Fuzzy Formal Concept Analysis for Identification of Weak and Strong Conjunction Relationships in Imprecision and Uncertainty Data

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Abstract
Formal concept analysis is a statistical based computational replica used for knowledge processing, reasoning and retrieval based operational innovations like knowledge management of data mining, robotics, big data, IoT and so on. FCA is used to accumulate the data and build the structure with the help of objects, concern attributes, co-related elements along with their efficient appropriate internal relationships based on the concern problems. It is best proficient environment implicated for knowledge processing data analysis purpose. Some of the technologies such as robotics, data mining, and big data are facing some difficulties in handling the vagueness and uncertainties in data processing stages. FCA is intermingling and originated with the superior advancements of fuzzy logic scenery, granularity based calculations, theory of probability along with fuzzy graphs, fuzzy set interval assessments and so on. The FCA is mainly hybridized with fuzzy logic based decision making futures implicated for managing the imprecision and uncertainty data. This paper describes the in-depth of description and computations of fuzzy implicated FCA functionalities for the identification of Weak and strong conjunction relationship in vagueness of data which is helpful to reduce the complex problems in handling the uncertainty data and it described the mixed inventions of fuzzy-FCA-fuzzy implications.

Keywords: Classical based Logic (CbL), Formal Concept Analysis (FCA), Fuzzy-FCA-Fuzzy (FFF), Fuzzy Logic (FL), Weak-Conjunction-Relationship (WCR), Strong-Conjunction-Relationship (SCR), Set of Objects (SO), Concept Lattice (CL), Set of Attributes (SA), Fuzzy based Formal Context (FbFC).

INTRODUCTION
The Formal concept analysis (FCA) is a statistical based computational replica designed based on lattice implicated theory described by Wille. In the present scenario, it is used for knowledge processing, reasoning and retrieval based operational innovations like knowledge management of data mining, robotics, big-data, IoT and so on. FCA is formed with occurrence matrix with rows by representing with objective of objects and columns by representing with attributes of concern objects and the central coordinating point of matrix is representing the relationship between the objects of objectives and attributes of objects as shown in the tables 2 and 3. Here the concept based lattices are considered as output responses which are deriving the generality and specialization between the formal concepts (FC) from the occurrence matrix, which is described by the Davey and Priestley. The Ganter and Wille has described that the FC are the fundamental component of thinking by holding the most important role in knowledge processing, reasoning and retrieval based operational by deriving the various levels of set of objects (SO) which is representing the distinct extents and set of attributes (SA) which is representing the intents by holding the concern common type of values [1]. In recent days FCA based applications are growing and involving the major role for solving the real time problems in knowledge executable systems, intangible data analysis and retrieval process [7]. The majority of the researches are focused on FCA along with concept lattice (CL) implicated construction [10][11], fuzzy based formal context (FbFC), CL based pruning [12][13], FCA rule acquisition methods [14], description of CL and rough set relationships [15][16] and FCA based applications [17][18]. Here the various levels of fuzzy extensions of FCA have been described along with their implications. In the advancement of FCA extend with the various different mathematical computations, algorithmic sequences and outcome responses are functioned for solving the various complex problems.

PAPER OBJECTIVES
FCA is intermingling and originated with the superior advancements of fuzzy logic scenery, granularity based calculations, theory of probability along with fuzzy graphs, fuzzy set interval assessments and so on. Majorly the FCA is hybridized with fuzzy logic based decision making futures implicated for managing the imprecision and uncertainty data. This paper proposes two major objectives such as: In the first objective described fuzzy extensions of FCA and mixed inventions of fuzzy-FCA-fuzzy implications along with their computations and in the second objective describing the in-
depth of description and computations of fuzzy implicated FCA functionalities for identification of Weak and strong conjunction relationship in vagueness data which is helpful to reduce the complex problems in handling the uncertainty data.

**METHODOLOGY EXECUTION SEQUENCES**

**Fuzzy-FCA for Identification of Weak and Strong Conjunction Relationships:**

In generalized fuzzy logic sequences, sometimes the computational scenarios are not flexible for executing the operations, but in FLC implicated FCA is deriving the future of fuzzy based SA [4]. The crisp based data implementation way is differ in FLC based Description Logic (DL) and FLC implicated FCA [2]. The generalized DL based concept descriptions are similarly implementing the FLC based DL, here the only semantics are fuzzy based but they are not pure fuzzy logic. The Gödel represented t-norm is used accurately in the section of FLC based FCA where the semantics will partly covers by the fuzzy based DL which is most probably consent to synergies as in the crisp case of scenarios. In Gödel based t-norm representations, sometimes it is holding the essential limit when the FLC based FCA implementation uses the weak-conjunction-relationship (WCR) for the semantics of SA while DL utilized the strong-conjunction-relationship (SCR) for its concern semantics. The SC and WC correspond solitary for the Gödel based t-norm. There is another limit in DL point of view which is the Gödel based t-norm is the solitary t-norm for which the standard DL based reasoning operations are acknowledged to be decisional scenario [3].

A set of degrees of truth-functions and truth-values are implicated with their concerns flexible operational objects and their concern attributes. Here the structure of truth degrees are formed with help of residuated lattices which is implicated with L. Such as L is implicated with set of (L, ∧, ∨, ⊙, →, 0, 1) which is a complete lattice with 0 and 1 being the least and greatest element of L which is complete if and only if the complete lattice implication (CLI) ⇐ (L, ∧, ∨, 0, 1) and the commutative monoid Implication (CMI) ⇐ (L, ⊙, 1).

The adjointness property is defined as p ⊙ q ≤ r if and only if r ⇐ p≤ q, V p, q, r ∈ L which is distinctly formed, defined and executed. The truth degrees values positions are consists the elements of ‘p’ of L, L ⊆ [0, 1] where the “⊙” denotes the truth function of fuzzy conjunction and ‘→’ denotes the truth function of fuzzy implication and L^U represents the collection of fuzzy sets in a universe set of ‘U’. Here the mapping is possible with implication of P of U to L along with a-cut functionality.

\[
a\text{-cut of } P \triangleq \{ u \in U \mid P(u) \geq p \} \quad \text{for } P \in L^U \text{ and } p \in L
\]

**FCA-Granularity based Calculations:**

Computed \(\rightarrow\) Weights (w) of concern concepts Attributes (Y) belongs to 0 ≤ w ≤ 1
Priority \(\rightarrow\) concepts whose weight is more than the chosen threshold θ (0 ≤ θ ≤ 1)
Threshold θ \(\rightarrow\) (0 ≤ θ ≤ 1)

Let us analyse any object \(x_i \in X\) of a given context and compute its probability \(P(y_j/x_i)\) for possessing the corresponding attribute \(y_i\). Then the average information weight \(E(y_i)\), of \(x_i\) to provide the attribute \(y_i \in Y\) can be computed as follows:

\[
E(y_i) = \frac{-\sum_{i=1}^{n} P(y_j/x_i) \log_2(P(y_j/x_i))}{\sum_{i=1}^{n} Y(y_i)}
\]

where \(m\) represents the total number of attributes

\[
\text{Weight (B)} = \frac{\sum_{i=1}^{n} W(y_i)}{m}
\]

Here B is treated as intent of object.
Fuzzy-FCA for Identification Implications of imprecision and uncertainty data

The FCA is hybridized with fuzzy logic based decision making futures implicated for managing the imprecision and uncertainty data.

FCA with fuzzy logic scenery:

FCA has been extended with a fuzzy setting for handling vagueness and uncertainty in data using the following implication variations are shown in fig. 1 used for identification of imprecision and uncertainty data as implicated in tables 4-6.

Fuzzy-FCA for Identification of Weak and strong relationships in imprecision and uncertainty data

Fuzzy-FCA is representing the fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from table 4 and fig. 3.
Relationship 1: Partial relationships (˜ R1) representation of fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from table 2 and fig.2 and fig.3.

Figure 2. Partial relationships (˜ R1) representation of fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from table 2.

Table 2: Partial relationships (˜ R1) representation of fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values

<table>
<thead>
<tr>
<th>Object / Attribute of concern object</th>
<th>Converted short form</th>
<th>Domestic</th>
<th>Wild</th>
<th>Forest</th>
<th>Village</th>
<th>Grass Eater</th>
<th>Meat Eater</th>
<th>Water</th>
<th>Insect Eater</th>
<th>Land</th>
<th>Desert</th>
<th>Sea</th>
<th>Trees</th>
<th>Den</th>
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<tbody>
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<td>X1</td>
<td>a</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>e</td>
<td>f</td>
<td>g</td>
<td>h</td>
<td>i</td>
<td>j</td>
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<td>l</td>
<td>m</td>
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<td>0.8</td>
<td>0.7</td>
<td>0</td>
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<td>0.4</td>
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<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
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<td>0.4</td>
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<td></td>
</tr>
<tr>
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<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
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</tr>
<tr>
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<td>0.8</td>
<td>0.7</td>
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<td>Crocodile (X8)</td>
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<td>0.8</td>
<td>0.7</td>
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<td>0.9</td>
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<td>0.9</td>
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<td>0.7</td>
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</tr>
</tbody>
</table>

Figure 3. It is representing the fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from figure 2.
Relationship 2: The Complete relationship is representing with CR˜ by combining the Weak-conjunction-relationship (WCR˜) and strong-conjunction-relationship (SCR˜) representation of fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from table 3 and fig.3.

\[
\text{WCR}\leftarrow \text{Weak-Conjunction-Relationship}
\]
\[
\text{SCR}\leftarrow \text{Strong-Conjunction-Relationship}
\]
\[
\text{CR}\leftarrow \text{Complete-Relationship}
\]

CR˜ \leftarrow WCR∗ * SCR∗

Table 3: Complete relationships (CR˜ \leftarrow WCR∗ * SCR∗) along with the Weak-conjunction-relationship (WCR∗) and strong-conjunction-relationship (SCR∗) representation of fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from table 2 and fig.4.

<table>
<thead>
<tr>
<th>Object/Attribute of concern object</th>
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<th>Wild</th>
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<th>Land</th>
<th>Desert</th>
<th>Sea</th>
<th>Trees</th>
<th>Den</th>
</tr>
</thead>
<tbody>
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<td>0.9</td>
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<td>0.8</td>
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<td>0</td>
<td>0</td>
<td>0.4</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Deer (X2)</td>
<td>X2</td>
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<td>0.9</td>
<td>0</td>
<td>0.8</td>
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</tr>
<tr>
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<td>X3</td>
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<td>0</td>
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<td>0.8</td>
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<tr>
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<td>0.8</td>
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<tr>
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<td>0.8</td>
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<tr>
<td>Crocodile (X8)</td>
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</tr>
</tbody>
</table>

Figure 4. Complete relationships (CR˜ \leftarrow WCR∗ * SCR∗) along with the WCR∗ and SCR∗ representation of fuzzy implicated attributes with concern logical relationships of fuzzy attributes of objects of objectives by considering the values from table 5.
**Complete Relationship Chart**

Let \( k = (G, M, I) \) is a formal context and \( X \subseteq G \). Here the consequences are represented as lower and upper approximation class of values. The casing based lower approximation indicated with \( \underline{R}(X) \) and upper approximation indicated with \( \overline{R}(X) \) of \( X \) is expressed as:

\[
\text{Lower Approximation } \underline{R}(X) = \{ x \in G : N(x) \subseteq X \} = \{ x \in G : \{ x \} \downarrow \subseteq X \}
\]

\[
\text{Upper Approximation } \overline{R}(X) = \{ x \in G : N(x) \cap X \neq \emptyset \} \sqsubseteq \{ x \in G : \{ x \} \downarrow \cap X \neq \emptyset \}
\]

\[
\text{WCR} \leftarrow \underline{R}(X)
\]

\[
\text{SCR} \leftarrow \overline{R}(X)
\]

**Figure 5.** Complete relationships \((CR \leftarrow WR \ast SR)\)

The accuracy computation degree \( ACD_c(X) \) of \( X \) which is expressed as

\[
ACD_c(X) = \frac{|\underline{R}(X)|}{|\overline{R}(X)|} = \left| \frac{\text{WCR}}{\text{SCR}} \right|
\]

The concern properties of the operators of \( \underline{R}(X) \) and \( \overline{R}(X) \) counting for the execution based on the considered sample data set which is consisting the uncertainty data of rough set.
Table 4: Complete relationship based fuzzy implicated FCA for deriving the Weak-conjunction-relationship (WCR˜) and strong-conjunction-relationship (SCR˜) based on StD1, StD2, Fuzzy-FCA-SCR˜ and Fuzzy-FCA-WCR˜

<table>
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<tr>
<th>x</th>
<th>a</th>
<th>b</th>
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<th>d</th>
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<th>f</th>
<th>g</th>
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<th>m</th>
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<td>0</td>
<td>0</td>
<td>0.3</td>
<td>0.55</td>
</tr>
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<td>0.1</td>
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<td>(X, b), (X, d), (X, f), (X, i), (X, m)</td>
</tr>
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</table>

CONCLUSION

Formal concept analysis is a statistical based computational replica and best proficient environment used for knowledge processing, data analysis, reasoning and retrieval based operational innovations like knowledge management of data mining, robotics, big data, IOT and so on. Majorly the FCA is hybridized with fuzzy logic based decision making futures implicated for managing the imprecision and uncertainty data. This paper describes the in-depth festivities of fuzzy implicated FCA functionalities for the identification of Weak and strong conjunction relationship in vagueness data which is helpful to reduce the complex problems in handling the uncertainty data and it described the mixed inventions of fuzzy-FCA-fuzzy implications.

REFERENCES


