Transient Analysis of $M^{[X_1]}, M^{[X_2]}/G_1, G_2/1$ retrial queueing system with priority services, Collisions, Orbital Search, Working Breakdown, Startup/Close down time, Feedback, modified Bernoulli vacation and Balking

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Abstract: This paper considers $M^{[X_1]}, M^{[X_2]}/G_1, G_2/1$ general retrial queueing system with priority services. The server serves two types of customers and follows the pre-emptive priority rule subject to collisions, orbital search, working breakdown, startup/closedown times and modified Bernoulli vacation with general (arbitrary) vacation periods. After completing the service, if there are no priority customers present in the system then the server may go for a vacation or close down the system. On completion of the close down, the server needs some time to set up the system. The priority customers who find the server busy are queued in the system. If a low-priority customer finds the server busy, they are routed to a retrial (orbit) queue that attempts to get the service. The system may become defective at any point of time when it is in operation. However, when the system is defective, instead of stopping service completely, the service continues only to the high priority customers at a slower rate.

We consider balking to occur at the low priority customer while the server is busy or idle. If, the server is busy with the low priority customer an arriving low priority customer may collide with the customer present in the system and both will be shifted to orbit. Using the supplementary variable technique, the steady-state distributions of the server state and the number of customers in the orbit are obtained. The stochastic decomposition property is investigated. Further, some particular cases of interest are discussed. Finally, numerical illustrations are provided.

AMS subject classification: 60K25, 68M30.

Keywords: Collisions, Orbital Search, Priority Queueing Systems, Retrial, Bernoulli Vacations, Working Breakdown, Startup/close down time, Balking.

INTRODUCTION

Pre-emptive priority queue was integrated by Jaiswal (1968). Several authors studied about priority queueing system with retrial, where the customers are joined the orbit and retry for their service. The perfectly reliable servers are not possible in real world phenomena. In fact, the servers may well be subject to lengthy and unpredictable breakdowns while serving a customer. For example, in manufacturing systems the machine may breakdown due to machine or job related problems. This results in a period of unavailable time until the servers are repaired. Such a system with repairable server has been studied as a queueing model and reliability model by many authors. However, they have assumed that the server breakdown even when the server is idle. In our paper, we have assumed that the server fails only in the operational state.

Studying queueing models with server breakdown, it is generally assumed that the server stops service when the server breaks down. However, in most of the models considered so far of queueing systems with server breakdowns, the underlying assumption has been that a server
breakdown disrupts the service completely in the system. In computer, the presence of a virus in the system may slow down the performance of the computer system. The computer system may still be able to perform various tasks but at a considerably slower rate. Here the failure of the computer system does not stop the work completely. Motivated by this factor, we have therefore considered in this paper a new class of queueing systems with the working breakdown policy with various parameters.

Kalidass et al. (2012) introduced the working breakdown policy, in which the server works at a lower service rate rather than stopping service during the breakdown period. In this policy the service can decrease complaints from the customers who should wait for the server to be, repaired and reduces the cost of waiting customers. Therefore, the working breakdown service is a more reasonable repair policy for unreliable queueing systems. Tao Li et al. (2017) and Zaiming Liu et al. (2014) describes more about working breakdowns. Recently, Kim B.K. et al. (2016) studied the M/G/1 queueing system with disasters and working breakdowns. They presented cycle analysis, and derived the system size distribution and the sojourn time distribution. Cheng-Dar Liou (2013) applied the matrix-geometric method to examine an infinite capacity Markovian queue with an unreliable server subject to working breakdowns and impatient customers.

Reflecting more practical situations, the server may turns off his service by a close down time and a start up time is required before starting for service. Dimitriou et al. (2009) studied about the queueing model with startup/closedown time and retrial customers. Customer impatience can be viewed as a potential loss of customers. Recently, Yang et al. (2017) studied the analysis of a finite capacity system with working breakdown and retention of impatient customers.

In physics a collision occurs when two or more objects hit each other. When objects collide, each object feels a force for a short amount of time. This force imparts an impulse, or changes the momentum of each of the colliding objects. A traffic collision occurs when a vehicle collides with another vehicle, pedestrian, animal, road debris, or other stationary obstruction, such as a tree, pole or building. Traffic collisions often result in injury, death, and property damage. Most chemical reactions occur only when reactants collide with one another.

In many situations involving data transmission from various sources there can be problem for a limited number of channels or other facilities. Uncoordinated attempts by several sources to use a single server facility can result in Collision leading to the loss of the transmission and hence the need for retransmission. An important problem concerns the development of workable procedures for removing the problem and corresponding message delay. Thus retrial queues with collisions are also appropriate for modeling the processes in computer and communications networks. Krishnakumar et al. (2010) and Peng Y. et al. (2013) studied about collisions with various parameters. Recently, Ayyappan et al. (2017) discussed the priority retrial queueing system with collision and orbital search.

This paper considers $M^{[X_1]}_1, M^{[X_2]}_2/G_1, G_2/1$ general retrial queueing system with priority services. Two different sorts of customers arrive at the system in two independent compound Poisson processes. Under the preemptive priority rule, the server providing general service to the arriving customers is subject to collisions, orbital search and modified Bernoulli vacation with general vacation time. We propose a retrial queueing model with the additional characteristics of server’s working breakdown, repair, startup/closedown times, balking. Arriving high priority customer who find the server busy with high(low) priority customer are queued (pre-empts the low priority service) and then are served in accordance with FCFS discipline. The arriving low-priority customer on finding the server busy cannot be queued they leave the service area and join the orbit as retrial customer. After completing service, if there is no high priority customer present in the system, the server may go for a vacation or close down the system. After completing the close down time the server need some time to set up the system. After completing vacation, repair, setup, if there is no high priority customer present in the system the server may search for the low priority customers in the orbit with probability $r$ or remains idle in the system. We consider balking to occur at the low priority customers during server’s busy or idle period.

The summary of the paper is as follows. Section 1 is an introduction to priority retrial queueing discipline and comprises literature review. Section 2 deals with model description, notations used, mathematical formulation and governing equations of the model. Section 3 elucidates the steady state solutions of the system. In section 4, stochastic decomposition is derived. Section 5 demonstrates the performance measures of the model. In Section 6, the numerical results and graphs are computed following which the conclusion is given.
MODEL DESCRIPTION

We consider an unreliable single server retrial queueing model with two types of customers namely, high priority and low-priority customers. The basic operation of the model can be described as:

Arrival and retrial process: Two class of customers arrive at the system in two independent compound Poisson processes with arrival rate $\lambda_1$ and $\lambda_2$ respectively. Let $\lambda_1 c_{1,i} \, dt$ and $\lambda_2 c_{2,i} \, dt \, (i = 1, 2, 3, \ldots)$ be the first order probability that a batch of ‘i’ customers arrives at the system during a short interval of time $(t, t + dt)$, where for $0 \leq c_{1,i} \leq 1$, $\sum_{i=1}^{\infty} c_{1,i} = 1$, $0 \leq c_{2,i} \leq 1$, $\sum_{i=1}^{\infty} c_{2,i} = 1$ and $\lambda_1 > 0$, $\lambda_2 > 0$. The arriving high priority customer who find the server busy is queued and then is served. The arriving low-priority customer on finding the server busy, are routed to a retrial queue and they follow constant retrial policy that attempts to get the service. The retrial time is generally distributed with distribution function $I(s)$ and the density function $i(s)$. Let $\eta(x)dx$ be the conditional probability of completion of retrial during the interval $(x, x + dx]$ where $x$ is the elapsed retrial time. If the server is busy with low priority customer then the arriving customer may collide with it and both are shifted to the orbit with probability $p_1$.

Service process: If a high priority customer arrives in batch and finds a low priority customer in service, they pre-empt the low priority customer who is undergoing service; thus the service of the pre-empted low priority customer begins only after the completion of service of all high priority customers present in the system. The service times for the high priority and low priority customers are generally (arbitrary) distributed with distribution functions $B_i(s)$ and the density functions $b_i(s)$, $i = 1, 2$ respectively. Let $\mu_{1i}(x)dx$ be the conditional probability of completion of the high priority and low priority customers service during the interval $(x, x + dx]$, where $x$ is the elapsed service time.

Modified Bernoulli Vacation: After completing all high priority customers and every service completion of low priority customer the server may take a vacation with probability $\theta$ or continue the service to the next customer with probability $1 - \theta$. Vacation time is generally distributed with distribution function $V(s)$ and the density function $v(s)$. Let $\beta(x)dx$ be the conditional probability of completion of vacation during the interval $(x, x + dx]$ where $x$ is the elapsed vacation time.

Working Breakdown state: The server may become inactive during busy period. At the time of breakdown the high priority customer who is in service will get service continuously by lower service rate $\mu_3$ and it follows exponential distribution. But, at the time of breakdown the low priority customer who is in service will send to the orbit.

Repair Process: The broken down server is sent for repair immediately so as to regain its functionality with exponential repair rate $\gamma$. Immediately after returning from the repair, the server starts to serve high priority/low priority customers.

Close down time: After completing vacation, repair, service, if there are no high priority customers present in the system the server will close down the system. Close down time is generally distributed with distribution function $C(s)$ and the density function $c(s)$. Let $\phi(x)dx$ be the conditional probability of a completion of close down time during the interval $(x, x + dx]$ where $x$ is the elapsed close down time.

Startup time: On completion of close down time, the server takes some time to start up the system to increase the efficiency of the service. Startup time is generally distributed with distribution function $M(s)$ and the density function $m(s)$. Let $\delta(x)dx$ be the conditional probability of a completion of startup time during the interval $(x, x + dx]$, where $x$ is the elapsed startup time.

Orbital Search: After completing vacation, setup, repair if there is no high priority customer present in the system the server may search the orbit to serve the low priority customer with probability $r$ or he may ideally present in the system.

Balking: If the server is busy or unavailable in the system, an arriving low-priority customers either join the orbit with probability $b$ or balks with probability $(1 - b)$.

Idle State: After completing the startup or vacation or repair if there is any high priority customer waiting in the system the server starts doing the service. Otherwise the server is simply present in the system for the customers to arrive.
Definitions and notations

Let $N_1(t), N_2(t)$ be the queue size and orbit size respectively at time $t$, $B_0^1(t), B_0^2(t)$ and $I^0(t)$ be the elapsed service time of the high, low priority customer and retrial time of the low priority customers, respectively at time $t$. In addition, let $V^0(t), C^0(t)$ and $M^0(t)$, be the elapsed vacation time, close down time and setup time of the server respectively at time $t$. The state of the server at time $t$ is given by,

$$Y(t) = \begin{cases} 
0, & \text{if the server is in idle state;} \\
1, & \text{if the server is busy with high priority customer;} \\
2, & \text{if the server is busy with low priority customer;} \\
3, & \text{if the server is in vacation;} \\
4, & \text{if the server is in working breakdown state;} \\
5, & \text{if the server is in repair process;} \\
6, & \text{if the server is in closedown state;} \\
7, & \text{if the server is in startup state.}
\end{cases}$$

We have $I(x), B_i(x), V(x), C(x)$ and $M(x)$ is continuous at $x = 0$, and $\eta(x) = \frac{dI(x)}{1-I(x)}$, $\mu_i(x)dx = \frac{dB_i(x)}{1-B_i(x)}$, $\beta(x)dx = \frac{dV(x)}{1-V(x)}$, $\phi(x)dx = \frac{dC(x)}{1-C(x)}$, and $\delta(x)dx = \frac{dM(x)}{1-M(x)}$ are the first order differential (hazard rate) functions of $I(.), B_i(.), V(.), C(.)$ and $M(.)$, respectively.

Queue size distribution

Since service time, vacation time, closedown time and startup time are not exponential, the process $\{Y(t), N_1(t), N_2(t)\}$ is not Markovian. In such case we introduce supplementary variables corresponing to elapsed times to make it Markovian (Cox(1955)). Joint distributions of the server state and queue size are defined.
The Kolmogorov forward equations which governs

\[ \mathcal{T}_{0,n}(s,t) = Pr(Y(t) = 0, N_1(t) = 0), \]
\[ N_2(t) = n, n \geq 0 \]
\[ P^{(1)}_{m,n}(x,t) = Pr(Y(t) = 1, x < B_0^0(t) \leq x + dx, \]
\[ N_1(t) = m, N_2(t) = n, m \geq 0, n \geq 0 \]
\[ P^{(2)}_{0,n}(x,t) = Pr(Y(t) = 2, x < B_0^0(t) \leq x + dx, \]
\[ N_1(t) = 0, N_2(t) = n, n \geq 0 \]
\[ V_{m,n}(s,x,t) = Pr(Y(t) = 3, x < V_0^0(t) \leq x + dx, \]
\[ N_1(t) = m, N_2(t) = n), m \geq 0, n \geq 0 \]
\[ \mathcal{C}_{m,n}(s,x,t) = Pr(Y(t) = 6, x \in C_0^0(t) \leq x + dx, \]
\[ N_1(t) = m, N_2(t) = n), m \geq 0, n \geq 0 \]
\[ M_{m,n}(s,x,t) = Pr(Y(t) = 7, x \in M_0^0(t) \leq x + dx, \]
\[ N_1(t) = m, N_2(t) = n), m \geq 0, n \geq 0 \]

\[ \mathcal{P}_{0,n}(x,t) + \frac{\partial}{\partial x} P^{(2)}_{0,n}(x,t) \]
\[ = -(\lambda_1 + \lambda_2 + \alpha + \mu_2(x)) P^{(2)}_{0,n}(x,t) \]
\[ + (1 - \delta_{0n}) \lambda_2 b (1 - p_1) \sum_{i=1}^{n} c_{2i} P^{(2)}_{0,n-i}(x,t) \]
\[ + \lambda_2 (1 - b) P^{(2)}_{0,n}(x,t); n \geq 0, \]

\[ \frac{\partial}{\partial t} C_{m,n}(x,t) + \frac{\partial}{\partial x} C_{m,n}(x,t) \]
\[ = -(\lambda_1 + \lambda_2 + \xi + \phi(x)) C_{m,n}(x,t) \]
\[ + (1 - \delta_{m0}) \lambda_1 \sum_{i=1}^{m} c_{1i} C_{m-i,n}(x,t) \]
\[ + (1 - \delta_{0n}) \lambda_2 b \sum_{i=1}^{n} c_{2i} C_{m,n-i}(x,t) \]
\[ + \lambda_2 (1 - b) C_{m,n}(x,t) + \xi C_{m+1,n}(x,t); m \geq 0, n \geq 0, \]

\[ \frac{\partial}{\partial t} M_{m,n}(x,t) + \frac{\partial}{\partial x} M_{m,n}(x,t) \]
\[ = -(\lambda_1 + \lambda_2 + \xi + \delta(x)) M_{m,n}(x,t) \]
\[ + (1 - \delta_{m0}) \lambda_1 \sum_{i=1}^{m} c_{1i} M_{m-i,n}(x,t) \]
\[ + (1 - \delta_{0n}) \lambda_2 b \sum_{i=1}^{n} c_{2i} M_{m,n-i}(x,t) \]
\[ + \lambda_2 (1 - b) M_{m,n}(x,t) + \xi M_{m+1,n}(x,t); m \geq 0, n \geq 0, \]

\[ \frac{d}{dt} Q_{m,n}(t) = -(\lambda_1 + \lambda_2 + \gamma + \mu_3) Q_{m,n}(t) \]
\[ + (1 - \delta_{m0}) \lambda_1 \sum_{i=1}^{m} c_{1i} Q_{m-i,n}(t) \]
\[ + (1 - \delta_{0n}) \lambda_2 b \sum_{i=1}^{n} c_{2i} Q_{m,n-i}(t) \]
\[ + \lambda_2 (1 - b) Q_{m,n}(t) + \mu_3 Q_{m+1,n}(t) \]
\[ + \alpha \int_{0}^{\infty} P^{(1)}_{m,n}(x,t) dx; m \geq 0, n \geq 0, \]

\[ \frac{d}{dt} R_{0,n}(t) = -(\lambda_1 + \lambda_2 + \gamma) R_{0,n}(t) \]
\[ + (1 - \delta_{0n}) \lambda_2 b \sum_{i=1}^{n} c_{2i} R_{0,n-i}(t) + \lambda_2 (1 - b) R_{0,n}(t) \]
\[ + \alpha \int_{0}^{\infty} P^{(2)}_{0,n-i}(x,t) dx; m \geq 0, n \geq 1, \]
The above set of equations are to be solved under the following boundary conditions at \( x = 0 \).

\[
I_{0,n}(0,t) = (1 - r)(\int_{0}^{\infty} M_{0,n}(x,t)\delta(x)dx
\]
\[
+ \int_{0}^{\infty} V_{0,n}(x,t)\beta(x)dx + \gamma R_{0,n}(t)); \ n \geq 1.
\]  

(10)

\[
P_{m,n}^{(1)}(0,t) = q \int_{0}^{\infty} P_{m+1,n}(x,t)\mu_{1}(x)dx
\]
\[
+ p \int_{0}^{\infty} P_{m+1,n}(x,t)\mu_{1}(x)dx + \lambda_{1} c_{1,m+1} I_{0,n}(t)
\]
\[
+ (1 - \delta_{0n})\lambda_{1} c_{1,m+1} \int_{0}^{\infty} P_{0,n-1}(x,t)dx
\]
\[
+ \int_{0}^{\infty} M_{m+1,n}(x,t)\delta(x)dx
\]
\[
+ \int_{0}^{\infty} V_{m+1,n}(x,t)\beta(x)dx; \ m \geq 0, \ n \geq 1.
\]  

(11)

\[
P_{0,n}^{(1)}(0,t) = q \int_{0}^{\infty} P_{1,0}(x,t)\mu_{1}(x)dx
\]
\[
+ p \int_{0}^{\infty} P_{1,0}(x,t)\mu_{1}(x)dx + \lambda_{1} c_{1,1} I_{0,0}(t)
\]
\[
+ \int_{0}^{\infty} M_{1,0}(x,t)\delta(x)dx
\]
\[
+ \int_{0}^{\infty} V_{1,0}(x,t)\beta(x)dx; \ n = 0,
\]  

(12)

\[
P_{0,n}^{(2)}(0,t) = \int_{0}^{\infty} I_{0,n+1}(x,t)\eta(x)dx
\]
\[
+ \lambda_{2} c_{2,n+1} I_{0,0}(t) + \lambda_{2} \sum_{i=1}^{n} c_{2,i} \int_{0}^{\infty} I_{0,n+1-i}(x,t)dx
\]
\[
+ r \int_{0}^{\infty} M_{0,n+1}(x,t)\delta(x)dx
\]
\[
+ \int_{0}^{\infty} V_{0,n+1}(x,t)\beta(x)dx + \gamma R_{0,n+1}(t)); \ n \geq 0.
\]  

(13)

\[
V_{0,n}(0,t) = \theta q \int_{0}^{\infty} P_{0,n}^{(1)}(x,t)\mu_{1}(x)dx
\]
\[
+ \theta \int_{0}^{\infty} P_{0,n}^{(2)}(x,t)\mu_{2}(x)dx; \ m = 0, \ n \geq 0.
\]  

(14)

\[
C_{0,n}(0,t) = (1 - \theta)q \int_{0}^{\infty} P_{0,n}^{(1)}(x,t)\mu_{1}(x)dx
\]
\[
+ (1 - \theta) \int_{0}^{\infty} P_{0,n}^{(2)}(x,t)\mu_{2}(x)dx; \ n \geq 0,
\]  

(15)

\[
M_{m,n}(0,t) = \int_{0}^{\infty} C_{m,n}(x,t)\phi(x)dx; \ m \geq 0, \ n \geq 0.
\]  

(16)

We assume that initially there are no customers in the system and the server is idle. Then the initial conditions are,

\[
P_{m,n}^{(1)}(0) = P_{0,n}^{(2)}(0) = V_{0,n}(0) = C_{m,n}(0) = M_{m,n}(0) \]
\[
= R_{0}(0) = Q_{0}(0) = 0; \ m \geq 0, \ n \geq 0
\]
\[
I_{0,n}(0) = 0 \quad \text{and} \quad I_{0,0}(0) = 1.
\]  

(17)

The Probability Generating Function(PGF) of this model:

\[
I(x,z_{2},t) = \sum_{n=1}^{\infty} z_{2}^{n} I_{0,n}(x,t), P_{0}^{(2)}(x,z_{2},t)
\]
\[
= \sum_{n=0}^{\infty} z_{2}^{n} P_{0,n}^{(2)}(x,0), R_{0}(z_{2},t) = \sum_{n=0}^{\infty} z_{2}^{n} R_{0,n}(t),
\]
\[
Q(z_{1},z_{2},t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} z_{1}^{m} z_{2}^{n} Q_{m,n}(t),
\]
\[
B(x,z_{1},z_{2},t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} z_{1}^{m} z_{2}^{n} B_{m,n}(x,t)
\]
where \( B = P^{(1)}, V, C, M, \)

By taking Laplace transforms from (1) to (17) and solve the equations, we get,

\[
T_{0}(x,s,z_{2}) = \big\{1 - T(f(a,s))\big\}e^{f(a,s)\frac{sx}{z_{2}}},
\]
\[
\overline{p}^{(1)}(x,s,z_{1},z_{2})
\]
\[
= \overline{p}^{(1)}(0,s,z_{1},z_{2})\big\{1 - \overline{B}_{1}(f_{1}(s,z_{1},z_{2}))\big\}e^{f_{1}(s,z_{1},z_{2})\frac{sx}{z_{2}}},
\]
\[
\overline{p}^{(2)}(x,s,z_{2}) = \overline{p}^{(2)}(0,s,z_{2})\big\{1 - \overline{B}_{2}(f_{2}(s,z_{2}))\big\}e^{f_{2}(s,z_{2})\frac{sx}{z_{2}}},
\]
\[
\overline{v}(x,s,z_{1},z_{2})
\]
\[
= \overline{v}(0,s,z_{1},z_{2})\big\{1 - \overline{V}(f_{3}(s,z_{1},z_{2}))\big\}e^{f_{3}(s,z_{1},z_{2})\frac{sx}{z_{2}}},
\]
\[
\overline{q}(s,z_{1},z_{2})
\]
\[
= \frac{a\overline{q}^{(1)}(0,s,z_{1},z_{2})\big\{1 - \overline{B}_{1}(f_{4}(s,z_{1},z_{2}))\big\}e^{f_{4}(s,z_{1},z_{2})\frac{sx}{z_{2}}}}{f_{5}(s,z_{1},z_{2})},
\]  

(18)

(19)

(20)

(21)

(22)
\[
\begin{align*}
R_0(s, z_2) &= \frac{\alpha z_2 P_0^2(0, s, z_2)(1 - B_2(f_2(s, z_2)))e^{-f_2(s, z_2)x}}{f_4(s, z_2)}, \\
\overline{M}(x, s, z_1, z_2) &= \overline{M}(0, s, z_1, z_2)[1 - \overline{M}(f_3(s, z_1, z_2))]e^{-f_3(s, z_1, z_2)x}, \\
\overline{C}(x, s, z_1, z_2) &= \overline{C}(0, s, z_1, z_2)[1 - \overline{C}(f_3(s, z_1, z_2))]e^{-f_3(s, z_1, z_2)x},
\end{align*}
\]

where,
\[
\begin{align*}
f(a, s) &= s + \lambda_1 + \lambda_2, \\
f_1(s, z_1, z_2) &= s + \lambda_1[1 - C_1(z_1)] + \lambda_2[b[1 - C_2(z_2)] + \alpha z_2 P_0^2(0, s, z_2)(1 - B_2(f_2(s, z_2)))e^{-f_2(s, z_2)x}], \\
f_4(s, z_1, z_2) &= s + \lambda_1[1 - C_1(z_1)] + \lambda_2[b[1 - C_2(z_2)] + \alpha z_2 P_0^2(0, s, z_2)(1 - B_2(f_2(s, z_2)))e^{-f_2(s, z_2)x}], \\
f_5(s, z_1, z_2) &= s + \lambda_1[1 - C_1(z_1)] + \lambda_2[b[1 - C_2(z_2)] + \alpha z_2 P_0^2(0, s, z_2)(1 - B_2(f_2(s, z_2)))e^{-f_2(s, z_2)x}].
\end{align*}
\]

By substituting (27) in required equations we get,
\[
\begin{align*}
T_0(0, s, z_2) &= 1 - (s + \lambda_1 + \lambda_2)T_{0,0}(s)G_4(s, z_1, z_2) \\
&\quad + (1 - r) \left\{ \int_0^\infty \overline{M}(x, s, z_2)b(x)dx + \gamma \int_0^\infty \overline{M}(x, s, z_2)b(x)dx \right\}
\end{align*}
\]

(28)

\[
\begin{align*}
z_2 P_0^2(0, s, z_2) &= \overline{T}_0(0, s, z_2) \left[ \frac{1 - \overline{I}(f(s, a, s))}{f(a, s)} \right] \\
&\quad + \lambda_2 b C_2(z_2) \left[ \frac{1 - \overline{I}(f(s, a, s))}{f(a, s)} \right] \\
&\quad + r \left\{ q P_0^2(0, s, z_2) \overline{N}_1(f_3(s, z_2)) \overline{G}_5(s, z_2) \right\}.
\end{align*}
\]

(29)

By solving the above equations, we get,
\[
\begin{align*}
\overline{P}^{(1)}(0, s, z_1, z_2) &= \left\{ \begin{array}{l}
\overline{T}_0(x, s, z_2) \left\{ \lambda_1 C_1(z_1) \overline{G}_4(x, g(z_2)) - \lambda_1 C_1(g(z_2)) \overline{G}_3(x, z_1, z_2) \right\} \\
+ \lambda_2 b C_2(z_2) \overline{G}_4(x, g(z_2)) \\
- \frac{\overline{G}_3(x, z_1, z_2) \overline{G}_5(x, g(z_2))}{\overline{G}_4(x, g(z_2))} \right\} (\gamma_0 + \rho \gamma_0) \overline{E}_1(f_1(s, z_1, z_2))
\end{array} \right.
\end{align*}
\]

(30)

\[
\begin{align*}
\overline{P}^{(2)}(0, s, z_2) &= \left\{ \begin{array}{l}
(1 - s + \lambda_1 + \lambda_2)T_{0,0}(s) \left\{ \overline{G}_3(x, g(z_2)) - \overline{G}_3(x, g(z_2)) \right\} + 1 - \frac{T_{0,0}(s)}{f(a, s)} \\
+ \lambda_2 b C_2(z_2) \left\{ \frac{1 - \overline{I}(f(a, s))}{f(a, s)} \right\} \overline{G}_5(x, g(z_2)) \\
\times \left\{ \overline{G}_3(x, g(z_2)) - \lambda_1 C_1(g(z_2)) \right\} + \lambda_2 b C_2(z_2) \overline{G}_3(x, g(z_2)) \\
- \left\{ \overline{G}_3(x, g(z_2)) \overline{G}_3(x, g(z_2)) + \overline{G}_3(x, g(z_2)) \overline{G}_5(x, g(z_2)) \right\} \overline{G}_5(x, g(z_2)) \\
\times \left\{ r + (1 - r) \left\{ T + \lambda_2 b C_2(z_2) \left\{ \frac{1 - \overline{I}(f(a, s))}{f(a, s)} \right\} \right\} \right\}
\end{array} \right.
\end{align*}
\]

(31)
\[ \tau_{0(0,s,z_2)} = \left[ \frac{1 - \rho < 1 \lambda_2 T_0(0,s,z_2) T_0(s,z_2)}{\sqrt{\tau_3(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_4(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_5(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_6(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_7(s,g(z_2))}} \right] \]

\[ \tau_{0(0,s,z_2)} = \left[ \frac{1 - \rho < 1 \lambda_2 T_0(0,s,z_2) T_0(s,z_2)}{\sqrt{\tau_3(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_4(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_5(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_6(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_7(s,g(z_2))}} \right] \]

\[ \tau_{0(0,s,z_2)} = \left[ \frac{1 - \rho < 1 \lambda_2 T_0(0,s,z_2) T_0(s,z_2)}{\sqrt{\tau_3(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_4(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_5(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_6(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_7(s,g(z_2))}} \right] \]

\[ \tau_{0(0,s,z_2)} = \left[ \frac{1 - \rho < 1 \lambda_2 T_0(0,s,z_2) T_0(s,z_2)}{\sqrt{\tau_3(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_4(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_5(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_6(s,g(z_2))}}, \frac{1 - \rho f_0(0,s,z_2)}{\sqrt{\tau_7(s,g(z_2))}} \right] \]
\[ \mathcal{Q}(s, z_1, z_2) = \frac{\alpha \mathcal{P}^{(1)}(0, s, z_1, z_2) [1 - \overline{B}_1(f_1(s, z_1, z_2))]}{f_1(s, z_1, z_2) f_4(s, z_1, z_2)}, \quad (41) \]

\[ \mathcal{R}(s, z_2) = \frac{\alpha z_2 \mathcal{P}^{(2)}_0(0, s, z_2) [1 - \overline{B}_2(f_2(s, z_2))]}{f_2(s, z_2) f_3(s, z_2)}, \quad (42) \]

\[ \mathcal{M}(s, z_1, z_2) = \left\{ \begin{array}{l}
(1 - \theta) \mathcal{P}^{(1)}(0, s, z_2) \\
\mathcal{P}^{(2)}_0(0, s, z_2) \left[ \overline{M}(f_5(s, z_1, z_2)) \right] \\
\mathcal{P}^{(2)}_0(0, s, z_2) \left[ \overline{M}(f_5(s, z_1, z_2)) \right] \times \overline{C}(f_5(z_1, z_2)), \quad (43) \end{array} \right. \]

**STEADY STATE ANALYSIS: LIMITING BEHAVIOUR**

By applying the well-known Tauberian property,

\[ \lim_{s \to 0} s \overline{S}(s) = \lim_{t \to \infty} f(t), \]

to the above equations, we obtain the steady-state solutions of this model.

\[ I_0(z_2) = I_0(0, z_2) \left[ \frac{1 - \overline{T}(f(a))}{f(a)} \right], \quad (44) \]

\[ P^{(1)}(z_1, z_2) = P^{(1)}(0, z_1, z_2) \left[ \frac{1 - \overline{B}_1(f_1(z_1, z_2))}{f_1(z_1, z_2)} \right], \quad (45) \]

\[ V(z_1, z_2) = \theta \mathcal{Q}^{(1)}(0, z_2) \mathcal{B}_1(f_1(z_2)) \]

\[ + \theta P^{(2)}_0(0, z_2) \mathcal{B}_2(f_2(z_2))) \left[ \frac{1 - \overline{V}(f_5(z_1, z_2))}{f_5(z_1, z_2)} \right], \quad (46) \]

\[ P^{(2)}_0(z_2) = P^{(2)}_0(0, z_2) \left[ \frac{1 - \overline{B}_2(f_2(z_2))}{f_2(z_2)} \right]. \quad (47) \]

\[ C(z_1, z_2) = \left\{ \begin{array}{l}
(1 - \theta) q P^{(1)}(0, z_2) \left[ \frac{1 - \overline{B}_1(f_1(z_2))}{f_1(z_2)} \right] \\
+ (1 - \theta) P^{(2)}_0(0, z_2) \left[ \frac{1 - \overline{B}_2(f_2(z_2))}{f_2(z_2)} \right] \\
\times \overline{C}(f_5(z_1, z_2)) \end{array} \right. \]

\[ Q(z_1, z_2) = \frac{\alpha P^{(1)}(0, z_1, z_2) [1 - \overline{B}_1(f_1(z_1, z_2))]}{f_1(z_1, z_2) f_4(z_1, z_2)}, \quad (50) \]

\[ R(z_2) = \frac{\alpha z_2 P^{(2)}_0(0, z_2) [1 - \overline{B}_2(f_2(z_2))]}{f_2(z_2) f_3(z_2)}, \quad (51) \]

\[ M(z_1, z_2) = \left\{ \begin{array}{l}
(1 - \theta) q P^{(1)}(0, z_2) \left[ \frac{1 - \overline{B}_1(f_1(z_2))}{f_1(z_2)} \right] \\
+ (1 - \theta) P^{(2)}_0(0, z_2) \left[ \frac{1 - \overline{B}_2(f_2(z_2))}{f_2(z_2)} \right] \\
\times \overline{C}(f_5(z_1, z_2)) \end{array} \right. \]

In order to determine \( I_{0,0} \), we use the normalizing condition

\[ P^{(1)}(1, 1) + V(1, 1) + P^{(2)}(1, 1) + Q(1, 1) + R(1) \]

\[ + M(1, 1) + C(1, 1) + I_0(0, 1) + I_{0,0} = 1. \]

For this, let \( P_q(z_1, z_2) \) be the probability generating function of the queue size irrespective of the state of the system. Then adding equations from (44) to (51), we obtain,

\[ P_q(z_1, z_2) = P^{(1)}(z_1, z_2) + V(z_1, z_2) + P^{(2)}_0(z_2) \]

\[ + M(z_1, z_2) + C(z_1, z_2) + Q(z_1, z_2) + R(z_2), \quad (53) \]

\[ P_q(z_1, z_2) = \frac{N_1(z_1, z_2)}{D_1(z_1, z_2)} + \frac{N_2(z_1, z_2)}{D_2(z_1, z_2)} + \frac{N_3(z_1, z_2)}{D_3(z_1, z_2)}, \]

where

\[ N_1(z_1, z_2) = I_0(0, z_2) \left[ \frac{1 - \overline{T}(f(a))}{f(a)} \right] \{G_4(z_2)f_5(z_1, z_2) \]

\[ + \lambda_1 C_1(g(z_2))[1 - \theta \overline{V}(f_5(z_1, z_2))] \]

\[ \times (1 - \theta) [1 - \overline{C}(f_5(z_1, z_2)) \mathcal{M}(f_5(z_1, z_2))]}, \]
\[
\begin{align*}
N_2(z_1, z_2) & = \mu(z_2) \left\{ G_4(z_2) \left[ 1 - \frac{\overline{B}_3(f_2(z_2)))}{f_2(z_2)} \right] \right. \\
& \times (f_3(z_2) + \alpha z_2) f_3(z_1, z_2) + \overline{B}_3(f_2(z_2))) \\
& \times f_3(z_2) \left[ 1 - \theta \overline{V}(f_3(z_1, z_2)) \right] \\
& \left. - (1 - \theta) \overline{C}(f_3(z_2)) \overline{M}(f_3(z_1, z_2)) \right] + f_3(z_2) \\
& \times (1 - \theta \overline{V}(f_3(z_1, z_2)) \\
& \left. - (1 - \theta) \left[ 1 - \overline{C}(f_3(z_1, z_2)) \overline{M}(f_3(z_1, z_2)) \right] G_3(g(z_2)) \right) , \\
N_5(z_1, z_2) & = P^{(1)}(0, z_1, z_2) \\
& \times \left\{ 1 - \frac{\overline{B}_1(f_1(z_1, z_2)))}{\alpha + f_4(z_1, z_2)} \right\} , \\
\text{(54)}
\end{align*}
\]

In order to obtain the probability of idle time \( I_{0,0} \), we use the normalizing condition,

\[
P^{(1)}(1, 1) + I_{0,0} = 1.
\]

From which we can have,

\[
\begin{align*}
(I_{0,0}) & = \frac{D_r}{n r} \\
D_r & = (\lambda_1 + \alpha)(\lambda_1 + \gamma) \gamma G_4(1)(\lambda_1 E(I_1) - \lambda_2 b E(I_2)) \\
& + \frac{\alpha}{\gamma} (- \lambda_1 E(I_1) - \lambda_2 b E(I_2) + \mu_3)) \\
& + \gamma P^{(2)}(0, 1)(\lambda_1 E(I_1) - \lambda_2 b E(I_2)) \\
& + \frac{\alpha}{\gamma} (- \lambda_1 E(I_1) - \lambda_2 b E(I_2) + \mu_3)) \left\{ G_4(1) \\
& \times \left[ \frac{\overline{B}_2(\lambda_1 + \alpha)(\lambda_1 + \gamma) \gamma}{(1 - \theta) E(C) + E(M)} \\
& + \theta E(V) \right] \left[ 1 - \overline{B}_2(\lambda_1 + \alpha + \gamma) \right] \\
& + [\lambda_1 + \alpha] \gamma \theta E(V) \\
& + (1 - \theta) E(C) + E(M)) \right\} G_3(1) \\
& + I_0(0, 1) \left[ 1 - \frac{\overline{I}(f(a))}{f(a)} \right] \\
& \times \left\{ G_4(1) + \lambda_1 \theta E(V) \\
& + (1 - \theta) E(C) + E(M)) \right\} (\lambda_1 + \alpha)(\lambda_1 + \gamma) \gamma \\
& - \lambda_1 E(I_1) \\
& - \lambda_2 b E(I_2) + \frac{\alpha}{\gamma} (- \lambda_1 E(I_1) - \lambda_2 b E(I_2) + \mu_3)) \right) \\
& + P^{(1)}(0, 1, 1) G_4(1)(E(B_1) \\
& - \lambda_1 E(I_1) - \lambda_2 b E(I_2) \\
& + \gamma \left( - \lambda_1 E(I_1) - \lambda_2 b E(I_2) + \mu_3) \right)(\lambda_1 + \alpha)(\lambda_1 + \gamma) \gamma.
\end{align*}
\]

**STOCHASTIC DECOMPOSITION**

**Theorem 2.** The number of customers in the system under steady state can be decomposed into two independent Probability generating functions, one of which is the PGF of the queue size distribution in the priority classical queue and unreliable server with with working breakdown, repair, modified Bernoulli vacation, closedown/startup time, collision, orbital search and balking and the other is the PGF of the conditional distribution of the number of customer in the orbit given that the system is idle. The existence of the stochastic decomposition property Artalejo et al. (1994) for our model can be demonstrated easily by showing that

\[
\Omega(z) = \Pi(z) \Psi(z)
\]

**Proof.** The probability generating function \( \Pi(z) \) of the system size in the classical priority queue and unreliable server with working breakdown, repair, modified Bernoulli vacation, closedown/startup time, collision, orbital search and balking is given by,

\[
\Pi(z) = \frac{nr}{dr},
\]

where

\[
nr = \left\{ - \lambda_1 - \lambda_2 b [1 - C_2(z_2)] I_{0,0} \\
\right. \left( (G_2(z_1, z_2)) G_4(1) G_1(z_1, z_2) G_3(g(z_2)) f_3(z_2) f_3(z_2) \\
\right) \times f_3(z_1, z_2) \right\} \left\{ (1 - \overline{B}_1(f_4(z_1, z_2)) + (\lambda_1 + \alpha) \overline{B}_1(f_1(z_1, z_2)) f_3(z_2) f_4(z_2) \\
\right\} \times \left\{ G_4(g(z_2)) \right\} \left\{ 1 - \overline{B}_2(f_2(z_2)) f_3(z_1, z_2) f_3(z_1, z_2) + \alpha z_2 \\
\right\} f_3(z_2) f_3(z_3) \overline{B}_2(f_2(z_2)) \\
\times (1 - \theta \overline{V}(f_3(z_1, z_2)) - (1 - \theta) \overline{C}(f_3(z_1, z_2)) \\
\times \overline{M}(f_3(z_1, z_2)) \right\} G_3(g(z_2)) f_3(z_2) \\
\times f_3(z_2) \left[ 1 - \theta \overline{V}(f_3(z_1, z_2)) - (1 - \theta) \overline{C}(f_3(z_1, z_2)) \right] \right\} \right\}
\]

\[
dr = \left\{ z_1 - (q + p z_2) f_4(z_1, z_2) f_3(z_2) f_3(z_2) f_3(z_2) f_4(z_2) f_3(z_1, z_2). \right. \\
\]

The probability generating function \( \psi(z) \) of the number of customers in the orbit when the system is idle is given by

\[
\Psi(z) = \frac{I_{0,0} + I(0, z_2)}{I_{0,0} + I(0, 1)}
\]

From the above, we see that \( \Omega(z) = \Pi(z) \Psi(z) \).
THE AVERAGE QUEUE LENGTH AND WAITING TIME

The Mean number of customers in the queue and orbit under the steady state condition is,

\[ L_{q1} = \frac{d}{dz} P_{q1}(z, 1)|_{z=1}, \]

\[ L_{q2} = \frac{d}{dz} P_{q2}(z, 2)|_{z=1}. \]  

(59)

then,

\[ L_{q1} = \frac{D_1^{(1, 1)} N_1^{(1, 1)} - D_2^{(1, 1)} N_2^{(1, 1)}}{2 D_2^{(1, 1)}}, \]

\[ L_{q2} = \frac{d_1^{(1, 1)} n_1^{(1, 1)} - d_2^{(1, 1)} n_2^{(1, 1)}}{2 (d_2^{(1, 1)})^2}. \]

By Little’s Law, Average waiting time of a customer in the high priority queue and low priority orbit is,

\[ W_{q2} = \frac{L_{q2}}{\lambda_2}, \]

\[ W_{q1} = \frac{L_{q1}}{\lambda_1}. \]  

(60)

(61)

Particular Cases

**Case 1** \( M^X/G/1 \) Queueing model:
If there are no priority arrival, no vacation, no working breakdown, no balking, no retrial, no closedown/startup time and batch arrival. The model under study becomes classical \( M^X/G/1 \) queueing system. In this case, the PGF of the busy state is given as,

\[ P(z) = \frac{-(1 - B(\lambda - \zeta C(z))) I_{0,0}}{z - B(\lambda - \zeta C(z))}. \]  

(61)

**Case 2** \( M/G/1 \) Queueing model:
If there are no priority arrival, no vacation, no working breakdown, no balking, no retrial, no closedown/startup time and single arrival. The model under study becomes classical \( M/G/1 \) queueing system. In this case, the PGF of the busy state is given as,

\[ P(z) = \frac{-(1 - B(\lambda - \zeta z)) I_{0,0}}{z - B(\lambda - \zeta z)}. \]  

(62)

The above two results are coincide with the results of Gross, D and Harris, M (1985).

NUMERICAL RESULTS

In order to see the effect of different parameters on the different states of the server we compute some numerical results. We consider the service time, vacation time and closedown/startup time to be exponentially distributed to numerically illustrate the feasibility of our results. By giving the following suitable values for the parameters which satisfy the stability condition, we compute the table values.

\[(\lambda_2, \mu_1, \mu_2, \mu_3, \eta, \theta, \alpha, \beta, \phi, \gamma, \delta, \rho, p_1, r, b)\]

\[(0.1, 7.7, 0.1, 10, 0.25, 20, 15, 15, 0.2, 0.6, 0.1, 0.8).\]

\[(\lambda_2, \mu_1, \mu_2, \mu_3, \eta, \theta, \alpha, \beta, \phi, \gamma, \delta, \rho, p_1, r, b)\]

\[(0.1, 7.7, 0.3, 11, 0.75, 16, 14, 15, 11, 0.1, 0.7, 0.1, 0.8).\]

\[(\lambda_2, \mu_1, \mu_2, \mu_3, \eta, \theta, \alpha, \beta, \phi, \gamma, \delta, \rho, p_1, r, b)\]

\[(0.1, 0.1, 0.1, 0.2, 0.1, 0.4, 0.5, 0.2, 1.5, 0.4, 0.4, 0.5, 0.3).\]

\[(\lambda_2, \mu_2, \mu_3, \eta, \theta, \alpha, \beta, \phi, \gamma, \delta, \rho, p_1, r, b)\]

\[(0.2, 0.2, 0.2, 0.1, 0.3, 10, 0.5, 20, 20, 20, 0.4, 0.1, 0.2, 0.8).\]

Table 1 and 4 clearly shows that as long as the arrival rate of high priority customers and vacation rate increases the servers idle time decreases. Simultaneously the utilisation factor, average queue length for both high priority and low priority queue characteristics

<table>
<thead>
<tr>
<th>( \lambda_1 )</th>
<th>( I_{0,0} )</th>
<th>( \rho )</th>
<th>( L_{q1} )</th>
<th>( W_{q1} )</th>
<th>( W_{q2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.9901</td>
<td>0.0099</td>
<td>0.0238</td>
<td>0.1971</td>
<td>0.0170</td>
</tr>
<tr>
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<td>0.9898</td>
<td>0.0102</td>
<td>0.0358</td>
<td>0.5023</td>
<td>0.0238</td>
</tr>
<tr>
<td>1.6</td>
<td>0.9896</td>
<td>0.0104</td>
<td>0.0490</td>
<td>0.7579</td>
<td>0.0306</td>
</tr>
<tr>
<td>1.7</td>
<td>0.9894</td>
<td>0.0106</td>
<td>0.0635</td>
<td>0.9665</td>
<td>0.0373</td>
</tr>
<tr>
<td>1.8</td>
<td>0.9893</td>
<td>0.0107</td>
<td>0.0792</td>
<td>1.1303</td>
<td>0.0440</td>
</tr>
<tr>
<td>1.9</td>
<td>0.9892</td>
<td>0.0108</td>
<td>0.1147</td>
<td>1.3305</td>
<td>0.0574</td>
</tr>
<tr>
<td>2.0</td>
<td>0.9892</td>
<td>0.0108</td>
<td>0.1345</td>
<td>1.3699</td>
<td>0.0641</td>
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<tr>
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<td>0.0108</td>
<td>0.1557</td>
<td>1.3703</td>
<td>0.0708</td>
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<tr>
<td>2.2</td>
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<td>0.0108</td>
<td>0.1557</td>
<td>1.3703</td>
<td>0.1303</td>
</tr>
</tbody>
</table>

**Table 2.** Effect of \( \lambda_2 \) on various queue characteristics

<table>
<thead>
<tr>
<th>( \lambda_2 )</th>
<th>( I_{0,0} )</th>
<th>( \rho )</th>
<th>( L_{q1} )</th>
<th>( L_{q2} )</th>
<th>( W_{q1} )</th>
<th>( W_{q2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.9976</td>
<td>0.0024</td>
<td>0.0444</td>
<td>1.5011</td>
<td>0.444</td>
<td>0.6004</td>
</tr>
<tr>
<td>2.6</td>
<td>0.9975</td>
<td>0.0025</td>
<td>0.0444</td>
<td>2.0266</td>
<td>0.444</td>
<td>0.7794</td>
</tr>
<tr>
<td>2.7</td>
<td>0.9975</td>
<td>0.0025</td>
<td>0.0444</td>
<td>2.5832</td>
<td>0.444</td>
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<tr>
<td>2.8</td>
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<td>0.0025</td>
<td>0.0444</td>
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<td>0.444</td>
<td>1.1319</td>
</tr>
<tr>
<td>2.9</td>
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<td>0.0026</td>
<td>0.0444</td>
<td>3.7835</td>
<td>0.444</td>
<td>1.3046</td>
</tr>
<tr>
<td>3.0</td>
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<td>0.0026</td>
<td>0.0444</td>
<td>4.4240</td>
<td>0.444</td>
<td>1.4747</td>
</tr>
</tbody>
</table>
Table 3. Effect of $\mu$ on various queue characteristics

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$I_{0,0}$</th>
<th>$\rho$</th>
<th>$L_{q_1}$</th>
<th>$L_{q_2}$</th>
<th>$W_{q_1}$</th>
<th>$W_{q_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.1</td>
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<td>0.3357</td>
<td>0.1069</td>
<td>1.4888</td>
<td>1.0692</td>
<td>14.8880</td>
</tr>
<tr>
<td>12.2</td>
<td>0.6645</td>
<td>0.3355</td>
<td>0.1054</td>
<td>1.3221</td>
<td>1.0545</td>
<td>13.2210</td>
</tr>
<tr>
<td>12.3</td>
<td>0.6646</td>
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Table 4. Effect of $\beta$ on various queue characteristics

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Graphical Study

We can plot the above data graphically to illustrate the feasibility of our results.

CONCLUSION

In this paper we have analysed a $M^{[X_1]}, M^{[X_2]}/G_1, G_2/1$ retrial queue with priority service under modified Bernoulli vacation subject to the server working breakdown along low priority customers are also increases. Table 2 reveals that as long as the arrival rate of low priority customers increases the servers idle time decreases. Simultaneously the utilisation factor, average queue length for low priority customers are increases. Since it is preemptive priority queueing system, the arrival of low priority customers does not affect the high priority queue. So it remains constant. Table 3 shows that as long as the service rate increases the server’s idle time increases and the utilisation factor, average queue length for both high priority and low priority customers are decreases.

FIGURE 2. Average queue sizes Vs High priority arrival rate $\lambda_1$

FIGURE 3. Average queue sizes Vs Low priority arrival rate $\lambda_2$

with it the close down/start up time also investigated. In addition, the effect of impatient behaviour of the customer on a service system is studied. The joint distribution of the number of customers in the queue and the number of customers in the orbit are derived. Numerical examples have been carried out to observe the trend of the mean number of customers in the system for varying parametric values. This paper analyzes a single-server retrial queue with constant retrial policy, preemptive repeat priority, collisions, orbital search, working breakdowns and repair in order to obtain analytical expressions for various performance measures of interest. The joint steady-state probability generating functions of the server state and the number of customers in the orbit are derived. The stochastic decomposition law for this system has also been established. Numerical examples have been carried out to observe the effects of several parameters
on the system. The novelty of this investigation is the discussion of the constant retrial policy, working breakdowns, retrial queueing system with preemptive repeat priority and collisions of customers. This makes the system more complex though realistic.

REFERENCES


