Exponential intuitionistic fuzzy entropy measure based image edge detection.

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INTRODUCTION

The concept of fuzzy set proposed by Zadeh [1, 2]. In recent times, many applications of this theory can be found, such that in signal and image processing, pattern recognition, operation research, artificial intelligence, expert systems, and robotics [3-7]. In fuzzy set theory, each elements has a degree of membership known as hesitation degree. A distance measure, called Exponential Intuitionistic Fuzzy Divergence have been introduced. Edge detection is carried out using the suggested distance measure. Experimental results reveal that the proposed method exhibits better performance and may efficiently be used for the detection of edges in images.

Keywords: Image edge detection, Attanassov's intuitionistic fuzzy set, exponential entropy, fuzzy divergence.

RELATED WORK

An edge in an image is defined a contour or boundary where a abrupt change occurs in some physical aspect such as gray level value of an image [18]. Edge detection is one of the most important tasks in image processing. Especially registration, segmentation, identification and recognition are based on edge detection algorithm. Many edge detection methods are available in the literature. Classical edge detection techniques, such as Sobel, Prewitt and Canny detectors were based on the concept of spatial derivative filtering [19, 20], where local gradient operators are used to detect edges of certain orientations only. Derivative filter suffer when the edges are noisy and blurred, and are not flexible. Many fuzzy and crisp edge detection methods have been proposed by researchers [20-26]. Some of the related work in fuzzy set theory is mentioned. Alshennawy et al. [20] suggested an edge detection approach based on fuzzy logic without determining the threshold value. They segmented the images into regions using 3 x 3 binary matrix. A fuzzy system mapped a range of values distinct from each other in the matrix to detect the edge. Lopera and Ilhami [27] used Jensen-Shannon divergence of gray level histogram obtained by sliding a double window over an image for edge detection. Khan and Thakur [24] proposed fuzzy logic based edge detection algorithm for gray scale images. Tao and Thomson [26] used gradient approximations as input variables. They used two fuzzy sets, small and large, as linguistic variables and 16 fuzzy rules from 16 contour structures were obtained. Ho and Ohnishi [22] used fuzzy edge detector for edge detection. They used several fuzzy templates and then edge detect an image using a similarity measure. In all fuzzy edge detection methods, the pixel degree of membership is equal to 1 minus the non-membership degree. However, it does not represent a
neutral of evidence which resulted in the real world there are many information cannot be represented and processed using fuzzy set theory. Intuitionistic fuzzy set has been proven to be highly useful to deal with uncertainty and vagueness. Recently, intuitionistic fuzzy set theory was used to improve accuracy in detection [6, 23, 28-30]. Chaiera [28] introduced intuitionistic fuzzy entropy in objective function of conventional clustering algorithm and applied to medical images. Hu and Li [23] introduced a novel method to detect hardwood leaves edges, which clusters, thresholds, and then detects edges of hardwood seedlings leaves using intuitionistic fuzzy set theory.

PLIMINARIES

In this section we present some basic concepts related to exponential entropy, fuzzy set theory and Atanassov’s intuitionistic fuzzy set, also we introduce some of the related distance measures.

Exponential Entropy.

Entropy is an uncertainty measure first introduced by Shannon [15, 16] into information theory to describe how much information is contained in a source governed by a probability law.

Let \( P = \{p_1, p_2, \ldots, p_n\} \) be the probability distribution of a discrete source. Therefore, \( 0 \leq p_i \leq 1 \), \( i = 1, 2, \ldots, n \) and \( \sum_{i=1}^{n} p_i = 1 \), where \( n \) is the total number of states and \( P \) is called the entropy of the distribution. The entropy of a discrete source is often obtained from the probability distribution. The Shannon Entropy can be defined as:

\[
H(P) = -\sum_{i=1}^{n} p_i \ln(p_i) \tag{1}
\]

This formalism has been shown to be restricted to the domain of validity of the Boltzmann–Gibbs–Shannon (BGS) statistics. Generally, systems that obey BGS statistics are called extensive systems. If we deem that a physical system can be decomposed into two statistical independent subsystems \( O \) and \( B \), the probability of the composite system is \( P^{O+B} = P^O \cdot P^B \), it has been verified that the Shannon entropy has the extensive property (additive) \( H(O + B) = H(O) + H(B) \). It is to be noted from the logarithmic entropic measure (1) that as \( p_i \rightarrow 0 \), its corresponding self information of this event, \( I(p_i) = -\ln(p_i) \rightarrow \infty \) but \( I(p_i=1) = -\ln(1) = 0 \) and \( I(p_i=0) = -\ln(0) \) is undefined.

Pal and Pal [17] proposed another measure called exponential entropy given by

\[
E(P) = \sum_{i=1}^{n} p_i e^{(1-p_i)} - 1 \tag{2}
\]

Exponential entropy (2) [31] has some advantages over Shannon’s entropy, which is widely acclaimed, we find that the measure of self information of an event with probability \( p_i \) is taken as \( \ln\left(\frac{1}{p_i}\right) = -\ln(p_i) \), a decreasing function of \( p_i \). The same decreasing character alternatively may be maintained by considering it as a function of \( 1 - p_i \) rather than of \( (1/p_i) \). Also for example, for the uniform probability distribution \( P = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}) \) exponential entropy has a fixed upper bound

\[
limit_{n \to \infty} H\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right) = e - 1
\]

which is not the case for Shannon’s entropy.

Basic Concept of Atanassov’s Intuitionistic fuzzy sets.

**Definition 1:**[2] A fuzzy set \( \tilde{A} \) in a finite set \( X = \{x_1, x_2, \ldots, x_n\} \) may be given mathematically as \( \tilde{A} = \{(x, \mu_A(x))|x \in X\} \) where the function \( \mu_A(x): X \rightarrow [0,1] \) is the membership function of an element \( x \) in the finite set \( X \), \( \mu_A(x) \in [0,1] \) is the degree of membership of \( x \) in \( X \). Exponential entropy for fuzzy set \( \tilde{A} \) corresponding to (2) has been introduced by Pal and Pal as:

\[
E(\tilde{A}) = \frac{1}{n(\sqrt{e} - 1)} \sum_{i=1}^{n} [\mu(\mu_A(x_i)e^{1-\mu(\mu_A(x_i))}(1-\mu_A(x_i))e^{\mu(\mu_A(x_i))} - 1] \tag{3}
\]

Further, Atanassov [8-11] suggested a generalization of fuzzy sets, called Intuitionistic Fuzzy Sets (IFS).

**Definition 2:** [11] An intuitionistic fuzzy set \( A \) in a finite set \( X = \{x_1, x_2, \ldots, x_n\} \) may be mathematically represented as \( A = \{(x, \mu_A(x), \gamma_A(x))|x \in X\} \) where the functions \( \mu_A(x), \gamma_A(x): X \rightarrow [0,1] \) are, respectively, the membership degree and the non-membership degree of an element \( x \) in a finite set \( X \) with the condition

\[
0 \leq \mu_A(x) + \gamma_A(x) \leq 1
\]

Obviously, each fuzzy set \( A \) is a particular case of intuitionistic fuzzy set \( A \), and may be represented as

\[
A = \{(x, \mu_A(x), 1 - \mu_A(x))|x \in X\}
\]

In [12, 13] Szmidt and Kacprzyk stressed the necessity of taking into consideration a parameter \( \pi(x) \), known as the intuitionistic index of \( x \) in \( A \) or degree of hesitation, which appears due to the lack of knowledge during calculate the distances between two fuzzy sets.

**Definition 3:** [9, 32] For each intuitionistic fuzzy set \( A \) in a finite set \( X \), which may be represented as \( A = \{(x, \mu_A(x), \gamma_A(x))|x \in X\} \),

if

\[
\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x), \tag{4}
\]

then \( \pi_A(x) \) is called the Atanassov’s intuitionistic index (or hesitation degree) of an element \( x \) in a finite set \( X \). In real world, the sum of membership and non-membership values may not be equal to 1. This is due to the occurrence of uncertainty in defining the membership function. This uncertainty is named as hesitation degree. Therefore the summation of membership, non-membership, and hesitation degree is equal to 1. It is obvious that

\[
0 \leq \pi_A(x) \leq 1, \text{ for each } x \in X
\]

In many theoretical and practical problems, in order to determine the difference between two objects, it is necessary to know the distance between two fuzzy sets. Szmidt and Kacprzyk [12, 13] presented some distance measures between two Intuitionistic Fuzzy Sets (IFS) \( A \) and \( B \) that take into account the degree of membership \( \mu \), the degree of non-
membership \( \gamma \), and the hesitation degree \( \pi \) in \( X = \{x_1, x_2, \ldots, x_n\} \).

**EXPERIMENTAL INTUITIONISTIC FUZZY ENTROPY DIVERGENCE**

Assume two intuitionistic fuzzy sets \( A \) and \( B \) are expressed as \( A = \{(x, \mu_A(x), \gamma_A(x), \pi_A(x)) | x \in X\} \) and \( B = \{(x, \mu_B(x), \gamma_B(x), \pi_B(x)) | x \in X\} \), respectively. Let the range of the membership degree of the two intuitionistic fuzzy sets \( A \) and \( B \) may be, respectively, introduced as \( \{\mu_A(x), (1-\pi_A(x))\} \) and \( \{\mu_B(x), (1-\pi_B(x))\} \) where \( \mu_A(x), \mu_B(x) \) are the degrees of membership and \( \pi_A(x), \pi_B(x) \) are the degrees of hesitation in the particular sets, with \( \pi_A(x) = 1-\mu_A(x) - \gamma_A(x) \), and \( \pi_B(x) = 1-\mu_B(x) - \gamma_B(x) \). This range due to the lack of knowledge in determining the membership values. The proposed distance measure takes into account the hesitation degrees:

\[
\Delta = (1-\pi_A(x))|_{x \in X} \quad \text{and} \quad \Delta = (1-\pi_B(x))|_{x \in X}
\]


due to the lack of knowledge in determining the membership values.

Fan and Xie [33] introduced fuzzy divergence from fuzzy exponential entropy using a row-vector. In this paper the divergence concept of Fan and Xie is extended to an image, exponential entropy using a row-vector. In this paper the proposed distance measure takes into account the lack of knowledge in determining the membership values.

According to (3), the exponential fuzzy entropy of an image \( A \) of size \( M \times M \) is defined as

\[
H(A) = \frac{1}{n} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \mu_A(a_{ij}) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} + (1 - \mu_A(a_{ij})) e^{\mu_B(b_{ij}) - 1} \]  

\[
(5)
\]

where \( n = M^2 \) and \( \mu_A(a_{ij}) \) is the membership values of the pixels in the image and \( a_{ij} \) is the \((i,j)\)th pixel of the image. According to (3), the exponential fuzzy entropy of an image \( A \) of size \( M \times M \) is defined as

\[
H(A) = \frac{1}{n} \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \mu_A(a_{ij}) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} + (1 - \mu_A(a_{ij})) e^{\mu_B(b_{ij}) - 1} \]  

\[
(5)
\]

where \( n = M^2 \) and \( \mu_A(a_{ij}) \) is the membership values of the pixels in the image and \( a_{ij} \) is the \((i,j)\)th pixel of the image. In order to determine the difference between two images \( A \) and \( B \) (i.e., at pixels \( a_{ij} \) and \( b_{ij} \)), it is necessary to know the amount of information between the membership degrees of images \( A \) and \( B \) which is given by:

\[
D_{\pi}(A, B) = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \left[ 1 - (1 - \mu_A(a_{ij})) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} - \mu_A(a_{ij}) e^{\mu_B(b_{ij}) - \mu_B(b_{ij})} \right]
\]

Similarly, the exponential fuzzy entropy divergence of \( B \) against \( A \) is:

\[
D_{\pi}(B, A) = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \left[ 1 - (1 - \mu_B(b_{ij})) e^{\mu_B(b_{ij}) - \mu_A(a_{ij})} - \mu_B(b_{ij}) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} \right]
\]

So, the total divergence between the pixels \( a_{ij} \) and \( b_{ij} \) of the images \( A \) and \( B \) is

\[
D_T = D_{\pi}(A, B) + D_{\pi}(B, A)
\]

\[
= \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \left[ 1 - (1 - \mu_A(a_{ij})) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} - (1 - \mu_B(b_{ij})) e^{\mu_B(b_{ij}) - \mu_A(a_{ij})} \right]
\]

\[
(6)
\]

According to (3), the exponential fuzzy entropy divergence between the pixels \( a_{ij} \) and \( b_{ij} \) of the images \( A \) and \( B \) is:

\[
D_{\pi} = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} \left[ (1 - \mu_A(a_{ij})) e^{\mu_B(b_{ij}) - \mu_B(b_{ij})} - (1 - \mu_B(b_{ij})) e^{\mu_A(a_{ij}) - \mu_A(a_{ij})} \right]
\]

\[
(7)
\]

Consequently, the overall Intuitionistic Exponential Fuzzy Divergence (IEFD), between the images \( A \) and \( B \) by adding equations (6) and (7), is given by:

\[
IEFD(A, B) = \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} 4 - (1 - \mu_A(a_{ij})) e^{\mu_B(b_{ij}) - \mu_A(a_{ij})} - (1 - \mu_B(b_{ij})) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})} - (1 - \mu_A(a_{ij})) e^{\mu_B(b_{ij}) - \mu_A(a_{ij})} - (1 - \mu_B(b_{ij})) e^{\mu_A(a_{ij}) - \mu_B(b_{ij})}
\]

\[
(8)
\]

**PROPOSED METHODOLOGY**

The process of spatial filtering consists simply of moving a filter mask \( \omega \) of order \( m \times n \) from point to point in an image. At each point \((i,j)\), the response of the filter at that point is calculated a predefined relationship. We will use the usual masks for detection the edges. For this purpose, smallest meaningful size of 8 fuzzy mask templates each of size \( 3 \times 3 \), as shown in Figure 1. Where the \( a \) and \( b \) are template parameters, \( a, b \in [0,1] \), which are changed with the different trained parameters.

\[
\begin{array}{cccc}
a & a & a & a \\
a & 0 & 0 & b \\
0 & 0 & 0 & b \\
\end{array}
\]

\[
\begin{array}{cccc}
a & b & b & b \\
a & 0 & 0 & b \\
0 & 0 & 0 & b \\
\end{array}
\]

\[
\begin{array}{cccc}
a & a & a & a \\
a & 0 & 0 & b \\
0 & 0 & 0 & b \\
\end{array}
\]

\[
\begin{array}{cccc}
0 & 0 & 0 & b \\
0 & 0 & 0 & b \\
0 & 0 & 0 & b \\
\end{array}
\]

\[
\begin{array}{cccc}
b & b & b & b \\
b & 0 & 0 & b \\
0 & 0 & 0 & b \\
\end{array}
\]

**Figure 1: Set of eight 3 \times 3 template**

Move each fuzzy template on the whole binary image and find The IEFD measure at each central pixel position \((i,j)\) of the image under the fuzzy template. IEFD\((i,j)\), is calculated between the image window (same size as the template) and the template, as given below:

\[
IEFD(i,j) = \max_N \left\{ \min_r \left( \text{IEFD}(A, B) \right) \right\}
\]

\[
(9)
\]

where \( N \) = number of templates, \( r \) = number of elements in the square template. It is noted that, IEFD\((i,j)\) is calculated for all pixel positions of the image. IEFD\((A, B)\), is calculated by determining the IEFD between each of the elements \( a_{ij} \) and \( b_{ij} \) of image window A and of template B using Equation (8). The IEFD matrix, which has the same size as that of image, is created with values of IEFD\((i,j)\) at each point of the matrix. This IEFD matrix is thresholded and thinned to get an edge.
detected image. The selection of threshold value is manually curbed for getting the final edge-detected result. In our experiment, for the two images A and B, the membership degree, non-membership degree and the intuitionistic fuzzy index (hesitation degree) of images A and B are, respectively, given by:

The membership degrees $\mu_A(a_{ij})$ are the normalized values of the $(i,j)$th pixel of image A, where the image A represents the chosen window in the test image. The membership degrees of the template pixels, $\mu_B(b_{ij})$ are the values of the template, where the image B is the fuzzy template. Due to the intuitionistic characteristics, the membership degrees of images A and B may take the values in the range $[\mu_A(x),(1-\pi_A(x))]$ and $[\mu_B(x),(1-\pi_B(x))]$, respectively.

From (4) we know that $\gamma_A(a_{ij}) = 1 - \mu_A(a_{ij}) - \pi_A(a_{ij})$.

So, the non-membership degree = $1 - $ membership degree - hesitation degree.

For calculating the hesitation degree or intuitionistic fuzzy index, we have assumed

$$h_c = h_c * (1 - $ membership $)$$

where $0 < h_c \leq 1$ is a hesitation constant. The value of $h_c$ should be such that the (4) holds.

The following Algorithm summarize the proposed technique for carrying out the edge detector.

**Algorithm : Edge Detection**

1. **Input:** A digital grayscale image A of size $M \times N$.
2. Create 8 fuzzy templates with values a and b.
3. Apply the fuzzy templates over the digital image by placing the center of each template at each point $(i, j)$ over the digital grayscale image.
4. Apply equation (8) to calculate the IEFD between each element of each template and the image window (same size as that of template) and choose the minimum IEFD value.
5. For all the 8 minimum IEFD values, that determined in step 4. Choose the maximum value between them by using equation (9).
6. Put the maximum value at the point where the template was centered over the image.
7. For all the pixel positions the max–min value has been determined and positioned.
8. A new intuitionistic exponential divergence matrix (IED-Matrix) has been formed.
9. Create a binary image and thin: For all i, j

   if $\text{IED-Matrix}(i, j) \leq \text{th}$ then $f(i, j) = 0$ else $f(i, j) = 255$,

   where th is the threshold value.

10. **Output:** The edge detection image $f$ of A.

**RESULTS AND DISCUSSION**

To demonstrate the efficiency of our proposed algorithm, the approach is tested over a number of different grayscale images and compared with classical operators. Results with different combination of $a$ and $b$ are shown for each images, but the edge-detected results are not at all good as shown in the Figures 2-6. The best combination is $a = 0.4$, $b = 0.6$ and with these values, the edge-detected results are found better. Also, it is observed that, on decreasing the number of templates, many edges can be missing and when increasing the number of templates, there is no change in the edge detection results. Results with different values of hesitation constant (hesitation degree) $h_c$ have been shown in Figures 2-6. To demonstrate the efficiency of our proposed algorithm, the proposed approach is tested over a number of different grayscale images and compared with Canny’s method [19], Laplacian’s method [34] and wavelet method [35] as shown in Figures 6 and 7. In each of the images in Figures 2-7, th and $h_c$ denote, the threshold value and the edge template hesitation constant, respectively.

a) "Blood" image. CT scanned Blood image of size 272×234 containing the gray level is shown in Figure 2a. The IEFD matrix has been thresholded and then thinned. The results with different values of $h_c$, are shown. The results with $h_c = 0.3$ and $h_c = 0.4$ have been found better, as shown in Figures 2c and 2e. The edges are clearly extracted using our proposed approach, with proper $h_c$, implies that the edge detection is completely dependent on the choice of $h_c$. Figure 2f, where the edges are not properly detected, is the result with $a = 0.9$, $b = 0.8$.

b) "Cameraman" image. Cameraman image of size 205×205 is shown in Figure 3a. The result of edge detection with varying values of $h_c$ are shown. Better results are obtained in Figures 3b and 3c.

![Figure 2: The results of the edge detection for blood image using IEFD](image)

![Figure 3: The results of the edge detection for Cameraman image using IEFD](image)
c) "Brain" image of size 256×256 is shown in Figure 4a. Edge-detected results with varying values of $h_c$ are shown. Figures 4b and 4c give a better result. Figure 4f, where the edges are not properly detected, is the result with different values of $a = 0.9$, $b = 0.8$.

d) "MRI" image is of size 256×210 is shown in Figure 5a. The edge detection results with different values of $h_c$ are shown in Figures 5b-5f. But the result in Figure 5b is better. The results with $a = 0.4$, $b = 0.65$ is shown in Figure 5f, where the edges are not properly detected.

For comparison with other methods, two results are shown.

e) "Lena" image is of size 257×257 is shown in Figure 6a. The results of our approach have been compared with a Canny's method, Laplacian's method and wavelet method. The edge detection results with different values of hesitation constants, $h_c$, are shown in Figures 6b and 6c. In Figure 6b, with $h_c = 0.5$, better result is obtained. Figure 6d is the result using Wavelet method. Figure 6e is the result using Canny's method. Figure 6f is the result using Laplacian's method.

f) "Rice" image is of size 236×213 is shown in Figure 7a. Figure 7b-7d are the result using proposed approach with $h_c = 0.4$. Figure 7e is the result using Canny's method. Figure 7f is the result using Laplacian's method.

CONCLUSION

In this paper, the unconventionality lies in using Atanassov's intuitionistic fuzzy set theory in image edge detection. A new distance measure, called exponential intuitionistic fuzzy divergence has been introduced. Edge detection was carried out by using the suggested distance measure. The result of edge detection is completely dependent on the selection of hesitation constant and thus be the hesitation degree. The dominant edges is clearly detected by using the proposed method, whereas removing the unwanted edges.

REFERENCES


