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# Optimized Multi Scale Image Fusion Technique Using Discrete Wavelet Transform and Particle Swarm Optimization for Colour Multi Focus Images

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**Abstract**

Wavelet transforms have emerged as a powerful tool in image fusion. However, the study and analysis of multi-scale image fusion is still challenging area of research. Therefore, in this paper, we propose an optimized multi-scale image fusion in discrete wavelet transform domain (DWT). The choice of fusion rule plays an important role in image fusion that determines which coefficient provides better information. In our proposed work, the source images (color multi focus images) are separated into Red, Green and Blue components respectively. The DWT is applied on each color band of the source images to get high and low frequency sub-bands and the high frequency components are fused using maximum selection rule. Low frequency sub-bands of each color band are fused using weighted average fusion strategy. In this work, particle swarm optimization (PSO) is aided to find the optimal weights for the fusion of wavelet coefficients of each color band that makes our work different from existing DWT based image fusion techniques. The inverse DWT is applied on the fused color bands to get the resultant image. Fusion results have been evaluated subjectively and objectively with gradient pyramid (GRAD), discrete wavelet transform (DWT) and DWT+Type-2 Fuzzy logic. The comparative analysis of fusion results with fusion quality metrics such as mutual information (MI), spatial Frequency (SF) and Petrovic metric ($Q^{AB/F}$) indicates that the proposed method provides better fused image.

**Keywords:** Multi focus image, DWT, PSO, GRAD, ROLP, Image quality metrics

## INTRODUCTION

Image fusion techniques can be classified into three categories: pixel level, decision level and feature level [1]. In pixel level fusion, the fused image is obtained from the corresponding pixel values of the source image whereas the feature level fusion segments the source image into regions and features such as edges, textures and intensity. The decision level fusion is based on the fuzzy logic, statistics, prediction, and so forth [2]. In our work, we have considered pixel level image fusion due to its effective computation and no loss of information.

Spatial and transform domain fusion techniques [3] gains the attention of researchers to fuse multi-focus images. Averaging [4], weighted Averaging, PCA [5], Linear Fusion, sharp fusion are spatial domain methods. The major disadvantages of spatial domain are that it introduces spatial distortions in the resultant fused image and does not provide any spectral information. Therefore, transform domain fusion methods have been proposed to overcome the drawback of spatial domain methods.

The pyramid and wavelet transforms are the two mainly used multi-scale image fusion techniques. Gradient pyramid [6], Laplacian pyramid [7-8], contrast pyramid [9], morphological pyramid techniques and [10], ratio of Laplacian pyramid [11] have been used for image fusion. The pyramid based approaches suffer from blocking effect [1] and they do not provide any directional information. Further the pyramid based fusion techniques have poor signal to noise ratio [2]. But the wavelet transforms provide better representation of detailed features of image than pyramid based fusion techniques.

The DWT is the most widely used wavelet transform for multi-focus image fusion. A novel contrast-based image fusion algorithm is proposed in the wavelet domain [12] for noisy source images. Li and Manjunath [13] proposed image fusion scheme using wavelet and area based maximum selection rule. But the area based maximum selection rule is suitable for images with shades. Redundant wavelet transform (RWT) is also applied for fusing images in paper [14]. The redundant nature of RWT increases the complexity of the fusion process [15]. To reduce the complexity of RWT, the authors in [16-17] used undecimated discrete wavelet transform for image decomposition. The authors in [18] applied discrete wavelet transform for image decomposition and designed the fusion rules based on the analysis of the directional information of the wavelet confidents. In [19], a new image fusion algorithm based on multiwavelet transform to fuse multisensor images is presented. In this fusion algorithm, a feature-based fusion rule is used to combine original sub images and to form a pyramid for the fused image. But multiwavelet also increases the complexity of the fusion algorithm. The authors in [20] proposed DWT based fusion scheme for CT and MRI images using Type-2 fuzzy logic. In this method, initially, the source images are decomposed into low-level and high-level sub-bands by discrete wavelet transformation (DWT). As the second step, for fusion, Type-2 fuzzy technique is applied on a low-level sub-band and average fusion method is applied on the high-level sub-bands in order to enhance the most prominent features present in CT image and MRI image. Finally, the fused low-level sub-band and highlevel sub-bands are reconstructed to form the final fused image using inverse-DWT.

The contourlet transform [21], curvelet transform [22], non-sub sampled contourlet transform [23] are advanced wavelet families that provide better results than wavelet transform. On the other hand, the advanced wavelet families are computationally costly and require huge amount of memory. Further, the study and analysis of DWT for multi-focus image fusion has not been studied well and it still needs attention of researchers. Since the optimal selection of fusion rules for a wavelet transform [13] has always been challenging problem. The fusion rules such as maximum selection, minimum selection and average fusion strategy is not suitable for all kind of images. Hence it is necessary to design optimal fusion strategy which is suitable for all kind of images. In this work we formulated fusion as an optimization problem and applied PSO to choose optimal weights for the fusion of wavelet coefficients. The proposed method having the following attractive features:

1) Source images are decomposed using DWT that reduces the computational complexity and avoids the spectral distortion in the fused images.

2) The newly designed optimal fusion strategy incorporates maximum information from source images.

3) Fusion scheme is applied on each colour channel separately that results in no spectral distortion on the fused images.

4) Moreover, the proposed fusion scheme is suitable for all kind of images.

The rest of the section is organized as follows. An optimized image fusion framework is discussed in section 2. The subjective and assessment of proposed image scheme is discussed in section 3. Finally, the conclusion of proposed approach is given at section 4.

## PROPOSED IMAGE FUSION FRAMEWORK

The schematic diagram of the proposed image fusion approach is given in Fig 1. Here DWT is employed for image decomposition and PSO is applied for the fusion of wavelet coefficients.

**Step 1:** First the source images (image1, image2) are separated into three different colour bands, namely red, green and yellow bands.

**Step 2:** The DWT is applied on the each colour band separately and decomposed into low (LL), and high frequency (HL, LH, and HH) coefficients.

**Step 3:** The high frequency coefficients of red colour band of image1 (I1) and image2 (I2) are fused using maximum selection rule.

**Step 4:** The low frequency coefficients of red colour band of I1 and I2 are fused using weighted average fusion rule. In this work, Swarm intelligence based particle swarm optimization is employed to find the optimal weights for the fusion of low frequency wavelet coefficients.

**Step 5:** After the wavelet coefficients are fused, the inverse DWT is applied on the fused low and high frequency components to get the fused red band image.

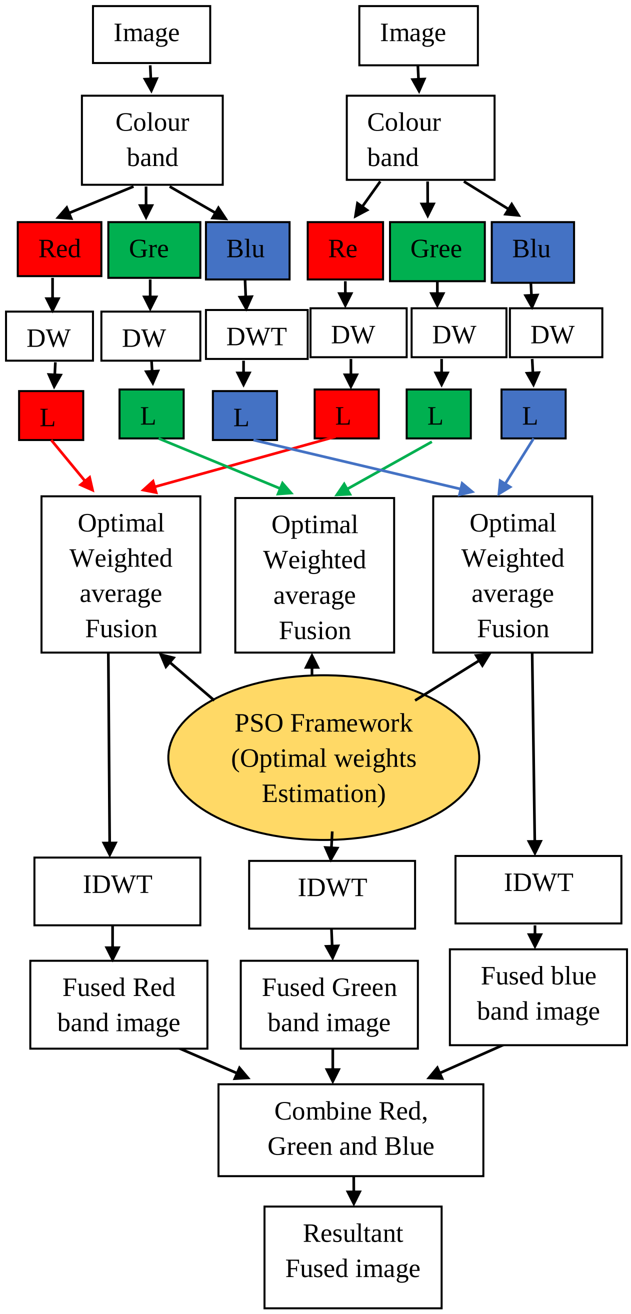


**Figure 1.** Schemetic diagram of proposed image fusion approach

**Step 6:** The same process can be applied to get green and blue band fused images. Finally, the three colour band images are combined to get the resultant fused image.

## Image decomposition using DWT

The discrete wavelet transform of a given signal $S(x)$ is obtained by analysis and synthesis process using scaling function $\varphi(x)$ and wavelet function $\psi(x)$ respectively [24].

The scaling function is defined as

$$\varphi(x) = \sqrt{2}\sum_k l(k)\varphi(2x-k) \tag{1}$$

Where $l(k)$ is the approximation or low pass coefficient of the signal $S(x)$. The value $\sqrt{2}$ maintains the norm of the scaling factor by a factor of two.

The wavelet function $\psi(x)$ computes the high frequency components or detailed coefficients are given by

$$\psi(x) = \sqrt{2}\sum_k h(k)\varphi(2x-k) \tag{2}$$

Where $h(k)$ are the high frequency or detailed wavelet coefficients.

The scaling coefficients $l(k)$ and the wavelet coefficients $h(k)$ are used in signal decomposition. Forward analysis of signal $S(x)$ at any scale $I$ is denoted by

$$S(x) = \sum_k P(i,k)\varphi_{i,k}(x) + \sum_i \sum Q(i,k)\varphi_{i,k}(x) \tag{3}$$

Where $P(i,k)$ and $Q(i,k)$ are scaling and wavelet coefficients at scale I. The scaling and wavelet coefficients at scale I can be computed using the Eq. (4) and Eq. (5)

$$P(i,k) = \sum_k l(k-2m)P(i+1,k) \tag{4}$$

$$Q(i,k) = \sum_k h(k-2m)P(i+1,k) \tag{5}$$

By combining the scaling and wavelet coefficients, the signal $S(x)$ can be reconstructed and mathematically it is represented as

$$P(i+1,k) = \sum_k P(i,k)l(m-2k) + \sum_k Q(j,k)h(m-2k) \tag{6}$$

The forward and backward analysis of signals provides the facility to have multi scale signal representations at varying scales [25].

Further, the DWT provides three spatial orientations namely, horizontal, diagonal, and vertical. This can be denoted by the following combination of scaling and wavelet functions.

$$\varphi_{LL}(x,y) = \varphi(x)\varphi(y) \tag{7}$$

$$\psi_{LH}(x,y) = \varphi(x)\psi(y) \tag{8}$$

$$\psi_{HL}(x,y) = \psi(x)\varphi(y) \tag{9}$$

$$\psi_{HH}(x,y) = \psi(x)\psi(y) \tag{10}$$

The source images $I_1$ and $I_2$ are decomposed into low and high frequency components by applying DWT. The coefficients of the both images are composed using the fusion rule. The fused image F is obtained by taking the inverse DWT to the fused coefficients.

$$F = (IDWT)\left[\phi\{DWT(I_1), DWT(I_2)\}\right], \tag{11}$$

Where $IDWT$ is the inverse $DWT$ . $\phi$ is the fusion rule given by

$$\phi(x,y) = w_1 \times I_1(x,y) + w_2 \times I_2(x,y), \tag{12}$$

Where, $I_1(x,y)$, $I_2(x,y)$ are the wavelet coefficients of source images and $\phi(x,y)$ is the fused wavelet coefficients. Here $w_1$ and $w_2$ determine the percentage of each image coefficient in the fused image, that minimize the information loss and change in spectral characteristics of an image. Many of the fusion rules are based on some fixed weights which lead to loss of contrast information that may degrade the outcome of fusion. Therefore our aim is to find optimal weights for fusion of wavelet coefficients that maximize information.

## OPTIMAL WEIGHT ESTIMATION USING PSO

PSO is a population-based optimization algorithm that mimics the behaviour of social organisms in which a set of initial population (particles) of solutions generated randomly and iteratively finds the optimum solution in a feasible solution space [26]. The movement of each particle depends on two main factors: $P_i(t)$ the best position (personal best) that ith candidate has found so far and $P_g(t)$ is the global best position found by the whole swarm. Based on personal best and global

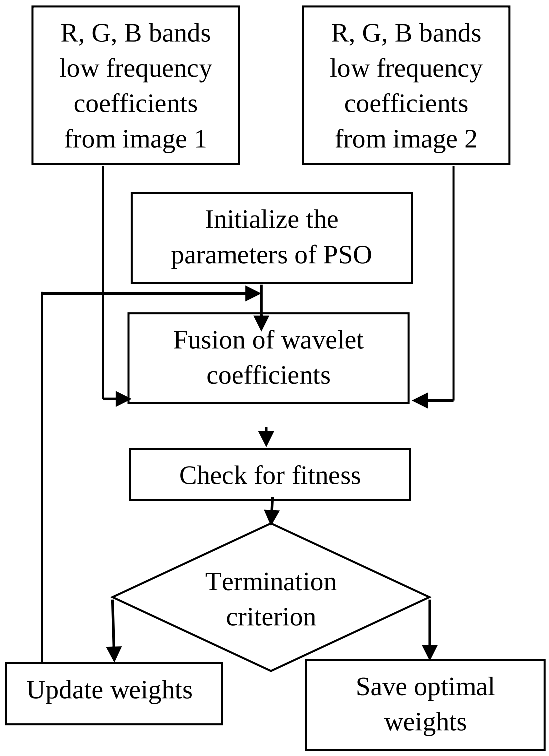


**Figure 2.** Optimal weight estimation using PSO framework for Low frequency subbands

best, each particle updates its velocity and position during each iteration (t) as follows:

$$v_i\left(t+1\right)=\omega v_i\left(t\right)+c_1r_1\left(P_i\left(t\right)-x_i(t)\right)+c_2r_2\left(P_g\left(t\right)-x_i(t)\right) \quad (13)$$

$$x_i\left(t+1\right)=x_i\left(t\right)+v_i\left(t+1\right) \quad (14)$$

Where $\omega$ is the inertia weight, which controls the convergence behaviour of PSO. The large inertia weight aids global search (searching new areas), while small inertia weight tends to aid local search. Current position of the $i^{th}$ particle and its velocity is indicated as $x_i(t)$ and $v_i(t)$ respectively. The parameters $c_1$ and $c_2$ are positive constants assumed to be 2, which stabilizes the influence of the individual best and global best position. The parameters $r_1$ and $r_2$ are applied to control the diversity of the population, and they are consistently distributed in the range [0, 1].In our proposed work the image is decomposed using DWT and the fused image is based on combining the different sub-band coefficients using the optimal weights.

In this paper image fusion is formulated as an optimization problem as given below: The set of particle values (weights) is defined as a set of $N$ particles

$$w=\left\{\begin{matrix} w_{11} & w_{21} \\ w_{12} & w_{22} \\ . & . \\ . & . \\ . & . \\ w_{1N} & w_{2N} \end{matrix}\right\}$$

Which maximizes the objective function (Entropy)

$$arg_t\, max(H)=-\sum_{i=0}^{255} p(i)\times log_2(p(i)) \quad (15)$$

Which subject to constraints $w_1+w_2=1$. Where, $p(i)$ is the probability of occurrence of $i^{th}$ intensity of the fused image. Each row in the particle set is substituted in Eq. (12) and the fused image is obtained. The particle set which gives the maximum entropy value for the fused image will be stored as gbest. After the maximum number of iterations is reached the value of gbest (optimal weights) is used to get the final fused image.

## EXPERIMENTAL RESULTS AND DISCUSSION

### Parameter settings

The proposed image fusion framework is compared with several other state-of-the-art multi resolution image fusion methods namely Gradient pyramid(GRAD)[6], DWT[13], and DWT+Fuzzy logic based fusion approaches[20]. The proposed fusion method is implemented using MatLab 2015. The number of decomposition level is set to 2 for DWT. The PSO parameters selected for the optimization problem are listed in Table 1 to obtain the weighted averaging fusion rule. The decomposition level for GRAD and DWT is set to 2. The high frequency components are fused by maximum selection rule whereas low frequency coefficients are fused using average fusion rule for GRAD and DWT.

**Table 1.** Parameters selected for PSO

| **Parameters** | **Values** |
|---|---|
| Particle size(N) | 30 |
| Dimension | 2 |
| Total iteration | 50 |
| Inertia(w) | w-initial=0.9,<br>w-final=0.4 |
| Learning Factors (c1,c2) | c1=2,c2=2 |

### Image quality assessment metrics

Evaluating the quality of the fused image is a challenging task as the reference image is not available to compare fusion results. Researchers have proposed several quality metrics to assess the quality of such image. In this work, we have chosen four metrics to evaluate the quality of the fused image.

**Standard deviation** is the square root of the variance, which reflects the spread in information and is given b

$$\sigma = \sqrt{\sum_{0}^{255}(i - i_{mean})^2 h(i)} \qquad (16)$$

Where $i$ and $i_{mean}$ is gray-level and mean intensity of the image, and $h(i)$ is the normalized histogram of the image. It is well-known that standard deviation is made out of the signal and noise parts. This metric would be more productive without noise. It gauges the contrast in the fused image hence; an image with high contrast would have a high standard deviation.

**Spatial Frequency** is used to measure the action level in an image. A large value of spatial frequency depicts the vast activity level in the image which represents the clarity of the image.

$$SF = \sqrt{(RF)^2 + (CF)^2} \qquad (17)$$

$$RF = \sqrt{\frac{1}{m \times n}\sum_{i=1}^{m}\sum_{j=2}^{n}\left[F(i,j) - F(i,j-1)\right]^2} \qquad (18)$$

$$CF = \sqrt{\frac{1}{m \times n}\sum_{i=1}^{m}\sum_{j=2}^{n}\left[F(i,j) - F(i-1,j)\right]^2} \qquad (19)$$

Here $RF$ and $CF$ are row frequency and column frequency. $F$ is the fused image; $m \times n$ is the size of the fused image.

**Mutual information:** MI quantifies the mutual dependence between the source and fused image which is given by

$$MI = MI_{AF} + MI_{BF} \qquad (20)$$

where $MI_{AF}$

$$= \sum_{m,n}\left(h_{AF}(m,n) log_2 \frac{h_{AF}(m,n)}{h_A(m) h_F(n)}\right)$$

and $MI_{BF}$

$$= \sum_{m,n}\left(h_{BF}(m,n) log_2 \frac{h_{BF}(m,n)}{h_B(m) h_F(n)}\right)$$

Where $MI_{AF}$ and $MI_{BF}$ are the mutual information between the source images $A$, $B$ and fused image. Where, $h_{AF}(m,n)$ is the joint probability distribution function of $A$ and $F$, and $h_A(m)$ and $h_F(n)$ are the marginal probability distribution functions of $A$ and $F$, respectively.

**Petrovic metrics:** $Q^{AB/F}$ compute the amount of edge information transferred from source image to fused image. The procedure for computing $Q^{AB/F}$ is given in [26], is adapted in our work to compute the Petrovic metric.

## RESULTS AND DISCUSSION

To evaluate the performance of the proposed system three set of multi focus images are taken from LIVE data base. In Fig 3, Image 1 is left side blurred and image 2 is right side blurred. While combining these two images using GRAD method the resultant image is not satisfactory. The DWT method amplifies the noise present the input images because the pyramid based approaches are more sensitive to noise and produces blocking effect in the fused image. The result of DWT+Type-2 Fuzzy logic is better than DWT and GRAD but the land is not clearly visible compared with proposed method. The proposed method effectively captures the information of leaves, stems and land into the fused image. The metrics values in table 2 indicates that the proposed method gives more information (MI=2.8221) and GRAD method gives less information (MI=0.9675) compared with DWT and DWT+Type-2 Fuzzy logic. The optimal values for fusing low frequency sub-bands are given in Table 3.

In Fig. 4, the input image 1 clearly captures the date and month while the letters printed on the book is effectively captured in image 2. When fusing these two images DWT gives better result than GRAD but it produces less contrast image.

**Table 2.** Objective assessment of proposed image fusion method

| **Metrics/ Methods** | **Proposed** | **DWT+ Fuzzy [20]** | **DWT [13]** | **GRAD [6]** |
|---|---|---|---|---|
| **Result of tree image** | | | | |
| Entropy | 7.2639 | 7.0664 | 7.1865 | 6.9481 |
| SD | 36.8123 | 32.0071 | 35.0391 | 29.3316 |
| SF | 34.1281 | 17.8101 | 32.9575 | 18.8573 |
| MI | 2.8221 | 2.6752 | 2.2987 | 2.0123 |
| $Q^{AB/F}$ | 1.0101 | 0.9754 | 0.9178 | 0.9675 |
| **Result of book image** | | | | |
| Entropy | 7.5897 | 6.1420 | 7.3855 | 6.2482 |
| SD | 68.0634 | 61.8467 | 62.6806 | 59.6358 |
| SF | 26.6565 | 14.4302 | 22.3716 | 13.8949 |
| MI | 3.3431 | 3.0123 | 3.0675 | 2.5908 |
| $Q^{AB/F}$ | 1.5230 | 1.3454 | 1.2343 | 1.0987 |
| **Result of Flower image** | | | | |
| Entropy | 7.9743 | 7.8602 | 7.6818 | 7.1922 |
| SD | 65.2443 | 61.5119 | 63.8265 | 56.2480 |
| SF | 46.1306 | 27.1892 | 43.6369 | 27.0663 |
| MI | 3.4264 | 3.1432 | 3.0213 | 2.9870 |
| $Q^{AB/F}$ | 1.2190 | 1.0865 | 1.0657 | 0.9871 |

The number 04 is not clearly visible in the DWT fused image. On the other hand, the letters printed in the book “IMAGE” is not clear in DWT+Type-2 Fuzzy logic. The proposed method successfully combined the date and month from image 1 and “IMAGE” letters from image 2. Moreover, it produces highly contrast fused image than state-of-arts. The metric QAB/F value (1.5230) indicates that the proposed method contains more edge information in the fused image. In Fig 5, in image 1 the flowers are blurred whereas in image 2, the lawn and table is blurred. The result of proposed method is better than state–of-art. The subjective and objective assessment of proposed method indicates that it outperforms than existing methods.

**Table 3.** Optimal weights

| **Image/weights** | **$w_1$** | **$w_2$** |
|---|---|---|
| Tree | 0.65 | 0.35 |
| Book | 0.56 | 0.44 |
| Flower | 0.38 | 0.62 |

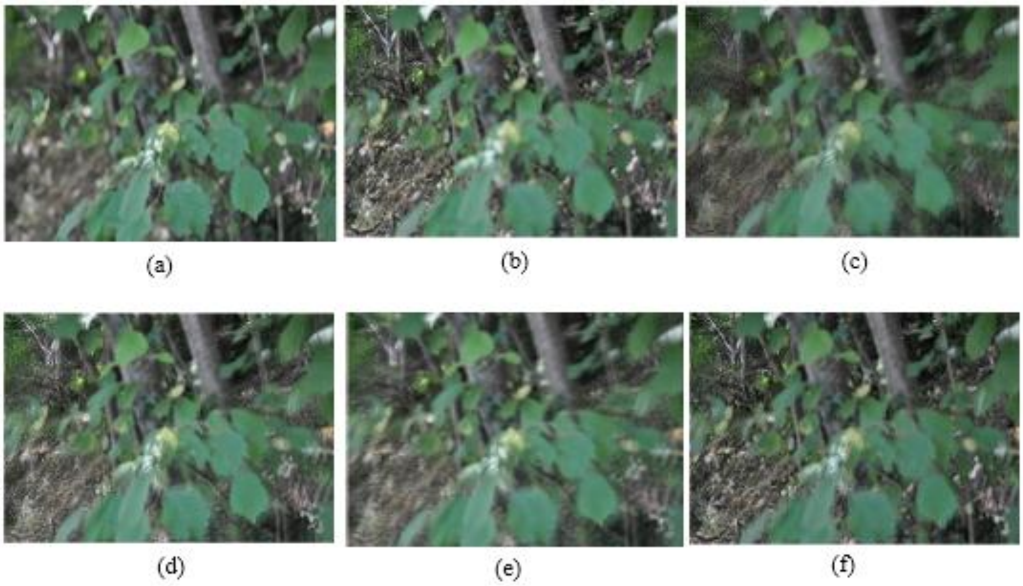


**Figure 3.** Fusion results of tree image using various image fusion techniques (a) Input Image1 (b) Input Image2 (c) GRAD [6] (d) DWT [13] (e) DWT+Type-2 Fuzzy logic [20] (f) Proposed


**Figure 4.** Fusion results of book image using various image fusion techniques (a) Input Image1 (b) Input Image2 (c) GRAD [6] (d) DWT [13] (e) DWT+Type-2 Fuzzy logic [20] (f) Proposed

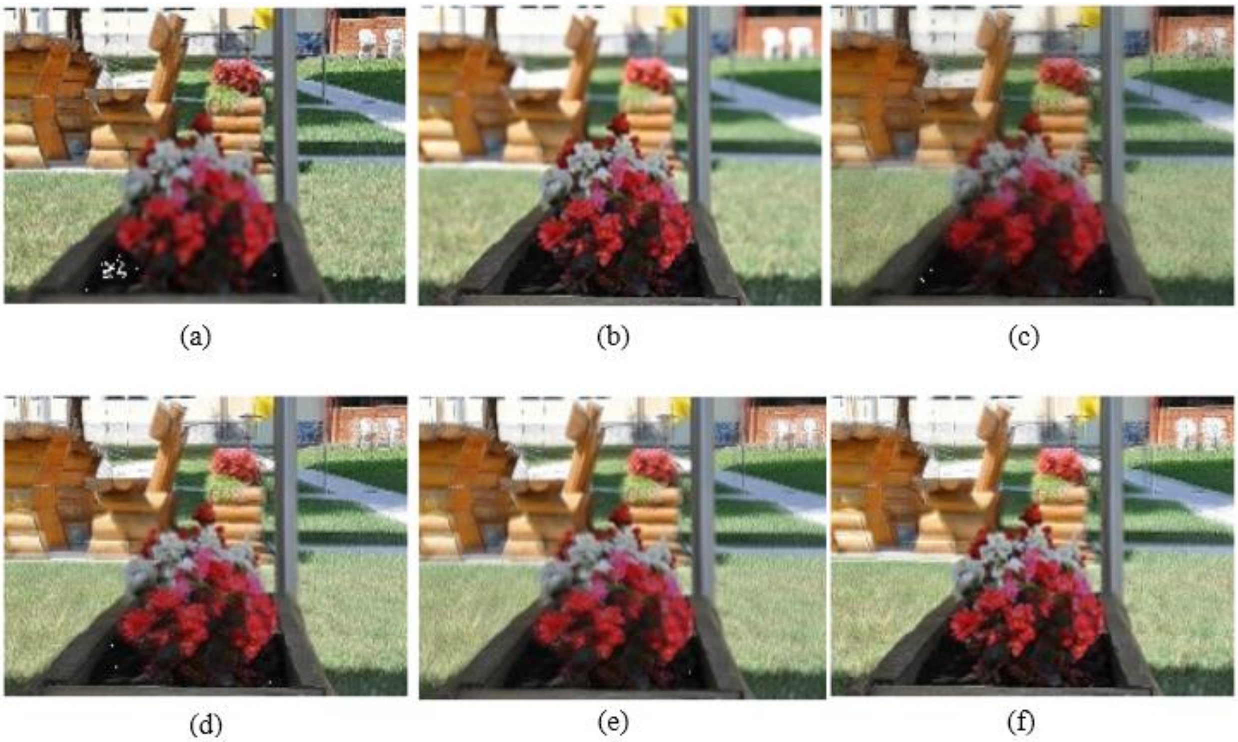


**Figure 5.** Fusion results of Flower image using various image fusion techniques (a) Input Image1 (b) Input Image2 (c) GRAD [6] (d) DWT [13] (e) DWT+Type-2 Fuzzy logic [20] (f) Proposed

## CONCLUSION

In this paper, we have proposed an optimal image fusion approach to fuse multi focus colour images in discrete wavelet transform domain. The DWT is applied for image decomposition and particle swarm optimization is applied to find the optimal weights for the fusion of wavelet coefficients. The high frequency sub-bands were fused using maximum selection rule whereas the low frequency sub-bands are fused using optimal weights obtained by PSO. To show the effectiveness of the proposed system, we have performed subjective and objective evaluation with GRAD, DWT and DWT+Type-2 Fuzzy logic. The comparative analysis of the fusion results are performed with stand deviation, spatial frequency, mutual information and edge quality metric. The visual and objective assessment indicates that the proposed gives better result than state-of-art methods. The performance of the proposed fusion can be improved by applying Curvelet based decomposition techniques.

The proposed method has the following attractive features

1) Source images are decomposed using DWT that reduces the computational complexity and avoids the spectral distortion in the fused images.
2) The newly designed optimal fusion strategy incorporates maximum information from source images.
3) Fusion scheme is applied on each color channel separately that results in no spectral distortion on the fused images.
4) Moreover, the proposed fusion scheme is suitable for all kind of images.

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