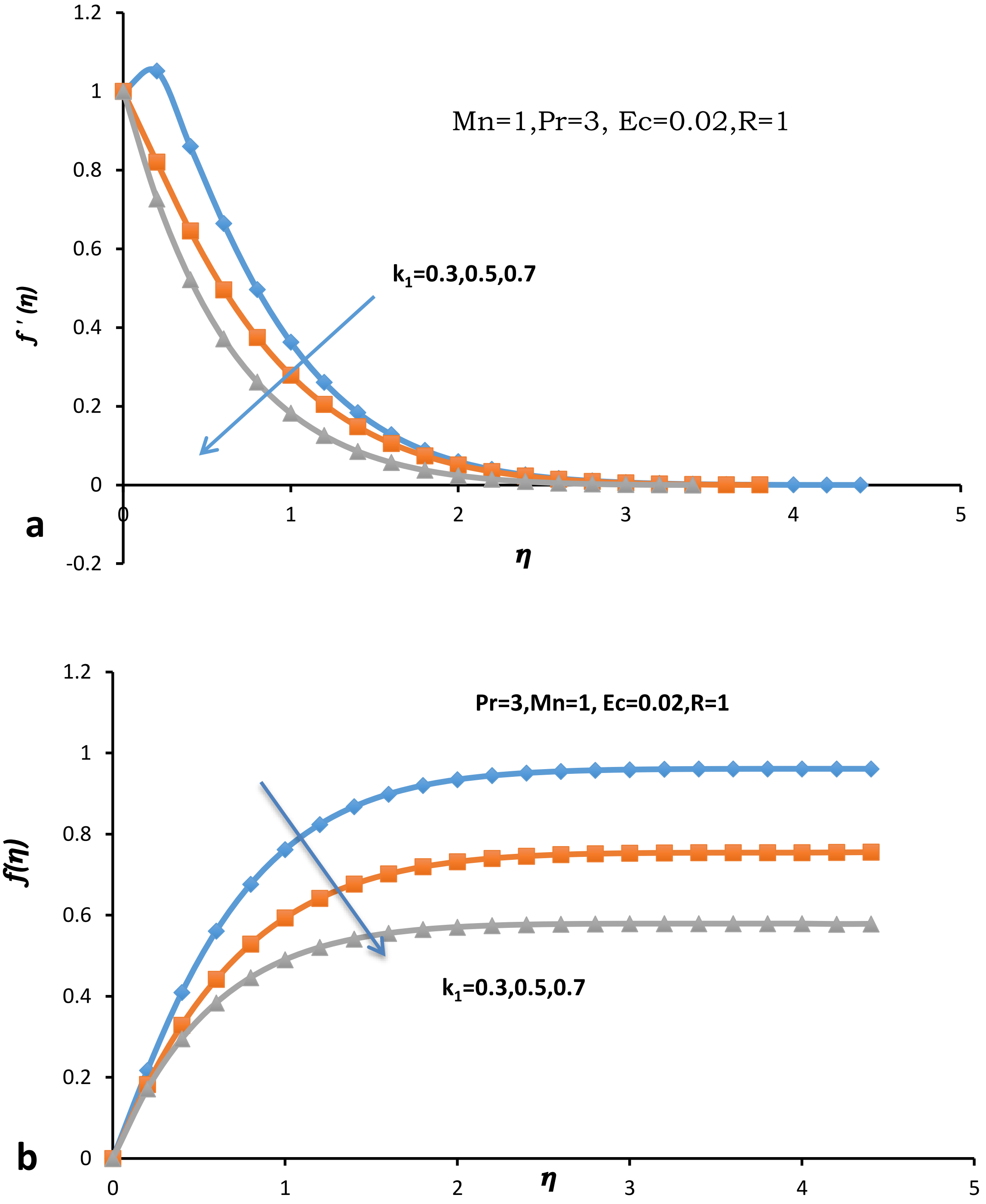

Figure2. Effect of viscoelasticity (k_1) on **(a)** transverse velocity component, **(b)** longitudinal velocity component

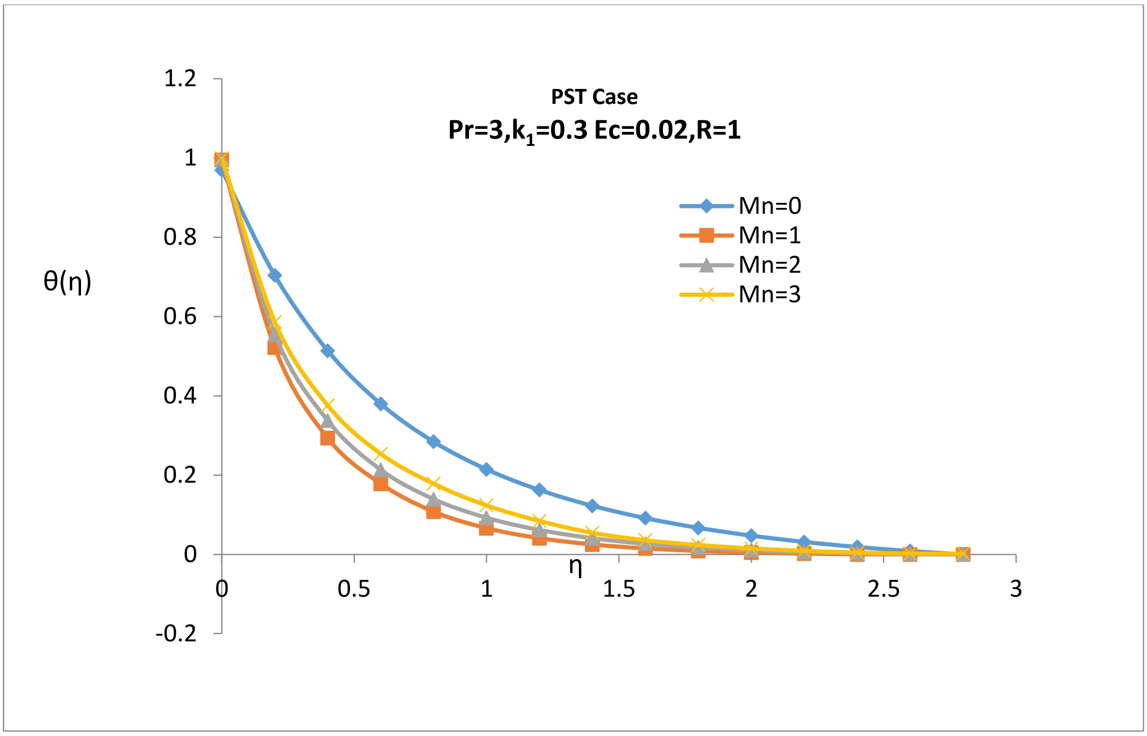

Figure 3. Effect of Magnetic field parameter (Mn) on temperature distribution θ (η)

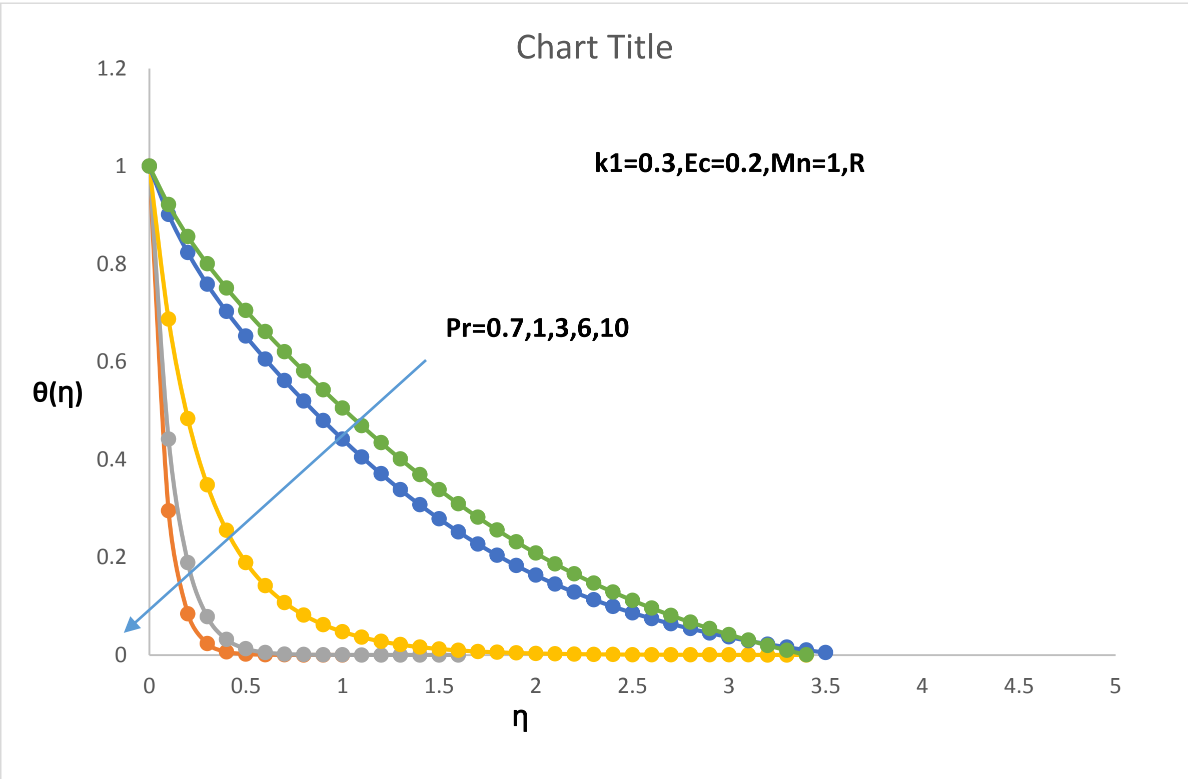

Figure 4. Variation of non-dimensional temperature θ (η) Vs η for different values of Prandtl number (Pr).

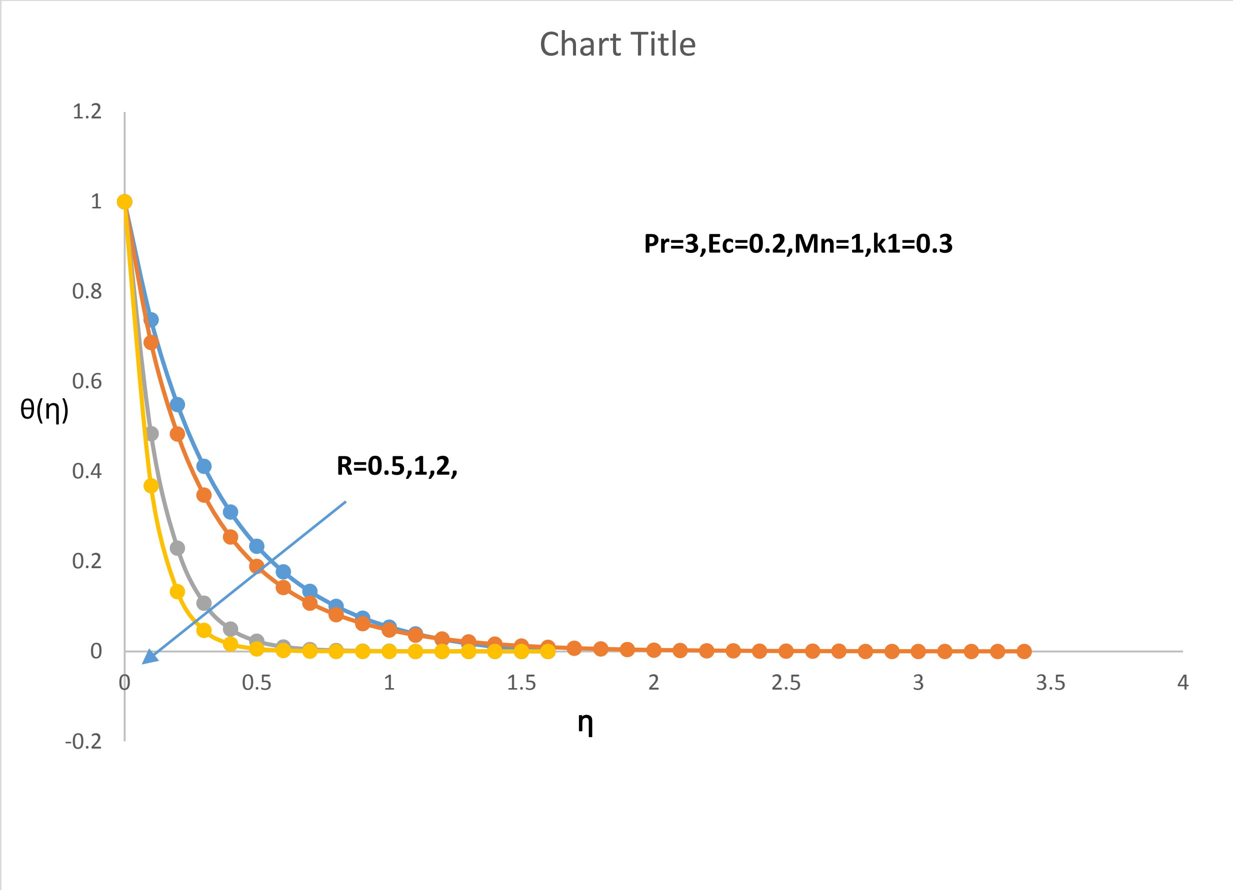

Figure 5. Effect of suction parameter (R) on temperature distribution θ (η).

Table1: Values of θ (η), θ' (η) in PST case with Pr=0.7, Mn=1, R=5, Ec=0.02, When work due to deformation is not taken into account

	η	θ(η)	θ'(η)
k1=0.3			
	0.0	1.00000	-3.19772
	0.2	0.54716	-1.55587
	0.4	0.31885	-0.81985
	0.6	0.19467	-0.46172
	0.8	0.12313	-0.27270
	1.0	0.08007	-0.16799
	1.4	0.03494	-0.07398
	1.8	0.01406	-0.03426
	2.0	0.00862	-0.02063
	2.4	0.00420	-0.00407
	2.8	0.00343	-0.00160
	3.0	0.00299	-0.00286
	3.4	0.00150	-0.00396
	3.8	0.00028	-0.00189
	4.0	0.00001	-0.00088
k1=0.5			
	0.0	1.00000	-3.44788
	0.2	0.51812	-1.61352
	0.4	0.28873	-0.78818
	0.6	0.17370	-0.40971
	0.8	0.11194	-0.22918
	1.0	0.07620	-0.13793
	1.4	0.03940	-0.06013
	1.8	0.02183	-0.03198
	2.0	0.01620	-0.02476
	2.4	0.00829	-0.01558
	2.8	0.00339	-0.00916
	3.0	0.00183	-0.00657
	3.4	0.00000	-0.00291
k1=0.7			
	0.0	1.00000	-4.03528
	0.2	0.43955	-1.83846
	0.4	0.18793	-0.80983
	0.6	0.07865	-0.34531
	0.8	0.03267	-0.14286
	1.0	0.01385	-0.05766
	1.4	0.00326	-0.00952
	1.8	0.00128	-0.00256
	2.0	0.00086	-0.00179
	2.4	0.00031	-0.00098
	2.8	0.00006	-0.00033
	3	0.00001	-0.00014

Table2: Values of θ (η), θ' (η) in PST case with Pr=0.7, Ec=0.02, Mn=1, R=5, when work due to deformation is taken into account

	η	θ(η)	θ'(η)
k1=0.3			
	0.0	1.00000	-3.56656
	0.2	0.50760	-1.62260
	0.4	0.28016	-0.76669
	0.6	0.16971	-0.38944
	0.8	0.11030	-0.22771
	1.0	0.07262	-0.15801
	1.4	0.02443	-0.08662
	1.8	0.00300	-0.02301
	2.0	0.00063	-0.00263
	2.4	0.00229	-0.00446
	2.8	0.00181	-0.00638
	3.0	0.00033	-0.00767
	3.4	-0.00161	-0.00085
	3.8	-0.00069	0.00392
	4.0	0.00000	0.00264
k1=0.5			
	0.0	1.00000	-3.64216
	0.2	0.49270	-1.68618
	0.4	0.25596	-0.79587
	0.6	0.14311	-0.38437
	0.8	0.08795	-0.19121
	1.0	0.05991	-0.10063
	1.4	0.03455	-0.04209
	1.8	0.02037	-0.03101
	2.0	0.01455	-0.02713
	2.4	0.00571	-0.01655
	2.8	0.00129	-0.00614
	3.0	0.00043	-0.00274
	3.4	0.00000	-0.00023
k1=0.7			
	0.0	1.00000	-4.08080
	0.2	0.43416	-1.84978
	0.4	0.18222	-0.80439
	0.6	0.07459	-0.33586
	0.8	0.03030	-0.13598
	1.0	0.01242	-0.05534
	1.4	0.00128	-0.01343
	1.8	-0.00228	-0.00550
	2.0	-0.00308	-0.00250
	2.4	-0.00296	0.00277
	2.8	-0.00136	0.00446
	3.0	-0.00054	0.00359
	3.1	-0.00021	0.00282
	3.2	0.00002	0.00189

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