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An Educational Rudder Sizing Algorithm for Utilization in Aircraft Design Software

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Abstract

The essential prerequisites of the rudder sizing process are the directional control and trim. The main roles of the rudder are: turn coordinate, asymmetric thrust on conventional transport aircraft, crosswind landing, adverse yaw, spin recovery, and glide slope adjustment. The asymmetric thrust and crosswind landing are the most crucial flight conditions that recognize the transport aircraft. This paper presents an educational rudder sizing algorithm of conventional aircraft, primarily, for aeronautical engineering students and also for fresh engineers to enhance their knowledge, understanding, and analysis. The paper introduces the necessary equations to guide the designer to satisfy the qualifications of the directional control and directional trim. A work out example has been added to explain the application of the algorithm.

Keywords: Algorithms, rudders, rudder sizing, control surface design, vertical tail design, preliminary design phase.

INTRODUCTION

Aircraft rudder is one of the primary control surfaces. Its main function is the directional control. As the rudder is deflected, a side force is generated, and in turn, a yawing moment is created about the aircraft cg along the aircraft z-axis. The essential prerequisites of the rudder sizing process are the directional control and trim. The directional control is managed basically via the yaw rate, whereas the directional trim is managed mainly via the maximum deflection of the rudder.

Some regulations related to directional control, which belong to the FAA [1], must be considered in the design of the rudder. For instance, the regulation of FAR part 25 section 147 [2] involves: "*It must be possible, with the wings level, to yaw into the operative engine and to safely make a reasonably sudden change in heading of up to 15 deg in the direction of the critical inoperative engine. This must be shown at 1.3 V_s for heading changes up to 15 deg, and with (i) the critical engine inoperative and its propeller in the minimum drag position; (ii) the power required for level flight at 1.3 V_s, but not more than maximum continuous power; (iii) the most unfavorable center of gravity; (iv) landing gear retracted; (v) flaps in the approach position; and (vi) maximum landing weight*". A similar regulation is founded in FAR part 23 [3] for general aviation and in MIL-STD [4] [5] for military aircraft.

It should be noted that there are meddling between aileron and rudder, and frequently employed concurrently. Therefore, directional and lateral dynamics are often coupled [6] [7]. Thus, it is a better rehearsal to size the aileron and rudder simultaneously. On the other hand, the aileron is a rate control device, while the rudder is similar to elevator as both are displacement control device. The sizing fundamentals of rudder and elevator are alike, but generally, the rudder sizing is more complicated [8].

However, a quick pass on the available developed software for educational aircraft design is publicized briefly. Starting with Roskam's software (AAA) [9], it is basically a coded version of his textbook [10]. Raymer in 1996 released his software package (ADS) [11] based also on his book [12]. Soon later, the comprehensive software CEASIOM [13] was developed in 2008 and still in the stage of improvement. Finally, Nicolosi released his software (ADAS) [14] in 2011. Unfortunately, all these software packages present the rudder sizing indirectly by offering just the stability and controllability derivatives, dimensional and non-dimensional. These derivatives are not so clear for students, and many practices must be exercised to be familiar with. Typically, the rudder sizing process is done in the detail phase.

On the research side view, most researches are intended for industry rather than for education. Struett [15] developed a Matlab program to size the empennage and to investigate its static and dynamic stability. His program is dedicated to Ryan Navion aircraft. Steer [16] established a design criteria for conceptual sizing of primary flight controls. It is based mostly on Concorde aircraft and mounted to offer equivalent un-supplemented stability and maneuver performance. The rudder is sized to offer an adequate power to overcome a double asymmetric engine failure through normal flight. For a full-scale vertical tail model, Whalen, et al., [17] elaborated an enhanced performance equipped with active flow control, while Andino, et al., [18] presented a flow separation control using sweeping jet actuators. Finally, Hettema [19] developed a rapid aerodynamic analysis method for the initial vertical tail design of conventional aircraft. He suggested to expand his module to include the calculations of the rudder design.

RUDDER SIZING FUNDAMENTALS

Rudder sizing can be done in the preliminary design phase mainly for aeronautical engineering students and also for fresh engineers to enhance their knowledge, understanding, and analysis. So, many evaluated or estimated parameters (related to the rudder) can be delivered from the conceptual design phase such as area, chord, span, and maximum deflection. Typical values for the rudder geometry are shown in Table 1 [8].

Table 1: Typical values for rudder's geometry [8]

Rudder's parameter	Value
Area S_r/S_v	0.15-0.3
Span b_r/b_v	0.7-1.0
Chord C_r/C_v	0.15-0.4
Maximum deflection	± 30 deg. (left/right)

Initially, the yawing moment (N_a) of a symmetric aircraft (with zero aileron deflection and zero sideslip angle) is determined as:

$$N_a = l_v L_v \tag{1}$$

Where: l_v is the vertical tail arm, and L_v is the vertical tail lift force which can be formalized as:

$$L_v = \bar{q} S_v C_{l_v} \tag{2}$$

Where: C_{l_v} is the lift coefficient of the vertical tail, which can be linearly modelled as:

$$C_{l_v} = C_{l_{v_o}} + C_{l_{v_\beta}}\beta + C_{l_{v_{\delta r}}}\delta_r \tag{3}$$

However, Equation 1 can be defined as:

$$N_a = \bar{q} S b C_n \tag{4}$$

Where: C_n is the coefficient of the yawing moment, which is linearly modelled as:

$$C_n = C_{n_o} + C_{n_\beta}\beta + C_{n_{\delta a}}\delta_a + C_{n_{\delta r}}\delta_r \tag{5}$$

Where: $C_{n_{\delta r}}$ is the coefficient of the aircraft yawing moment as a result of rudder deflection, which can be determined in a directional form as:

$$C_{n_{\delta r}} = -\, C_{l_{\alpha_v}} \bar{V}_v\, \eta_v\, \tau_r \frac{b_r}{b_v} \tag{6}$$

Where: $C_{l_{\alpha_v}}$ is the curve slope coefficient of the vertical tail lift, $\bar{V}_v$ is the volume coefficient of the vertical tail, and τ_r is obtained from Figure 1 [20]; the mathematical model of Figure 1 using MATLAB [21], CF tool is:

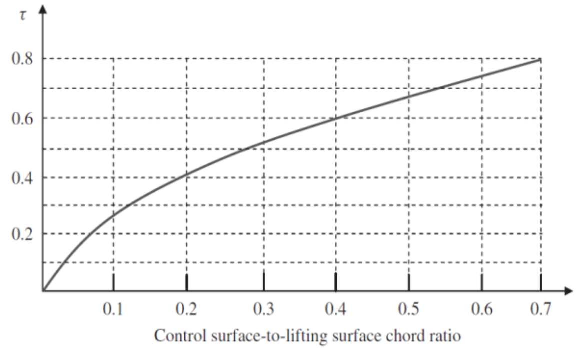

Figure 1: A general representation of the control surface effectiveness [20].

$$\tau_r = 1.129 \times \left(\frac{C_r}{C_v}\right)^{0.4044} - 0.1772 \tag{7}$$

In general, the main roles of the rudder are: turn coordinate, asymmetric thrust on conventional transport aircraft, crosswind landing, adverse yaw, spin recovery, and glide slope adjustment. Based on the aircraft flight and configuration, one or more of these roles is the most significant and critical. For example, in a single-engine aircraft, the most crucial function is the maximum crosswind landing, while for a multi-engine aircraft, the directional control for balancing asymmetric thrust is the most critical condition as well as crosswind landing. However, the designer must initially define the most critical role of the rudder to employ within the aircraft mission. This paper presented the roles of conventional aircraft, which are asymmetric thrust and crosswind landing with their algorithms. A brief description of its requirements, the necessary equations, and its procedure to size the aircraft rudder is introduced, as well.

Asymmetric thrust

In conventional transport aircraft, the crucial asymmetric thrust situation often happens when the engines of the aircraft (on one side) stop working at low speed. In this condition, the created yawing moment by the asymmetric thrust assembly must be defeated by a very robust rudder. FAR part 25 Section 149 states that the transport (multi-engine) aircraft should be able to directional control at a crucial speed, which in turn should not be more than 1.13 of the aircraft stall speed (v_s) at the most crucial takeoff arrangement and at on the whole of critical cg location. This crucial speed is called the minimum controllable speed (v_{mc}). Moreover, the lateral control should be powerful to roll the aircraft in less than 5 seconds, from the initial stable flight within 20 deg in the route to set off a rotation further than the failed engines. For safety consideration, the minimum controllable speed should be at least 0.8 of the aircraft stall speed [8].

On a trimmed flight, the operative engine thrust (T_L) is equal to the drag of the aircraft. In addition, the yawing moment summation is zero. Hence,

$$N_a = -T_L y_T \tag{8}$$

Where: y_T Is the distance from the operating engine to the fuselage center line, and?

$$N_a = \bar{q}Sb\left(C_{n_o} + C_{n_\beta}\beta + C_{n_{\delta a}}\delta_a + C_{n_{\delta r}}\delta_r\right) \quad (9)$$

Assuming that the conventional transport aircraft is symmetric about the xz plane means that $C_{n_o} = 0$. If there is no aileron deflection ($\delta_a = 0$), then the sideslip angle (β) is zero, as well. Hence, to trim a 2-engine aircraft directionally in the asymmetric thrust situation, the required rudder deflection is:

$$\delta_r = \frac{T_L y_T}{-\bar{q}SbC_{n_{\delta r}}} \quad (10)$$

In case of a multi-engine aircraft, Equation 10 will be:

$$\delta_r = \frac{\sum_{i=1}^{n=2} T_L y_T}{-\bar{q}SbC_{n_{\delta r}}} \quad (11)$$

Where: n is the total operative engines on one side.

Crosswind landing

All aircraft types are needed to land safely while a crosswind is blowing. The rudder should be very influential to allow the pilot to trim for the particular crosswinds. For instance, the rudder needs to provide a sideslip angle to keep alliance with the centre line of the landing strip. The sideslip angle (β) is denoted as the angle between the direction of the relative wind (v_w) and the aircraft path (v_f), i.e.:

$$\beta = \tan^{-1}\left(\frac{v_w}{v_f}\right) \quad (12)$$

Hence, the total aircraft speed (v_t) is equal to the vector summation of the path speed (v_f) and the crosswind speed (v_w), i.e.:

$$v_t = \sqrt{v_f^2 + v_w^2} \quad (13)$$

During a crabbed landing and the aircraft is trimmed directionally, the aerodynamic side force of the aircraft (F_{a_y}) is:

$$F_{a_y} = \bar{q}\, S\, \left(C_{y_o} + C_{y_\beta}(\beta - \sigma) + C_{y_{\delta r}}\delta_r\right) \quad (14)$$

Where:

$$C_{y_\beta} = -K_{f1}C_{l_{\alpha_v}}\left(1 - \frac{d\sigma}{d\beta}\right)\eta_v \frac{S_v}{S} \quad (15)$$

And

$$C_{y_{\delta r}} = C_{l_{\alpha_v}}\eta_v \tau_r \frac{b_r S_v}{b_v S} \quad (16)$$

The force created by the crosswind (F_w) is defined as:

$$F_w = \frac{1}{2}\, \rho\, v_w^2\, S_s\, C_{d_y} \quad (17)$$

Where: S_s is the projected side area of the aircraft and C_{d_y} is the coefficient of the aircraft side drag. For conventional aircraft, the side drag coefficient has typically a value of 0.55-0.8.

Since F_{a_y} is equal to the crosswind force (F_w), therefore:

$$\frac{1}{2}\, \rho\, v_w^2\, S_s\, C_{d_y} = \bar{q}\, S\, \left(C_{y_o} + C_{y_\beta}(\beta - \sigma) + C_{y_{\delta r}}\delta_r\right) \quad (18)$$

While the moment equilibrium equation is:

$$N_a + F_w\, d_c\, \cos\sigma = 0 \quad (19)$$

Where: N_a is the aircraft yawing moment, and DC is the distance between the aircraft projected side area's center and the aircraft cg.

$$N_a = \bar{q}\, S\, b\, \left(C_{n_o} + C_{n_\beta}(\beta - \sigma) + C_{n_{\delta r}}\delta_r\right) \quad (20)$$

Since:

$$C_{n_\beta} = K_{f2}C_{l_{\alpha_v}}\left(1 - \frac{d\sigma}{d\beta}\right)\eta_v \frac{l_v S_v}{bS} \quad (21)$$

The parameters K_{f1} and K_{f2} are the fuselage contribution to the derivatives C_{y_β} and C_{n_β}, respectively. For conventional aircraft, the typical value for the parameter K_{f1} is between 0.65-0.75, and for the parameter K_{f2} is between 1.3-1.4. The factor d_σ / d_β is the sidewash gradient of the vertical tail. It should be noted that if the sideslip angle is negative (i.e., the crosswind from the left), the rudder deflection is utilized negative in all equations.

Substituting Eq. 20 into Eq. 19 and reforming Eq. 18 yields:

$$\frac{1}{2}\, \rho\, v_t^2\, S\, b\, \left(C_{n_o} + C_{n_\beta}(\beta - \sigma) + C_{n_{\delta r}}\delta_r\right) + F_w\, d_c \cos\sigma = 0 \quad (22)$$

$$\frac{1}{2}\, \rho\, v_w^2\, S_s\, C_{d_y} - \bar{q}\, S\, \left(C_{y_o} + C_{y_\beta}(\beta - \sigma) + C_{y_{\delta r}}\delta_r\right) = 0 \quad (23)$$

Solving Eq. 22 and Eq. 23 simultaneously yields the two unknown variables, i.e. the rudder deflection (δ_r) and the crab angle (σ).

On the other side, to evaluate the aircraft projected side area's center (ac), the side view of the aircraft is partitioned into a number of segments (n), each segment of standard geometric shape like a rectangle or a triangle or a circle. The center of these standard shapes is known and can easily obtain from any standard mathematical handbooks such as Ref [22]. For conventional aircraft, the projected side area is partitioned into two major segments, the fuselage side area and the vertical side area. Therefore, the distance (x_i) between the reference line (for example, the fuselage nose) and the a_c is obtained by the following formula:

$$x_{a_c} = \frac{\sum_{i=1}^{n=2} A_i x_i}{\sum_{i=1}^{n=2} A_i} = \frac{(l_f D_f)x_f + S_v x_v}{l_f D_f + S_v} \quad (24)$$

Where: A_i is the area of the i segment.

RUDDER SIZING ALGORITHM

As mentioned above, the asymmetric thrust and crosswind landing are the most crucial flight conditions that recognize the conventional aircraft. To achieve the crosswind landing prerequisites, the rudder sizing algorithm steps are as follows:

1. From the conceptual design phase, record all the data related to rudder sizing process which include vertical tail geometry, rudder-to-vertical tail span and cord ratios as in Table 1, and aircraft cg positions (normal, forward, and aft). Also, identify the main crucial restriction, which is the most adverse cg position and altitude, for directional control.
2. From FAR regulations, find the maximum crosswind speed and the aircraft approach speed.
3. Applying Eq. 13 to find the aircraft total speed, assuming perpendicular crosswind to the landing strip.
4. Determine the aircraft projected side area and its center by employing Eq. 24.
5. Calculate the distance from the center of the aircraft projected area to the cg of the aircraft.
6. Applying Eq. 17 to find the aircraft side force created by the crosswind.
7. Using Eq. 12, calculate the aircraft sideslip angle.
8. Applying Eq. 21 and Eq. 15 to find the aircraft derivatives C_{n_β} and C_{y_β}, respectively.
9. Based on the selected value of the rudder-to-vertical chord ratio, determine the rudder angle of attack effectiveness from Figure 1.
10. If τ_r is more than one, it is required to redesign the vertical tail and relocate the aircraft cg. Return to step 1.
11. Applying Eq. 6 and Eq. 16 to find the aircraft control derivatives $C_{n_{\delta r}}$r and $C_{y_{\delta r}}$, respectively.
12. Calculate the rudder deflection angle and the crab angle by solving simultaneously the Eq. 22 and Eq. 23.
13. It is recommended to expand the rudder chord ratio up to 1 (i.e. all-moving tail) if the deflection angle of the rudder is greater than 30 deg.

Now, with conventional aircraft, to achieve the asymmetric thrust prerequisites, the rudder sizing algorithm steps are as follows:

1. From the conceptual design phase, record all the data related to rudder sizing process which include vertical tail geometry, rudder-to-vertical tail span ratio and the maximum rudder deflection from Table 1, and aircraft cg positions (normal, forward, and aft). Also, identify the main crucial restriction, which is the most adverse cg position, altitude, and inoperative engines, for directional control.
2. From FAR regulations, find the minimum aircraft controllable speed, which is suggested to be 0.8 of the aircraft landing stall speed.
3. Employing Eq. 8 to find the maximum yawing moment.
4. Calculate the control derivative $C_{n_{\delta r}}$ using Eq. 9.
5. Calculate the rudder deflection using Eq. 10 (for 2-engine aircraft) or Eq. 11 (for multi-engine aircraft).
6. If the maximum rudder deflection is less than the calculated rudder deflection, either resizing the rudder, or increasing the minimum aircraft controllable speed, but not exceeding 1.13 of the stall speed (FAR part 25 Section 147). Return to step 4.
7. Applying Eq. 6 to determine the rudder angle of attack effectiveness.
8. If τ_r is more than one, it is required to redesign the vertical tail and relocate the aircraft cg. Return to step 1.
9. Using Figure 1 to find the equivalent rudder-to-vertical tail chord ratio.
10. It is recommended to go for an all-moving tail if the calculated chord ratio from step 6 is greater than 0.5.

TESTING THE ALGORITHM

The algorithm can be coded in a high level language and encapsulated in any preliminary aircraft design software to speed up the rudder sizing process. The following example illustrates the application of this algorithm as an instructional use.

Example

For an 80 passenger conventional two wing-engine transport aircraft with the following data delivered from the conceptual design phase:

m_{to} = 36000 kg, v_s = 53.65 m/s, T_{max} = 98.8 kN, l_f = 34.3 m, D_f = 2.9 m, C_{yo} = 0, S = 66 m^2, b = 24.8 m, C_{no} = 0, S_v = 7 m^2, b_v = 4.58 m, b_r / b_v = 1, C_r / C_v = 0.3, $\bar{V}_v$= 0.084, η_v = 0.95, $C_{l\alpha v}$ = 4.5 1/rad, $d\sigma/d\beta$ = 0, l_v = 19.6 m, x_{cg} = 15.26 m.

It is required that the aircraft is not able to spin and is able to land safely with a crosswind of 20.6 m/s (40 knots). Design the aircraft rudder.

Solution

Step 1- Initially, the critical sizing requirements must be identified. The aircraft is a conventional transport, so it is not spin-able. The crosswind landing is the most crucial flight condition that recognizes the conventional transport (wing installed engines) aircraft. The asymmetric thrust is also examined.

Step 2- From the example statement, the maximum crosswind speed is v_w = 20.6 m/s, and the approach aircraft speed is 1.1*v_s = 1.1*59 = 64.9 m/s.

Step 3- Applying Equation 106 to find the total aircraft speed, v_t = 68.15 m/s.

Step 4- For a conventional transport aircraft, the fuselage and vertical tail side area represent the the aircraft projected side area, while the wing and most of the engines are projected into the fuselage. Typically, it is proper to add 2% to justify the landing gear. Therefore, the aircraft projected side area is:

$$S_s = 1.02 \times (S_f + S_v) = 1.02 \times (l_f D_f + S_v) = 98.96\ m^2$$

From the cg analysis model of the conceptual phase, the fuselage center is = 16.62 m and the vertical tail center is = 33.56 m. Using Eq. 24, the overall center of the aircraft projected side area =18.34 m

Step 5- The distance d_c between the aircraft cg and the center of the projected side area is = 3.09 m.

Step 6- The aircraft side force is = 16705.8 N.

Step 7- The aircraft sideslip angle β is = 0.307 rad = 17.6 deg.

Step 8- The derivative C_{n_β} = 0.43 1/rad and the derivative C_{y_β} = - 0.61 1/rad.

Step 9- For the ratio C_r / C_v = 0.3, τ_r = 0.517.

Step 10- τ_r is less than one.

Step 11- The derivative $C_{y_{\delta r}}$ = 0.233 1/rad and the derivative $C_{n_{\delta r}}$ = - 0.181 1/rad.

Step 12- The rudder deflection δ_r = 0.437 rad = 25 deg, σ = 0.213 rad = 12.18 deg.

Step 13- As long as the calculated rudder deflection is beneath the maximum deflection (i.e., 25 < 30), so, the aircraft lands safely and able to deal with the crosswind of 20.6 m.

Now, the designed rudder should be examined to check if it is satisfying the other design requirement, which is the asymmetric thrust, starting with step 2.

Step 2- The minimum aircraft controllable speed (v_{mc}) = 0.8*v_s = 42.92 m/s.

Step 3- The maximum yawing moment = - 192297.57 Nm.

Step 4- The control derivative $C_{n_{\delta r}}$ = - 0.181 1/rad.

Step 5- The rudder deflection = 0.564 rad = 32.33 deg.

Step 6- Since the calculated rudder deflection is slightly greater than the maximum deflection, therefore, increase the minimum aircraft controllable speed. To find the new minimum controllable speed, apply Equation 10 with a 30 deg as a maximum rudder deflection, yields:

$$v_{mc} = \sqrt{\frac{T_L y_T}{-0.5 \rho S b C_{n_{\delta r}} \delta_r}}$$

$$= \sqrt{\frac{49800 \times 3.81}{-0.5 \times 1.225 \times 66 \times 24.8 \times (-0.184) \times 0.5233}}$$

$$= 44.33\ m/s$$

$$v_{mc} = \frac{44.33}{53.65} \times v_s = 0.83 \times v_s$$

Hence, the minimum controllable speed is increased from *0.8* v_s to *0.83* v_s.

Step 7-10 Done in crosswind landing analysis.

CONCLUSIONS

An instructive algorithm for the rudder sizing of conventional transport aircraft has been introduced. It is established largely for aeronautical engineering students and also for fresh engineers to augment their knowledge and analysis. The paper introduced the required equations to guide the student step-by-step to satisfy the criterion of the directional control and directional trim. A work out example has been added to describe the application of the algorithm.

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