Influence of Time Duration between Successive Earthquakes on the Nonlinear Response of SDOF Structure

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Abstract
The repetition of earthquake ground motion of medium and strong intensities at brief time intervals has been often observed and interested recently. In this work, the influences of successive earthquakes on the response of purely elastic and elasto-plastic SDOF structure are analyzed. An extensive parametric study for SDOF structure under repeated earthquakes has been conducted, in terms of the time duration between multiple earthquakes, the maximum amplitude of mainshock with respect to foreshock and aftershock amplitudes, inelastic displacement ratio, ductility demand, input and hysteretic energies and structural resistance function. It is observed that the successive ground motion concept has a large influence on the inelastic maximum displacement of SDOF structure. Further it is concluded that this inelastic displacement relative to elastic one and the yield value is greatly affected by the value of the structural resistance function and on the time duration between successive earthquakes.

Keyword: SDOF; successive earthquakes; nonlinear response; inelastic displacement; input and hysteretic energies.

INTRODUCTION
An assumption in the building seismic design, which assume that the earthquake is often happen as a one event. The practice situation explained that the ground motion never happen unique. Earthquake with a strong strength have more and large both aftershocks which happen before the mainshock and earthquake that happen after the mainshock which named foreshocks. The sequences of these three ground motion continue for years or even longer [1]. Unpredictable aftershocks ground motion could collapse some buildings that cracked from the mainshock earthquake. The repetition of medium-strength earthquake ground motions after any interval of time is the definition of successive earthquake ground motions. This time can be taken minutes, hours, days or years. Adding of foreshock, mainshock and aftershock data tables in multiple earthquakes around the world are available in many references [2-5]. Form these tables; it can observe that the successive ground motions are not necessarily take place within a day only. The second observation proved that the successive ground motions are sourced from different ruptured fault [2].

Always in successive ground motions, the damaged unrepaired structure after the first ground motions may become at the end of the repeated earthquakes completely inadequate [5]. In spite of the evidence that multiple earthquakes hazard is clearly threatening, the influence of successive ground motions on the structures has not attracted much attention [3]. Author is tried to review the previous attempts on the repeated earthquakes effect on buildings throughout this introduction. A little research has investigated the successive earthquakes effects on buildings. Many works investigated on the SDOF response under single event [3,4,6]. Only some of the studies concentrated on the SDOF response with multiple earthquakes ground motions with purely elastic system [7-17].

In 2003 Amadio C. et al [1] studied the influence of successive seismic ground motions on the nonlinear SDOF response. It was concluded in has work that the model of elasto-perfectly plastic is the weakest model under multiple earthquakes. While in 2009 Hatzigeorgiou G. D. and Beskos D. E. [3] investigated the SDOF response under successive seismic events in term of inelastic displacement ratio. The purpose of this research is to use a new procedure for the inelastic displacement ratio. Hatzigeorgiou G. D. and Liolios A. A. [4] in 2010 studied the nonlinear response of eight reinforced concrete planar building frames under strong successive ground motions (forty five sequential ground motion). This work conducts a details parametric study on eight reinforced concrete planar building frames under strong five ground motions. From this research, it can be shown that multiple earthquakes have a large influence on both the displacement response and on the reinforcement concrete frames design. Finally in 2013, Faisal A. et al [2] conducted a study for the ductility demand at story level of concrete frames behave inelastic manner under multiple earthquakes. From this study, it can be observed that the successive earthquakes largely increase the ductility demand at story level of inelastic concrete building.

The significantly focus of this paper is to find the influence of time duration between successive earthquakes on the purely elastic and elasto-plastic response of SDOF structure. The present study also aims to investigate the influence of the structure resistance function on the total response of SDOF
structure. Different scales of the maximum amplitude of mainshock with respect to foreshock and aftershock amplitudes have been investigated also.

**ONE-DEGREE OF FREEDOM ELASTIO-PERFECTLY PLASTIC STRUCTURE**

In linear elastic systems, the load displacement curve is drawn by straight line with constant slope $k$ and unlimited upper value. Usually in real practical situations the linear behavior become nonlinear. The nonlinear system can be solved simply using numerical analysis by defining the resistance as function of displacement only. Figure.1 shows the dynamic response of a SDOF structure in the elastic region and plastic region. The nonlinear behavior is offend used for structure that have considerable ductility [18].

Assume the SDOF structure shown in Figure.1a, the columns stiffness assumed to have the resistance function shown in Figure.1b. From this Figure, it can be seen that the resistance increases linearly with a slope of $k$ as the displacement increases from zero till to the yield displacement. Then the resistance is assumed to remain constant at $R_m$ as the displacement increases further. The $R_m$ value will be continued until the ductility limit of the structure is reached [19].

![Figure.1: Resistance function for elastic-perfectly plastic System. (a): SDOF (b) definition of the resistance function](image)

In this case, the spring force which is named the structural resistance is denoted by $R$ because this value changes throughout total behavior of inelastic system. Since the equations of motion become as follows [14, 15]:

(a) \[ M \ddot{y} + R - F(t) = 0 \]
(b) \[ M \ddot{y} + ky - F(t) = 0 \] \[ 0 < \dot{y} < y \]
(c) \[ M \ddot{y} + R_m - F(t) = 0 \] \[ y < \dot{y} < y_m \]
(d) \[ M \ddot{y} + R_m - k(y_m - y) - F(t) = 0 \]
\[ (y_m - 2y) < \dot{y} < y_m \]

(1)

Where Eq.(a) is expressed the general equation of motion, while Eq.(b) is used for elastic part. Eq.(c) is fitted for perfectly plastic part. Finally Eq.(d) covers the elastic behavior after $y_m$. The structure parameters were considered in this study are the mass of the structure is $M = 0.82$ kNsec$^2$/m, the stiffness of the structure is $k = 240$ kN/m, the yield force or the structural resistance is denoted as $R$ in which this value is illustrated in the Table 1. 0.05 viscous damping is used in this work.

**SEISMIC INPUT**

This paragraph is concerned with the procedure of assembling of multiple earthquakes records. The objective is to study the influence of successive earthquakes on structural response relative to single ground motion. A combination of the double and triple artificial successive earthquakes is used in the present study. The mainshock used in the study is based on EL CENTRO Earthquake of 40 second duration (USGS STATION 117) as shown in Figure.2. The beforshock and aftershock assembly method is based on the study of Hatzigeorgiou and Beskos [3] as shown in Figure.3. The amplitude ratio of the assembled earthquake is scaled based on the peak ground acceleration (PGA) ratio. Based on above, the assembled earthquakes will be three values of amplitude ratios. These three type are named case1 which is defined as single earthquake event (mainshock only) with a ratio of PGA amplitude equal to (1, 0, 0). While, Case 2 is defined as double earthquake events (either foreshock–mainshock or mainshock–aftershock) with a ratio of PGA amplitude equal to (1, 1, 0). In the Case3, the sequence is represented by triple earthquake events (foreshock–mainshock–aftershock) with a ratio equal to (1, 1, 1). The final case (case4) is simulated the sequence as triple earthquakes with amplitude ratio equal to (0.853, 1.000, 0.853). The time duration between two consecutive ground motions was denoted as $T$. This parameter was assumed to change as a percentage from the total earthquake duration as 50%, 75% and 100% as shown in Table1.
Figure 2: EL CENTRO Earthquake (USGS STATION 117)

(a) Single event
(b) Double event
(c) Triple event with the same scale
(d) Triple event with different scale

Figure 3: Artificial Seismic Sequences for EL CENTRO (USGS STATION 117)

<table>
<thead>
<tr>
<th>Analysis Name</th>
<th>Type</th>
<th>T(Sec)</th>
<th>Trel</th>
<th>Rv</th>
<th>y</th>
<th>ymax</th>
<th>T(sec)</th>
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<tr>
<td>Case 1 - ∞ - 0.8R</td>
<td>Case-1</td>
<td>∞</td>
<td>0.0456</td>
<td>10.9</td>
<td>8.7</td>
<td>0.036</td>
<td>0.044</td>
</tr>
<tr>
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<td>0.0456</td>
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<td>6.5</td>
<td>0.027</td>
<td>0.038</td>
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<tr>
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<td>0.0456</td>
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<td>0.051</td>
</tr>
<tr>
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<td>2.1</td>
<td>0.009</td>
<td>0.044</td>
</tr>
<tr>
<td>Case 2 - 30 - 0.8R</td>
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<td>10.8</td>
<td>6.6</td>
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<td>0.0407</td>
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<td>11</td>
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<td>0.009</td>
<td>0.064</td>
</tr>
<tr>
<td>Case 3 - 20 - 0.6R</td>
<td>Case-3</td>
<td>20</td>
<td>0.045</td>
<td>10.8</td>
<td>6.4</td>
<td>0.027</td>
<td>0.0406</td>
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<td>0.027</td>
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<td>Case 4 - 40 - 0.6R</td>
<td>Case-4</td>
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<td>0.052</td>
<td>12.4</td>
<td>7.5</td>
<td>0.031</td>
<td>0.0406</td>
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</tbody>
</table>
VERIFICATION

LINEAR SOLUTION FORMULATION

Solving the differential equation of motion represents in Eq.(1) is exact for a function of exciting represented by linear parts. The solution method requires that the loading function must be expressed at equal time periods $\Delta t$. This can be obtained by simulating the point of loading function by linear interpolation. Therefore, the total time of the loading function is divided into $N$ equal time periods of duration $\Delta t$. For each $\Delta t$, the response is found by taking the primary conditions at the beginning of that time period and the linear loading function during the interval. The displacement and velocity at the end of the preceding time interval is used as the initial conditions for the next time interval [20]. This algorithm was programmed on excel sheet to verify the result of SAP2000 in case of single earthquake and double one. Two earthquake events are subjected to the SDOF structure shown in Figure.1 the first one is a single event while the second one is a double events with 6 sec time separation. These events representation are shown in Figure.4, the linear time-displacement history for single event and double events using both SAP2000 and Manual calculation are drawn in Figure.5 and Figure.6 respectively. It shown identical behavior between two procedures in both the two type of events.

Figure 4: Input Earthquakes

(a) Single Event "EL CENTRO"
(b) Double Events "EL CENTRO" 6 sec

Figure 5: Linear time-displacement history for single event

(a) SAP 2000
(b) Manual Calculation
4.2 NONLINEAR SOLUTION FORMULATION

There are many methods to find the solution of the nonlinear equation of motion of structure subjected to time history loading. The most effective method is named step-by-step integration method. The response of any structure is found at a sequence of increments $\Delta t$ of time in this method. Equal time lengths are usually used in these methods. The condition of dynamic equilibrium is established at commence of each interval. The response at the end of time increment $\Delta t$ is found based on an assumption that the coefficients $k(y)$ and $c(y)$ remain constant during the interval $\Delta t$. These coefficients are recalculate at the start of each time increment to include the nonlinear behavior. Then the response found by using the displacement and velocity that calculated at the end of the time period which are used as the initial conditions for the next time step. So it can be defined the nonlinear behavior in this method is approximately resulted from changing linear systems. The constant and the linear acceleration methods are the two popular methods available in the literature [21]. In this study the linear acceleration method is used by using NONLIN program. This program was developed by Dr. Finley A. Charney. The results of this program was compared with the results of SAP2000 to calibrate the final one. Single earthquake named EL CENTRO (USGS STATION 117) was adopted in this nonlinear comparison as shown in Figure.7 (a) and Figure.8(a) based on SAP2000 and NONLIN respectively. Nonlinear Time displacement history by SAP2000 Figure.7(b) is identical with the time displacement history found by NONLIN.
RESULTS AND DISCUSSION

Table 1 shows the analysis cases investigate in this study. These analysis is divided into four cases. Case 1 represents single earthquake event with four different resistance. Two repeated earthquakes events with 30 sec time duration between them is represented in case 2 with also four different resistance (R). While case 3 simulates three successive earthquakes events with the same amplitude at structural resistance equal to 0.6 R_e with three different time duration between them. Finally, case 4 shows the behavior of SDOF structure under three repeated earthquakes events with different amplitudes between the mainshock with respect to before shock and after shock with same case three parameters in term of R and T. The displacement-time history with input and hysteretic energies of case 1 are shown in Figure 9 to Figure 12. From Figure 13 to Figure 17 the results of case 2 are shown. While the results of case 3 are shown in Figure 18 to Figure 22. Finally, the results of case 4 are illustrated in Figure 23 to Figure 27.

This work shows a tool which is effective for measuring the different responses of SDOF structural system under different repeated earthquake events. Detailed study are done to study the influence of successive earthquakes events in term of the maximum responses and when it is occurs with different tools for measurement of ductility of structure due to these multiple earthquakes. It is indicated from case 1 (single events) that the steady state range of amplitude reduced as the resistance decreased from 0.8 to 0.2 times the structural elastic resistance function (Figure 9 to Figure 12). It is also observed from the same Figures that the difference between input energy and hysteretic energy decreased as the structural resistance decreases further. Finally, it is shown that the maximum amplitude occurs at the early time of loading in this case. Figures of case 2 when a double earthquakes events come to the picture as shown in Figure 13 to Figure 16 with 30 second time interval between them indicate similar patterns of case 1 behavior in terms of steady state rang of amplitude and difference between input energy and hysteretic energy. The clearly difference is the time at which the maximum amplitude occurs is found to be in later case after the first event was finished. It is concluded from case 3 and case 4, as shown in Figures 17 to 19 and Figures 20 to 22 respectively, that the maximum displacement occurs at different times and with different values. When comparing Figure 10 to Figures 14, 18 and 21, it can be shown that the ductility demand for the single event changed in comparison with multiple events. In order to measure the ductility demand, the ductility measure is found by dividing the maximum displacement y_m on the yield displacement y. In addition to that the inelastic displacement ratio which is found by dividing the ratio of the maximum inelastic displacement y_m on the maximum elastic displacement y_el is used also as measured tool for comparison of single event together with other double and triple ones either with same amplitudes or different amplitudes.

Figure 23 shows the variation of inelastic displacement ratio (IDR) with the structural resistance function for case 1 and case 2 loading. It is observed that the IDR is approach 1 at resistance equal to 0.2 time the structural elastic resistance. After that the IDR is drop to about 0.66 at structural resistance equal to 0.4 time the structural elastic resistance. When the resistance goes up the IDR approach one again (Figure 23a). Similar pattern it was observed in case of double repeated earthquakes. The variation of IDR with time between successive earthquakes for case 3 and case 4 is shown in Figure 24. From these Figures it is clearly shown that the IDR reduced as the time between successive earthquakes increases. In case of triple earthquake of same amplitudes the curve is concave up while in case of triple earthquakes of different amplitudes the curve is concave down. Figure 25 shows the variation of ductility with structural resistance for SDOF subjected to case 1 and case 2 loading. It is clearly shown from these Figures that the ductility is reduced sharply when the resistance is increasing from 0.2 to 0.4 time the elastic resistance function. After that when the structural resistance increased the ductility still reducing in slightly manner. The variation of ductility with time between successive earthquakes is drawn in Figure 26 for case 3 and case 4 loading. It is clear observed from these Figures the ductility value for the equal maximum amplitudes triple repeated earthquakes case 3 is greater than the ductility value for the different maximum amplitudes triple successive earthquakes case 4 for all three different time between successive earthquakes.
Figure 9: Case -1 - ∞ - 0.8

Figure 10: Case -1 - ∞ - 0.6

Figure 11: Case -1 - ∞ - 0.4

Figure 12: Case -1 - ∞ - 0.2
Figure 13. Case -2 - 30 - 0.8

(a) Displacement-time history  
(b) Input and hysteretic energies

Figure 14. Case -2 - 30 - 0.6

(a) Displacement-time history  
(b) Input and hysteretic energies

Figure 15. Case -2 - 30 - 0.4
Figure 16. Case -2 - 30 - 0.2

Figure 17. Case -3 - 20 - 0.6

Figure 18. Case -3 - 30 - 0.6
Figure 19. Case 3 - 40 - 0.6

Figure 20. Case 4 - 20 - 0.6

Figure 21. Case 4 - 30 - 0.6
Figure 22. Case -4 - 40 - 0.6

Figure 23: Variation of IDR with Structural Resistance ($R_y$)

Figure 24: Variation of IDR with time between successive earthquakes (Tsec)
6.0 CONCLUSIONS

The present work investigates the effect of successive ground motions on the inelastic displacement SDOF structure. The effect of time duration between repeated earthquakes, structural resistance and different patterns of multiple earthquakes is studied. The major part of this paper is to find the inelastic displacement ratio of the successive ground motions in term of ductility demand and inelastic displacement ratio. This detailed study lead to followings:

1- The increasing in the steady state range of amplitudes resulted from the increase in the structural resistance function irrespectively whether SDOF structure is under single or multiple earthquakes

2- The decrease in the structural resistance of the SDOF structure always leads to a decrease in the difference between input and hysteretic energy independently either a single or triple earthquakes.

3- The time which the maximum displacement amplitude occurs at early stage of loading in a single event but it is shifted when a multiple earthquake is applied.

4- The maximum displacement of SDOF structure in term of ductility demand or in term of inelastic displacement ratio was found to be significantly affected by the successive ground motions, structural resistance function of SDOF, time between multiple earthquakes and finally on the pattern of the repeated earthquakes.

REFERENCES


Figure 25: Variation of Ductility with Structural Resistance (R_y)

Figure 26: Variation of Ductility with time between successive earthquakes (Tsec)


