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Intuitionistic fuzzy γ generalized closed mappings

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Abstract

In this paper, we introduce intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings and intuitionistic fuzzy M-γ generalized closed mappings investigate some of their properties. We discuss the relation between intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings and intuitionistic fuzzy M-γ generalized closed mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings and intuitionistic fuzzy M - γ generalized closed mappings.

Subject classification code: 03F55, 54A40.

INTRODUCTION

Atanassov [1] introduced the notion of intuitionistic fuzzy sets using the notion of intuitionistic fuzzy sets. Coker [2] introduced the notion of intuitionistic fuzzy topological spaces. Prema and Jayanthi [6] introduced intuitionistic fuzzy γ generalized closed sets. In this paper we introduce intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings and intuitionistic fuzzy M - γ generalized closed mappings investigate some of their properties. We discuss the relation between intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings and intuitionistic fuzzy M - γ generalized closed mappings.

PRELIMINARIES

Definition 1: [1] An intuitionistic fuzzy set (IFS for short) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \longrightarrow [0,1]$ and $\nu_A : X \longrightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS (X), the set of all intuitionistic fuzzy sets in X. An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A\rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2: [1] Let A and B be two IFSs of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$. Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 3: [2] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 4: [6] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ generalized closed set (IFγGCS for short) if $\gamma cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFγOS in (X, τ).

The complement A^c of an IFγGCS A in an IFTS (X, τ) is called an intuitionistic fuzzy γ generalized open set (IFγGOS for short) in X.

Definition 5: [6] A mapping f: $(X, \tau) \to (Y, \sigma)$ is called an intuitionistic fuzzy γ generalized continuous (IFγG continuous for short) mapping if $f^{-1}(V)$ is an IFγGCS in (X,

τ) for every IFCS V of (Y, σ).

Definition 6: [6] An IFTS (X ,τ) is an $IF\gamma_\gamma T_{1/2}$ space if every IFγGCS is an IFγCS in X.

Definition 7: [6] An IFTS (X ,τ) is an $IF\gamma_c T_{1/2}$ space if every IFγGCS is an IFCS in X.

Definition 8: [7] A mapping f: X→ Y is called an intuitionistic fuzzy closed mapping (IFCM for short) if f(A) is an IFCS in Y for each IFCS A in X.

Definition 9: [3] An intuitionistic fuzzy point (IFP in short) written as $p_{(\alpha,\beta)}$ is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 10: [7] Let $p_{(\alpha,\beta)}$ be an IFP in (X, τ). An IFS A of X is called an intuitionistic fuzzy neighbourhood (IFN for short) of $p_{(\alpha,\beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition 11: [5] Let $p_{(\alpha,\beta)}$ be an IFP in (X, τ). An IFS A of X is called an intuitionistic fuzzy γ neighbourhood (IFγN for short) of $p_{(\alpha,\beta)}$ if there exists an IFγOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Definition 12: [2] Let X and Y be two non empty sets and f: X → Y be a mapping. If $A = \{\langle x, (\mu_A(x), \nu_A(x) / x \in X\rangle\}$ is an IFS in X, then the image of A under f, denoted by f(A), is the IFS in Y defined by $f(A) = \{\langle y, f(\mu_A)(y), f_-(\nu_A)(y) / y \in Y\rangle\}$, where $f_-(\nu_A) = 1 - f(1-\nu_A)$.

Definition 13: [2] Let X and Y be two non empty sets and f: X → Y be a mapping. If $B = \{\langle y, \mu_B(y), \nu_B(y) / y \in Y\rangle\}$ is an IFS in Y, then the preimage of B under f is denoted and defined by $f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x) / x \in X\rangle\}$ where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ for every $x \in X$.

Corollary 14: [4] Let A, A_i (i ∈ J) be intuitionistic fuzzy sets in X and B, B_j (j ∈ K) be intuitionistic fuzzy sets in Y and f: X → Y be a mapping. Then

a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]

d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]

e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

g) $f^{-1}(0_\sim) = 0_\sim$

h) $f^{-1}(1_\sim) = 1_\sim$

i) $f^{-1}(B^c) = (f^{-1}(B))^c$

INTUITIONISTIC FUZZY γ GENERALIZED CLOSED MAPPINGS

In this section we introduce intuitionistic fuzzy γ generalized closed mappings, intuitionistic fuzzy γ generalized open mappings, intuitionistic fuzzy M - γ generalized closed mappings and study some of their properties.

Definition 15: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy γ generalized closed mapping (IFγG closed mapping for short) if f (V) is an IFγGCS in Y for every IFCS V of X.

Example 16: Let X = {a, b}, Y = {u, v} and $G_1 = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b)\rangle$, $G_2 = \langle y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)\rangle$, $G_3 = \langle y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)\rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, G_3, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS $G_1^c = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b)\rangle$ is an IFCS in X. Then $f(G_1^c) = \langle y, (0.3_u, 0.2_v), (0.7_u, 0.8_v)\rangle$ is an IFS in Y. Now

$IF\gamma O(Y) = \{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0,1], \nu_u \in [0,1], \nu_v \in [0,1] /$ either $\nu_u < 0.4$ or $\nu_v < 0.3$, $\nu_u \geq 0.5$ whenever $\nu_v < 0.6$, $0.4 \leq \nu_u \leq 0.5$ whenever $\nu_v \leq 0.4$, $\mu_u \geq 0.5$, $\mu_v \geq 0.6$, $0.5 \leq \nu_u < 0.6$ whenever $\nu_v \geq 0.6$, $\mu_u \geq 0.4$, $\mu_v \geq 0.3$ and $\nu_u \geq 0.6$ whenever $\nu_v \geq 0.4$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$and

$IF\gamma C(Y) = \{0_\sim, 1_\sim, \mu_u \in [0,1], \mu_v \in [0, 1], \nu_u \in [0,1], \nu_v \in [0,1] /$ either $\mu_u < 0.4$ or $\mu_v < 0.3$, $\mu_u \geq 0.5$ whenever $\mu_v < 0.6$, $0.4 \leq \mu_u \leq 0.5$ whenever $\mu_v \leq 0.4$, $\nu_u \geq 0.5$, $\nu_v \geq 0.6$, $0.5 \leq \mu_u < 0.6$ whenever $\mu_v \geq 0.6$, $\nu_u \geq 0.4$, $\nu_v \geq 0.3$ and $\mu_u \geq 0.6$ whenever $\mu_u \geq 0.4$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1\}$.

Hence $f(G_1^c)$ is an IFγGCS in Y. Therefore f is an IFγG closed mapping.

Theorem 17: Every IF closed mapping is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF closed mapping. Let A be an IFCS in X. Then f(A) is an IFCS in Y. Since every IFCS is an IFγGCS [6], f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 18: In Example 3.2, f is an IFγG closed mapping, but since $f(G_1^c)$ is not an IFCS in Y, as $cl(f(G_1^c)) = G_2^c \neq f(G_1^c)$, f is not an IF closed mapping.

Theorem 19: Every IF semi closed mapping [7] is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF semi closed mapping. Let A be an IFCS in X. Then f(A) is an IFSCS in Y. Since every IFSCS is an IFγGCS [6], f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

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Example 20: In Example 3.2, f is an IFγG closed mapping, but since f (G_1^c) is not an IFSCS in Y, as int(cl(f(G_1^c))) = int(G_2^c) = $G_3 \not\subseteq$ f (G_1^c), f is not an IF semi closed mapping.

Theorem 21: Every IF pre closed mapping [7] is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IF pre closed mapping. Let A be an IFCS in X. Then f(A) is an IFPCS in Y. Since every IFPCS is an IFγGCS [6], f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 22: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.6_a, 0.7_b), (0.4_a, 0.3_b)⟩, G_2 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩, G_3 = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_1^c = ⟨x, (0.4_a, 0.3_b), (0.6_a, 0.7_b)⟩ is an IFCS in X. Then f (G_1^c) = ⟨y, (0.4_u, 0.3_v), (0.6_u, 0.7_v)⟩ is an IFS in Y. Now

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.4$ or $\nu_v < 0.3$, $\nu_u \geq 0.5$ whenever $\nu_v < 0.6$, $0.4 \leq \nu_u \leq 0.5$ whenever $\nu_v \leq 0.4$, $\mu_u \geq 0.5$, $\mu_v \geq 0.6$, $0.5 \leq \nu_u < 0.6$ whenever $\nu_v \geq 0.6$, $\mu_u \geq 0.4$, $\mu_v \geq 0.3$ and $\nu_u \geq 0.6$ whenever $\nu_v \geq 0.4$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}and

IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0, 1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\mu_u < 0.4$ or $\mu_v < 0.3$, $\mu_u \geq 0.5$ whenever $\mu_v < 0.6$, $0.4 \leq \mu_u \leq 0.5$ whenever $\mu_v \leq 0.4$, $\nu_u \geq 0.5$, $\nu_v \geq 0.6$, $0.5 \leq \mu_u < 0.6$ whenever $\mu_v \geq 0.6$, $\nu_u \geq 0.4$, $\nu_v \geq 0.3$ and $\mu_u \geq 0.6$ whenever $\mu_v \geq 0.4$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}.

Hence f (G_1^c) is an IFγGCS in Y. Therefore f is an IFγG closed mapping. But f is not an IF pre closed mapping, as f (G_1^c) is not an IFPCS in Y, that is as cl(int(f (G_1^c))) = cl(G_3) = $G_2^c \not\subseteq$ f (G_1^c).

Theorem 23: Every IFα closed mapping [7] is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFα closed mapping. Let A be an IFCS in X. Then f(A) is an IFαCS in Y. Since every IFαCS is an IFγGCS [6], f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 24: In Example 3.2, f is an IFγG closed mapping, but not an IFα closed mapping, since f(G_1^c) is not an IFαCS in Y, as cl(int(cl(f (G_1^c)))) = cl(int(G_2^c)) = cl(G_3) = G_2^c $\not\subseteq$ f (G_1^c).

Theorem 25: Every IFγ closed mapping [5] is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFγ closed mapping. Let A be an IFCS in X. Then f(A) is an IFγCS in Y. Since every IFγCS is an IFγGCS [6], f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 26: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)⟩, G_2 = ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩, G_3 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_1^c = ⟨x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)⟩ is an IFCS in X. Then f (G_1^c) = ⟨y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)⟩ is an IFS in Y. Now

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.5$ or $\nu_v < 0.6$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}

IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.6$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}.

Hence f(G_1^c) is an IFγGCS in Y. Therefore f is an IFγG closed mapping. But since f (G_1^c) is not an IFγCS in Y, as int(cl(f(G_1^c))) ∩ cl(int(f (G_1^c))) = $1_\sim$ ∩ $1_\sim$ = $1_\sim$ $\not\subseteq$ f (G_1^c), f is not an IFγ closed mapping.

Theorem 27: Every IFSP closed mapping [8] is an IFγG closed mapping but not conversely in general.

Proof: Let f: (X, τ) → (Y, σ) be an IFSP closed mapping. Let A be an IFCS in X. Then f(A) is an IFSPCS in Y. Since every IFSPCS is an IFγGCS [6], f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

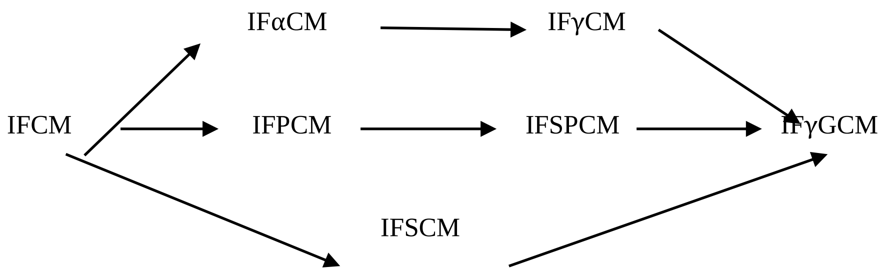
Example 28: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)⟩, G_2 = ⟨y, (0.5_u, 0.7_v), (0.5_u, 0.3_v)⟩, G_3 = ⟨y, (0.5_u, 0.6_v), (0.5_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$ G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. The IFS G_1^c = ⟨x, (0.6_a, 0.6_b), (0.4_a, 0.4_b)⟩ is an IFCS in X. Then f (G_1^c) = ⟨y, (0.6_u, 0.6_v), (0.4_u, 0.4_v)⟩ is an IFS in Y. Now

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.5$ or $\nu_v < 0.6$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}and IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.6$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}.

Hence f (G_1^c) is an IFγGCS in Y. Therefore f is an IFγG closed mapping.

Since IFPC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0, 1]$, $\mu_v \in [0, 1]$, $\nu_u \in [0, 1]$, $\nu_v \in [0, 1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.6$, $0 \leq \mu_u + \nu_u \leq 1$ and $0 \leq \mu_v + \nu_v \leq 1$}, there exists no IFPCS B in Y such that int(B) ⊆ f (G_1^c) ⊆ B in Y, f(G_1^c) is not an IFSPCS in Y. Hence f is not an IFSP closed mapping.

The relation between various types of intuitionistic fuzzy closed mappings is given in the following diagram. In this diagram 'CM' means closed mappings. The reverse implications are not true in general.



Theorem 29: Let f: X → Y be a bijective mapping. Then the following are equivalent if Y is an IFγ_γ $T_{1/2}$ space:

(i) f is an IFγG closed mapping

(ii) γcl(f(A)) ⊆ f(cl(A)) for each IFS A of X

(iii) f^{-1} (γcl(B)) ⊆ cl(f^{-1} (B)) for every IFS B of Y

Proof: (i) ⟹ (ii) Let A be an IFS in X. Then cl(A) is an IFCS in X. (i) implies that f(cl(A)) is an IFγGCS in Y. Since Y is an IFγ_γ $T_{1/2}$ space, f(cl(A)) is an IFγCS in Y. Therefore γcl(f(cl(A))) = f(cl(A)). Now γcl(f(A)) ⊆ γcl(f(cl(A))) = f(cl(A)). Hence γcl(f(A)) ⊆ f(cl(A)) for each IFS A of X.

(ii) ⟹ (i) Let A be any IFCS in X. Then cl(A) = A. (ii) implies that γcl(f(A)) ⊆ f(cl(A)) = f(A). But f(A) ⊆ γcl(f(A)). Therefore γcl(f(A)) = f(A). This implies f(A) is an IFγCS in Y. Since every IFγCS is an IFγGCS, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

(ii) ⟹ (iii) Let B be an IFS in Y. Then f^{-1} (B) is an IFS in X. Since f is onto, γcl(B) = γcl(f(f^{-1}(B))) and (ii) implies γcl(f(f^{-1}(B))) ⊆ f(cl(f^{-1}(B))).Therefore γcl(B) ⊆ f(cl(f^{-1}(B))). Now f^{-1} (γcl(B)) ⊆ f^{-1} (f(cl(f^{-1} (B)))) = cl(f^{-1}(B)), since f is one to one. Hence f^{-1} (γcl(B)) ⊆ cl(f^{-1}(B)).

(iii) ⟹ (ii) Let A be any IFS of X. Then f(A) is an IFS of Y. Since f is one to one, (iii) implies that f^{-1} (γcl(f(A))) ⊆ cl(f^{-1} (f(A))) = cl(A). Therefore f(f^{-1} (γcl(f(A)))) ⊆ f(cl(A)). Since f is onto γcl(f(A)) = f(f^{-1} (γcl(f(A)))) ⊆ f(cl(A)).

Theorem 30: A bijective mapping f: X → Y is an IFγG closed mapping if and only if for every IFS B of Y and for every IFOS U containing f^{-1} (B), there is an IFγGOS A of Y such that B ⊆ A and f^{-1} (A) ⊆ U.

Proof: Necessity: Let B be any IFS in Y. Let U be an IFOS in X such that f^{-1} (B) ⊆ U, then U^c is an IFCS in X. By hypothesis f(U^c) is an IFγGCS in Y. Let A = $(f(U^c))^c$, then A is an IFγGOS in Y and B ⊆ A, since for a bijective mapping $f(U)^c = (f(U))^c$. Now f^{-1} (A) = $f^{-1}(f(U^c))^c$ = $(f^{-1}(f(U^c)))^c$ ⊆ U.

Sufficiency: Let A be any IFCS in X, then A^c is an IFOS in X and f^{-1} (f(A^c)) ⊆A^c. By hypothesis there exists an IFγGOS B in Y such that f(A^c) ⊆ B and f^{-1} (B) ⊆ A^c. Hence B^c ⊆ f(A) ⊆ $f(f^{-1}(B))^c$ ⊆ B^c. This implies that f(A) = B^c. Since B^c is an IFγGCS in Y, f(A) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Theorem 31: If f: (X, τ) → (Y, σ) is an IFCM and g: (Y, σ) → (Z, δ) is an IFγG closed mapping, then g ∘ f : (X, τ) → (Z, δ) is an IFγG closed mapping.

Proof: Let A be an IFCS in X, then f(A) is an IFCS in Y, since f is an IF closed mapping. Since g is an IFγG closed mapping, g(f(A)) is an IFγGCS in Z. Therefore g ∘ f is an IFγG closed mapping.

Theorem 32: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ_γ $T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping

(ii) f(B) is an IFγGCS in Y for every IFCS B in X

(iii) (int(cl(f(B))) ∩ cl(int(f(B)))) ⊆f(cl(B)) for every IFS B in X

Proof: (i) ⇒ (ii) is obvious.

(ii) ⇒ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFγGCS in Y. Since Y is an IF$\gamma_\gamma T_{1/2}$ space, f(cl(B)) is an IFγCS in Y. Therefore f(cl(B)) = γcl(f(cl(B))) ⊇ f(cl(B)) ∪ (int(cl(f(cl(B))))∩cl(int(f(cl(B))))) ⊇ (int(cl(f(cl(B)))) ∩ cl(int(f(cl(B))))) ⊇ (int(cl(f(B))) ∩ cl(int(f(B)))).

(iii) ⇒ (i) Let A be an IFCS in X. By hypothesis, f(cl(A)) = f(A) ⊇ (int(cl(f(A))) ∩ cl(int(f(A)))). This implies f(A) is an IFγCS in Y and hence an IFγGCS in Y. Therefore f is an IFγG closed mapping.

Theorem 33: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following conditions are equivalent if Y is an IFγ_γ $T_{1/2}$ space:

(i) f is an IFγG closed mapping

(ii) (int(cl(f(B))) ∩ cl(int(f(B))))⊆ f (γcl(B)) for each IFCS B in X

(iii) f(γint(B)) ⊆ (int(cl(f(B))) ∪ cl(int(f(B))))for each IFOS B of X

(iv) f^{-1} (cl(int(A)) ∩ int(cl(A))) ⊆ cl(f^{-1} (A)) for each IFS A of Y

Proof: (i) ⇒ (ii) Let B be an IFCS in X. Then f (B) is an IFγGCS in Y by hypothesis. Since Y is an IFγ$_γ$ $T_{1/2}$ space, f(B) is an IFγCS in Y. Therefore (int(cl(f(B))) ∩ cl(int(f(B)))) ⊆ f(B) = f (γcl(B)).

(ii) ⇒ (iii) can be easily proved by taking complement in (ii).

(iii) ⇒ (iv) Let A ⊆ Y. Then B = f^{-1}(A) ⊆ X. Now A = f(f^{-1}(A)) = f(B). Here int(f^{-1} (A)) = int(B) is an IFOS in X. Then (iii) implies that f (γint(int(B))) ⊆ (int(cl(f(int(B))) ∪ cl(int(f(int(B))))) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))). Now $(int(cl(A^c)) \cup cl(int(A^c)))^c$ ⊆ $(int(cl(f(B)^c)) \cup cl(int(f(B)^c)))^c$ ⊆ $(f(γint(int(B^c))))^c$. Therefore (cl(int(A)) ∩ int(cl(A))) ⊆ f(γcl(cl(B)). Now f^{-1}(cl(int(A)) ∩ int(cl(A))) ⊆ f^{-1} (f (γcl(cl(B)))) ⊆ cl(B) = cl(f^{-1}(A)).

(iv) ⇒ (i) Let B be any IFCS in X, then f(B) is an IFS in Y. By hypothesis f^{-1}(cl(int(f(B))) ∩ int(cl(f(B)))) ⊆ cl(f^{-1}(f(B))) ⊆ cl(B) = B. Now (cl(int(f(B))) ∩ int(cl(f(B)))) ⊆ f (f^{-1}(cl(int(f(B))) ∩ int(cl(f(B))))) ⊆ f (B). This implies f(B) is an IFγCS and hence it is an IFγGCS in Y. Thus f is an IFγG closed mapping.

Theorem 34: Let f: X → Y be an IFγG closed mapping. Then for every IFS A of X, f(cl(A)) is an IFγGCS in Y.

Proof: Let A be any IFS in X. Then cl(A) is an IFCS in X. By hypothesis f(cl(A)) is an IFγGCS in Y.

Theorem 35: Let f: X → Y be an IFγG closed mapping where Y is an IFγ$_γ$ $T_{1/2}$ space, then f is an IF closed mapping if every IFγCS is an IFCS in Y.

Proof: Let f be an IFγG closed mapping. Then for every IFCS A in X, f(A) is an IFγGCS in Y. Since Y is an IFγ$_γ$ $T_{1/2}$ space, f(A) is an IFγCS in Y and by hypothesis f(A) is an IFCS in Y. Hence f is an IF closed mapping.

Theorem 36: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ$_c$$T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping

(ii)f(B) is an IFγGCS in Y for every IFCS B in X

(iii) int(cl(f(B))) ⊆ f(cl(B)) for every IFS B in X

Proof: (i) ⇔ (ii) is obvious.

(ii) ⇒ (iii) Let B be an IFS in X, then cl(B) is an IFCS in X. By hypothesis f(cl(B)) is an IFγGCS in Y. Since Y is an IFγ$_c$$T_{1/2}$ space, f(cl(B)) is an IFCS in Y. Therefore f(cl(B)) = cl(f(cl(B))) ⊇ int(cl(f(cl(B)))) ⊇ int(cl(f(B))).

(iii) ⇒ (i) Let A be an IFCS in X. By hypothesis, f(cl(A)) = f(A) ⊇ int(cl(f(A))). This implies f(A) is an IFSCS in Y and hence an IFγGCS in Y. Therefore f is an IFγG closed mapping.

INTUITIONISTIC FUZZY γ GENERALIZED OPEN MAPPINGS

Definition 37: A mapping f: X→ Y is said to be an intuitionistic fuzzy γ generalized open mapping (IFγG open mapping for short) if f(A) is an IFγGOS in Y for each IFOS A in X.

Theorem 38: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following are equivalent if Y is an IFγ$_γ$ $T_{1/2}$ space:

(i) f is an IFγG open mapping

(ii) f(int(A)) ⊆ γint(f(A)) for each IFS A of X

(iii) int(f^{-1} (B)) ⊆ f^{-1} (γint(B)) for every IFS B of Y

Proof: This theorem can be easily proved by taking complement in Theorem 3.15.

Theorem 39: A mapping f: (X, τ) → (Y, σ) is an IFγG open mapping if f(γint(A)) ⊆ γint(f(A)) for every A ⊆ X.

Proof: Let A be an IFOS in X. Then int(A) = A. Now f(A) = f(int(A)) ⊆ f(γint(A)) ⊆ γint(f(A)), by hypothesis. But γint(f(A)) ⊆ f(A). Therefore f(A) is an IFγOS in Y and hence is an IFγGOS in Y. Thus f is an IFγG open mapping.

Theorem 40: A mapping f: (X, τ) → (Y, σ) is an IFγG open mapping if and only if int(f^{-1} (B)) ⊆ f^{-1} (γint(B)) for every B ⊆Y, where Y is an IFγ$_γ$ $T_{1/2}$ space.

Proof: Necessity: Let B ⊆ Y. Then f^{-1} (B) ⊆ X and int(f^{-1} (B)) is an IFOS in X. By hypothesis, f(int(f^{-1} (B))) is an IFγGOS in Y. Since Y is an IFγ$_γ$ $T_{1/2}$ space, f(int(f^{-1} (B)) is an IFγOS in Y. Therefore f(int(f^{-1} (B))) = γint(f(int(f^{-1} (B)))) ⊆ γint(B). This implies int(f^{-1} (B)) ⊆ f^{-1}(f(int (f^{-1} (B)))) ⊆ f^{-1} (γint(B)).

Sufficiency: Let A be an IFOS in X. Therefore int(A) = A and f(A) ⊆ Y. By hypothesis int(f^{-1} (f(A))) ⊆ f^{-1} (γint(f(A))). Therefore A = int(A) ⊆ int(f^{-1} (f(A))) ⊆ f^{-1} (γint(f(A))). This implies f(A) ⊆ f(f^{-1} (γint(f(A)))) ⊆ γint(f(A)) ⊆ f(A). Thus f(A) is an IFγOS in Y and hence an IFγGOS in Y. Thus f is an IFγG open mapping.

Theorem 41: Let (X, τ) be an IFTS where X is an IFγ$_γ$ $T_{1/2}$ space. An IFS A is an IFγGOS in X if and only if A is an

IFγN of $p_{(\alpha,\beta)}$ for each $p_{(\alpha,\beta)} \in A$.

Proof: Necessity: Let $p_{(\alpha,\beta)} \in A$. Let A be an IFγGOS in X. Since X is an IFγ_γ $T_{1/2}$ space, A is an IFγOS in X. Then clearly A is an IFγN of $p_{(\alpha,\beta)}$ as $p_{(\alpha,\beta)} \in A \subseteq A$.

Sufficiency: Let $p_{(\alpha,\beta)} \in A$. Since A is an IFγN of $p_{(\alpha,\beta)}$, there is an IFγOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$. Now $A = \cup\{p_{(\alpha,\beta)} / p_{(\alpha,\beta)} \in A\} \subseteq \cup\{B_{p(\alpha,\beta)} / p_{(\alpha,\beta)} \in A\} \subseteq A$. This implies $A = \cup\{B_{p(\alpha,\beta)} / p_{(\alpha,\beta)} \in A\}$. Since each B is an IFγOS, A is an IFγOS and hence is an IFγGOS in X.

Theorem 42: For any IFS A in an IFTS (X, τ) where X is an IFγ_γ $T_{1/2}$ space, A ∈ IFγGO(X) if and only if for every IFP $p_{(\alpha,\beta)} \in A$, there exists an IFγGOS B in X such that $p_{(\alpha,\beta)} \in B \subseteq A$.

Proof: Necessity: If A ∈ IFγGO(X), then we can take B = A so that $p_{(\alpha,\beta)} \in B \subseteq A$ for every IFP $p_{(\alpha,\beta)} \in A$.

Sufficiency: Let A be an IFS in X and assume that there exists B ∈ IFγGO(X) such that $p_{(\alpha,\beta)} \in B \subseteq A$. Since X is an IF$\gamma_\gamma$ $T_{1/2}$ space, B is an IFγOS of X. Then $A = \cup_{p(\alpha,\beta) \in A}\{p_{(\alpha,\beta)}\} \subseteq \cup_{p(\alpha,\beta)\in A} B \subseteq A$. Therefore A is an IFγOS and hence an IFγGOS in X. Thus A ∈ IFγGO(X).

Theorem 43: Let f: (X, τ) → (Y, σ) be a mapping that satisfies f(int(B)) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))) for every IFS B in X. Then f is an IFγG open mapping.

Proof: Let B be an IFOS in X. Then int(B) = B. By hypothesis f(B) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))). This implies f(B) is an IFγOS in Y. Therefore it is an IFγGOS in Y and hence f is an IFγG open mapping.

Theorem 44: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ_γ $T_{1/2}$ space. Then f is an IFγG open mapping if and only if for any IFP $p_{(\alpha,\beta)} \in Y$ and for any IFN B of $f^{-1}(p_{(\alpha,\beta)})$, there is an IFγN A of $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$.

Proof: Necessity: Let $p_{(\alpha,\beta)} \in Y$ and B be an IFN of $f^{-1}(p_{(\alpha,\beta)})$. Then there is an IFOS C in X such that $f^{-1}(p_{(\alpha,\beta)}) \in C \subseteq B$. Since f is an IFγG open mapping, f(C) is an IFγGOS in Y. Since Y is an IFγ_γ $T_{1/2}$ space, f(C) is an IFγOS in Y and $p_{(\alpha,\beta)} \in f(f^{-1}(p_{(\alpha,\beta)})) \subseteq f(C) \subseteq f(B)$. Put A = f(C). Then A is an IFγN of $p_{(\alpha,\beta)}$ and $p_{(\alpha,\beta)} \in A \subseteq f(B)$. Thus $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq f^{-1}(f(B)) = B$.

Sufficiency: Let B ⊆ X be an IFOS. If $f(B) = 0_\sim$ then there is nothing to prove. Suppose that $p_{(\alpha,\beta)} \in f(B)$ then $f^{-1}(p_{(\alpha,\beta)}) \in B$ and B is an IFN of $f^{-1}(p_{(\alpha,\beta)})$. By hypothesis there is an IFγN A of $p_{(\alpha,\beta)}$ such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$. Therefore there is an IFγOS C in Y such that $p_{(\alpha,\beta)} \in C \subseteq A = f(f^{-1}(A)) \subseteq f(B)$. Hence $f(B) = \cup\{p_{(\alpha,\beta)} / p_{(\alpha,\beta)} \in f(B)\} \subseteq \cup\{C / p_{(\alpha,\beta)} \in f(B)\} \subseteq f(B)$. Thus $f(B) = \cup\{C / p_{(\alpha,\beta)} \in f(B)\}$. Since each C is an IFγOS, f(B) is also an IFγOS and hence is an IFγGOS in Y. Therefore f is an IFγG open mapping.

Theorem 45: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an IFγ_γ $T_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping

(ii) f(B) is an IFγGOS in Y for every IFOS B in X

(iii) f(int(B)) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))) for every IFS B in X

Proof: (i) ⟹ (ii) is obvious as $f(A^c) = (f(A))^c$ for a bijective mapping.

(ii) ⟹ (iii) Let B be an IFS in X, then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFγGOS in Y. Since Y is an IFγ_γ $T_{1/2}$ space, f(int(B)) is an IFγOS in Y. Therefore f(int(B)) = γint(f(int(B))) ⊆ f(int(B)) ∩ (int(cl(f(int(B))) ∪ cl(int(f(int(B)))) ⊆ int(cl(f(int(B))) ∪ cl(int(f(int(B)))) ⊆ (int(cl(f(B))) ∪ cl(int(f(B)))).

(iii) ⟹ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, $f(int(A^c)) = f(A^c) \subseteq (int(cl(f(A^c))) \cup cl(int(f(A^c))))$. That is (cl(int(f(A))) ∩ int(cl(f(A)))) ⊆ f(A). This implies f(A) is an IFγCS and hence an IFγGCS. Therefore f is an IFγG closed mapping.

Theorem 46: Let f: (X, τ) → (Y, σ) be a bijective mapping. Then the following conditions are equivalent if X and Y are IF$\gamma_\gamma T_{1/2}$ spaces.

(i) f is an IFγG closed mapping,

(ii) f (B) is an IFγGOS in Y for each IFOS B in X,

(iii) for each IFP $p_{(\alpha,\beta)}$ in Y and for every IFOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, there exists an IFγOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$.

Proof: (i) ⇔ (ii) is obvious as $f(A^c) = (f(A))^c$ for a bijective mapping.

(ii) ⇒ (iii) Let B be any IFOS in X and let $p_{(\alpha,\beta)} \in Y$. Given $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis f(B) is an IFγGOS in Y. As Y is an IF$\gamma_\gamma T_{1/2}$ spaces, f(B) is an IFγOS in Y. Take A = f(B). Then $p_{(\alpha,\beta)} \in f(B) = A$ and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) ⇒ (i) Let A be an IFCS in X. Then its complement, say B is an IFOS in X. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \in B$. By hypothesis there exists an IFγOS C in Y such that $p_{(\alpha,\beta)} \in C$ and $f^{-1}(C) \subseteq B$. This implies $p_{(\alpha,\beta)} \in C \subseteq f(f^{-1}(C)) \subseteq f(B)$. That is $p_{(\alpha,\beta)} \in f(B)$. Since C is an IFγOS, C = γint(C) ⊆

γint(f(B)). Therefore $p_{(\alpha,\beta)} \in$ γint(f(B)). But f(B) = $\bigcup_{p(\alpha,\beta)\in f(B)} p_{(\alpha,\beta)} \subseteq$ γint(f(B)) ⊆ f(B). Hence f(B) is an IFγOS in Y and is an IFγGOS in Y. Thus f(A) is an IFγGCS in Y and f is an IFγG closed mapping.

Theorem 47: A bijective mapping f: (X, τ) → (Y, σ) is an IFγG open mapping if (int(cl(f(A))) ∪ cl(int(f(A)))) ⊆ f (cl(A)) for every IFS A in X.

Proof: Let A be an IFOS in X then A^c is an IFCS in X. By hypothesis, (int(cl(f(A^c))) ∩ cl(int(f(A^c)))) ⊆ f(cl(A^c)) = f (A^c). Now (int(cl(f(A))) ∪ cl(int(f(A))))^c = (int(cl(f(A^c))) ∩ cl(int(f(A^c)))) ⊆ f (A^c) = (f (A))^c. This implies f (A) ⊆ (int(cl(f(A))) ∪ cl(int(f(A)))). Hence f(A) is an IFγOS and hence it is an IFγGOS. Therefore f is an IFγG open mapping.

Theorem 48: Let f: (X, τ) → (Y, σ) be a bijective mapping where Y is an $IF\gamma_cT_{1/2}$ space. Then the following are equivalent:

(i) f is an IFγG closed mapping

(ii)f(B) is an IFγGOS in Y for every IFOS B in X

(iii)f(int(B)) ⊆ cl(int(f(B))) for every IFS B in X

Proof: (i) ⇒ (ii) is obvious as f(A^c) = (f(A))^c for a bijective mapping.

(ii) ⇒ (iii) Let B be an IFS in X, then int(B) is an IFOS in X. By hypothesis f(int(B)) is an IFγGOS in Y. Since Y is an $IF\gamma_cT_{1/2}$ space, f(int(B)) is an IFOS in Y. Therefore f(int(B)) = int(f(int(B))) ⊆ cl(int(f(int(B)))) ⊆ cl(int(f(B))).

(iii) ⇒ (i) Let A be an IFCS in X. Then A^c is an IFOS in X. By hypothesis, f(int(A^c)) = f(A^c) ⊆ cl(int(f(A^c))). That is int(cl(f(A))) ⊆ f(A). This implies f(A) is an IFSCS in Y and hence an IFγGCS in Y. Therefore f is an IFγG closed mapping.

INTUITIONISTIC FUZZY M-γ GENERALIZED CLOSED MAPPINGS

In this section we introduce intuitionistic fuzzy M-γ generalized closed mappings and study some of their properties.

Definition 49: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy M-γ generalized closed mapping (IFMγG closed mapping for short) if f(V) is an IFγGCS in Y for every IFγGCS V in X.

Example 50: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨y, (0.5_u, 0.4_v), (0.5_u, 0.6_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then

IFγO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.4$, $\nu_a \geq 0.5$ whenever $\nu_b \geq 0.6$, $0.5 \leq \nu_a \leq 0.6$ whenever $0.4 < \nu_b < 0.6$ and $\mu_a \geq 0.5$, $\mu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.4$, $\mu_a \geq 0.5$ whenever $\mu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $0.4 \leq \mu_b < 0.6$ and $\nu_a \geq 0.5$, $\nu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.5$ or $\nu_v < 0.4$, $\nu_u \geq 0.5$ whenever $\nu_v \geq 0.6$, $0.5 \leq \nu_u \leq 0.6$ whenever $0.4 < \nu_v < 0.6$ and $\mu_u \geq 0.5$, $\mu_v \geq 0.4$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0, 1]$, $\mu_v \in [0, 1]$, $\nu_u \in [0, 1]$, $\nu_v \in [0, 1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.4$, $\mu_u \geq 0.5$ whenever $\mu_v \geq 0.6$, $0.5 \leq \mu_u < 0.6$ whenever $0.4 \leq \mu_v < 0.6$ and $\nu_u \geq 0.5$, $\nu_v \geq 0.4$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

Let A = ⟨x, (0.4_a, 0.4_b), (0.6_a, 0.6_b)⟩ in X. Then A is an IFγGCS in X and f(A) = ⟨y, (0.4_u, 0.4_v), (0.6_u, 0.6_v)⟩ is also an IFγGCS in Y. Therefore f is an IFMγG closed mapping.

Theorem 51: Let f: (X, τ) → (Y, σ) be an IFMγG closed mapping, then f is an IFγG closed mapping but not conversely in general.

Proof: Let f be an IFMγG closed mapping. Let V be any IFCS in X. Then V is an IFγGCS and by hypothesis f(V) is an IFγGCS in Y. Hence f is an IFγG closed mapping.

Example 52: Let X = {a, b}, Y = {u, v} and G_1 = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩, G_2 = ⟨y, (0.6_u, 0.8_v), (0.4_u, 0.2_v)⟩, G_3 = ⟨y, (0.5_u, 0.5_v), (0.4_u, 0.4_v)⟩. Then τ = {$0_\sim$, G_1, $1_\sim$} and σ = {$0_\sim$, G_2, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f: (X, τ) → (Y, σ) by f(a) = u and f(b) = v.

IFγO(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$/ either $\nu_a < 0.5$ or $\nu_b < 0.4$, $\nu_a \geq 0.5$ whenever $\nu_b \geq 0.6$, $0.5 \leq \nu_a \leq 0.6$ whenever $0.4 < \nu_b < 0.6$ and $\mu_a \geq 0.5$, $\mu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0, 1]$, $\nu_a \in [0, 1]$, $\nu_b \in [0, 1]$/ either $\mu_a < 0.5$ or $\mu_b < 0.4$, $\mu_a \geq 0.5$ whenever $\mu_b \geq 0.6$, $0.5 \leq \mu_a < 0.6$ whenever $0.4 \leq \mu_b < 0.6$ and $\nu_a \geq 0.5$, $\nu_b \geq 0.4$, $0 \leq \mu_a+\nu_a \leq 1$ and $0 \leq \mu_b+\nu_b \leq 1$}.

IFγO(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\nu_u < 0.5$ or $\nu_v < 0.5$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

IFγC(Y) = {$0_\sim$, $1_\sim$, $\mu_u \in [0,1]$, $\mu_v \in [0,1]$, $\nu_u \in [0,1]$, $\nu_v \in [0,1]$/ either $\mu_u < 0.5$ or $\mu_v < 0.5$, $0 \leq \mu_u+\nu_u \leq 1$ and $0 \leq \mu_v+\nu_v \leq 1$}.

Then f is an IFγG closed mapping but not an IFMγG closed mapping.

Since the IFS A = ⟨x, (0.5_a, 0.5_b), (0.5_a, 0.4_b)⟩ is an IFγGCS in X but f(A) = ⟨y, (0.5_u, 0.5_v), (0.5_u, 0.4_v)⟩ is not an IFγGCS in Y, since γcl(f(A)) = $1_\sim$ ⊈ G_2, G_3 but f(A) ⊆ G_2, G_3.

Theorem 53: The composition of two IFMγG closed mappings is an IFMγG closed mapping in general.

Proof: Let f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, δ) be IFMγG closed mappings. Let V be an IFγGCS in X. Then f(V) is an IFγGCS in Y by hypothesis. Since g is an IFMγG closed mapping, g(f(V)) is an IFγGCS in Z. Hence g o f is an IFMγG closed mapping.

Theorem 54: Let f: (X, τ) → (Y, σ) be an IFγG closed mapping and g: (Y, σ) → (Z, δ) is an IFMγG closed mapping then g o f : (X, τ) → (Z, δ) is an IFγG closed mapping.

Proof: Let V be an IFCS in X. Then f (V) is an IFγGCS in Y. Since g is an IFMγG closed mapping, g(f(V)) is an IFγGCS in Z. Hence g o f is an IFγG closed mapping.

Theorem 55: If f: (X, τ) → (Y, σ) is a bijective mapping, then the following are equivalent:

(i) f is an IFMγG closed mapping

(ii) f(A) is an IFγGCS in Y for every IFγGCS A in X

(iii) f(A) is an IFγGOS in Y for every IFγGOS A in X

Proof: As $f(A^c) = (f(A))^c$ is true for a bijective mapping the proof is obvious.

Theorem 56: If f: (X, τ) → (Y, σ) is a bijective mapping and Y is an IFγ_γ $T_{1/2}$ space then the following are equivalent:

(i) f is an IFMγG closed mapping

(ii) f(A) is an IFγGOS in Y for every IFγGOS A in X

(iii) for every IFP $p_{(\alpha,\beta)} \in Y$ and for every IFγGOS B in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$, there exists an IFγGOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B$

Proof: (i) ⇒ (ii) is obvious by Theorem 55.

(ii) ⇒ (iii) Let $p_{(\alpha,\beta)} \in Y$ and let B be an IFγGOS in X such that $f^{-1}(p_{(\alpha,\beta)}) \in B$. This implies $p_{(\alpha,\beta)} \in f(B)$. By hypothesis, f(B) is an IFγGOS in Y. Let A = f(B). Therefore $p_{(\alpha,\beta)} \in f(B)$ = A and $f^{-1}(A) = f^{-1}(f(B)) \subseteq B$.

(iii) ⇒ (i) Let B be an IFγGCS in X. Then B^c is an IFγGOS in X. Let $p_{(\alpha,\beta)} \in Y$ and $f^{-1}(p_{(\alpha,\beta)}) \in B^c$. This implies $p_{(\alpha,\beta)} \in f(B^c)$. By hypothesis there exists an IFγGOS A in Y such that $p_{(\alpha,\beta)} \in A$ and $f^{-1}(A) \subseteq B^c$, then $A = f(f^{-1}(A)) \subseteq f(B^c)$. Hence by Theorem 4.6, $f(B^c)$ is an IFγGOS in Y. As f is a bijective mapping, $f(B^c) = (f(B))^c$. Therefore f(B) is an IFγGCS in Y. Thus f is an IFMγG closed mapping.

Theorem 57: If f: (X, τ) → (Y, σ) is a bijective mapping, where X and Y are IFγ_γ $T_{1/2}$ spaces, then the following are equivalent:

(i) f is an IFMγG closed mapping

(ii) f(A) is an IFγGOS in Y for every IFγGOS A in X

(iii) f(γint(B)) ⊆ γint(f(B)) for every IFS B in X

(iv) γcl(f(B)) ⊆ f(γcl(B)) for every IFS B in X

Proof: (i) ⇔ (ii) is obvious.

(ii) ⇒ (iii) Let B be any IFS in X. Since γint(B) is an IFγOS, it is an IFγGOS in X. Then by hypothesis, f(γint(B)) is an IFγGOS in Y. Since Y is an IFγ_γ $T_{1/2}$ space, f(γint(B)) is an IFγOS in Y. Therefore f(γint(B)) = γint(f(γint(B))) ⊆ γint(f(B)).

(iii) ⇒ (iv) can easily proved by taking complement in (iii).

(iv) ⇒ (i) Let A be an IFγGCS in X. By hypothesis, γcl(f(A)) ⊆ f(γcl(A)). Since X is an IFγ_γ $T_{1/2}$ space, A is an IFγCS in X. Therefore, γcl(f(A)) ⊆ f(γcl(A)) = f(A) ⊆ γcl(f(A)). Hence f(A) is an IFγCS in Y and hence an IFγGCS in Y. Thus f is an IFMγG closed mapping.

Theorem 58: Let f : X → Y be a bijective mapping, where X is an IFγ_γ $T_{1/2}$ space. If f is an IFMγG closed mapping then for each IFP $p_{(\alpha,\beta)} \in Y$ and every IFγN A of $f^{-1}(p_{(\alpha,\beta)})$, there exists an IFγGOS B inY such that $p_{(\alpha,\beta)} \in B \subseteq f(A)$.

Proof: Let $p_{(\alpha,\beta)} \in Y$ and let A be the IFγN of $f^{-1}(p_{(\alpha,\beta)})$. Then there exists an IFγOS C in X such that $f^{-1}(p_{(\alpha,\beta)}) \in C \subseteq$ A. Since every IFγOS is an IFγGOS, C is an IFγGOS in X. Then by hypothesis, f(C) is an IFγGOS in Y. Now $p_{(\alpha,\beta)} \in f(C) \subseteq f(A)$. Put B = f(C). This implies $p_{(\alpha,\beta)} \in B \subseteq f(A)$.

Theorem 59: Let f : X → Y be a bijective mapping, where X is an IFγ_γ $T_{1/2}$ space. If f is an IFMγG closed mapping then for each IFP $p_{(\alpha,\beta)} \in Y$ and every IFγN A of $f^{-1}(p_{(\alpha,\beta)})$, there exists an IFγGOS B in Y such that $p_{(\alpha,\beta)} \in B$ and $f^{-1}(B) \subseteq A$.

Proof: Let $p_{(\alpha,\beta)} \in Y$ and let A be the IFγN of $f^{-1}(p_{(\alpha,\beta)})$. Then there exists an IFγGOS B in X such $p_{(\alpha,\beta)} \in B \subseteq f(A)$. Now $f^{-1}(B) \subseteq f^{-1}(f(A)) = A$. That is $f^{-1}(B) \subseteq A$.

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