Experimental Flight Testing and System Identification of Eurocopter AS365N2 Dauphin Helicopter using CIFER®

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Abstract

In this paper, the test helicopter is modified by installing data acquisition system and air data boom. The test helicopter should be approved by the local civil aviation authority with a special airworthiness certificate. Initially, Eurocopter will perform the mandatory checks for test helicopter like weighing of helicopter, Electromagnetic compatibility (EMC)/Electromagnetic interference (EMI) testing, engine checks and Functional Check Flight (FCF). Once the flight test is complete, Eurocopter will again perform the mandatory checks like weighing, FCF, etc. The local civil authority should issue a standard airworthiness certificate after making a full check up to the helicopter. By using the obtained flight data, system identification is performed by frequency-response method with the help of commercial software known as Comprehensive Identification from Frequency Responses (CIFER®).

Keywords: Air data boom, CIFER®, Flight Test, Helicopters, System Identification

INTRODUCTION

Basic air data is all about measurements that can give information about physical characteristics of the air mass that surrounds the aircraft. The air is assumed to be dry, so nothing must be done to account for moisture content. With the value of temperature, pressure and using commonly available data it's possible to retrieve all the physical characteristics of dry air. The following data is needed for a minimal air data system: Direct measured data, Outside air temperature (OAT), Outside static pressure Ps, Total pressure Pt, Angle of attack (AOA) and angle of sideslip (AOS).

We designed and installed an instrumentation system comprised of sensors, signal conditioning circuits, power supplies, a data acquisition system (DAS), and associated mounting brackets and wiring. Sensors, mounting brackets, and wiring were installed at locations throughout the helicopter, including the cockpit and transmission deck. The signal conditioning circuits, power supplies, and DAS [1] were installed in an equipment rack fastened to a base plate, which was bolted to the cabin floor through seating attachment points.

A sensor produces data for each measured parameter. In some cases, an appropriate sensor was already installed on the helicopter (e.g. engine parameter sensors, radar altimeter, etc.) and we chose to interface directly to that sensor to obtain the required signal. In other cases, a special purpose sensor (e.g. pilot control position sensors, angle of attack and sideslip sensors, etc.) were installed to measure the parameter of interest. Signal conditioning circuits converted the various outputs of each sensor (e.g. voltage, frequency, resistance, etc.) into a DC voltage compatible with the analog to digital converters in the DAS. Each signal conditioning circuit was designed with high input impedance to prevent loading of the sensor signal and to maintain isolation between the DAS and the helicopter.

The DAS used here for flight testing was Fifth Generation Data Acquisition System (5GDAS). The primary role of the 5GDAS was to log data during helicopter maneuvers. The 5GDAS uses 12-bit (1 part in 4096) analog-to-digital converters to digitize analog parameters, TTL-compatible input circuits to receive discrete parameters, and an ARINC-429 bus receiver channel to receive parameters from data buses. The complete set of measured parameters (analog, digital and bus) were sampled within a one-millisecond interval to effectively eliminate time skews. Data logging occurs at sample intervals of 0.003, 0.015, 0.045 and 0.090 seconds depending on the parameter and the requirements of the test.

Aircraft system identification is a highly versatile procedure for rapidly and efficiently extracting accurate dynamic models of an aircraft from the measured response to specific control inputs. Models might be desired to characterize the aircraft dynamics as a whole or to characterize an aircraft subsystem, such as an actuator, rotor system, or the engine. Key applications of aircraft system-identification results include piloted simulation models, comparison of wind-tunnel versus flight measurements, validation and improvement of physics-based simulation models, flight-control system development and validation, and handling-qualities specification compliance testing [2]. Frequency-response-based methods will provide a unified flow of information regarding system performance around the entire life cycle from specification and design through development and flight test, as seen in figure 1.
The Eurocopter AS365N2 Dauphin is a medium-weight, multipurpose, twin-engine helicopter originally developed and manufactured by the French firm Aérospatiale, later merged with Eurocopter Group which is currently produced by Airbus Helicopters [3]. The N2 version (commercial debut in 1990) of the Dauphin features several improvements over the original version of the helicopter (commercial debut in 1978), including being equipped with more powerful Arriel 1C2 turboshaft engines, an uprated gearbox, increased maximum takeoff weight, a redesigned Fenestron tail rotor inside an enlarged tail fin, greater usage of composite materials, a better cabin arrangement with redesigned doors and a revised interior, and retractable landing gear. The AS365 series of helicopters is one of the most successful designs by Eurocopter and has been widely used throughout the world in both civil and military applications, with missions including corporate transport, air taxi/ferry operations, airborne law enforcement, emergency medical services, search and rescue, firefighting, and electronic news gathering, as well as several other purpose-built military variants. It can travel long ranges and performs well in high ambient temperature operational climates as well as operations at significant altitude. A 3-view layout of the AS365N2 helicopter is presented in figure 2.

The automatic flight control system (AFCS) (consisting of cyclic, collective, and anti-torque controls) is used to regulate the helicopter attitude, altitude, and direction of flight. The flight controls are both electronically and hydraulically boosted to reduce pilot effort and to counteract control feedback forces. Control inputs from the cyclic and collective control sticks in the cockpit are transmitted by high-speed limited-authority twin-motor electric actuators (pitch and roll axis) to dual-chamber hydraulic servo actuators mounted on the top deck. A single-motor electric actuator transmits inputs to a single dual-chamber hydraulic servo actuator (yaw axis) for tail rotor pitch control. These position-controlled actuators are series-mounted in the flight control linkage downstream of the anchoring point. This arrangement allows them to drive the downstream part of the control linkage (towards the blades) without moving the cockpit controls. The actuators operate the cyclic and collective levers, which raise, lower and tilt the swashplate.

Figure 1: Roles of system identification in the flight-vehicle development process

Figure 2: Three-View Layout of AS365N2 Helicopter
The 3-axis flight control system features an integrated autopilot and stability augmentation system (SAS) with the ability to hold trim inputs to the stick, as well as speed, heading, and altitude hold functions which are integrated into the cyclic control grip. Various additional functions can be controlled through buttons on the cyclic grip, including a 4-way beep trim switch for pitch and roll trim, radio operations, cargo load release, force trim release (FTR), and other functions depending on the operational configuration. The collective pitch lever control grip also features abilities to control rotor RPM trim, windshield wiper control, landing lights, emergency flotation gear, and a winch cable cutter amongst other functions, depending on the operational configuration. The collective pitch lever also features a friction locking device that is adjustable to hold the collective in place during operations. The cyclic control stick does not have a friction locking device installed, instead relying on the force trim system and the SAS.

AIR DATA BOOM (ADB) SYSTEM

The ADB system used for this experiment is a Space Age Control Inc. Model 100510 Swivel Head Air Data Boom [4]. The air data boom has two vanes which are attached to potentiometers. One measures angle of attack (AOA) while the other measures angle of sideslip (AOS). The system is also equipped with a static and total pressure ports which provide the necessary readings to calculate air speed. Information regarding the data boom is tabulated below:

\[ P_t = \text{total pressure: the sum of local atmospheric pressures plus dynamic pressures. Algebraically, total pressure } (P_t) \text{ equals the sum of static pressure } (P_s) \text{ plus impact pressure } (q_c). \]

\[ P_s = \text{static pressure: the absolute pressure of still air surrounding a body; the atmospheric pressure at the flight level of the aircraft.} \]

\[ q_c = \text{impact pressure: a calculated value } (q_c), \text{ it is the difference between total pressure and static pressure; it is the pressure created by the forward speed of the aircraft.} \]

\[ \alpha = \text{angle of attack: the angle measured in the XZ plane between the X axis and the relative air flow; also designated as AOA, alpha, or } \alpha; \text{ angle of attack is not the same concept as "pitch" which indicates the rotation of the aircraft relative to three imaginary lines running through an airplane and intersecting at right angles at the airplane's center of gravity.} \]

\[ \beta = \text{angle of sideslip: the angle measured in the XY plane between the Y axis and the relative air flow; also designated as AOS, beta, or } \beta; \text{ angle of sideslip is not the same concept as "yaw" which indicates the rotation of the aircraft relative to three imaginary lines running through an airplane and intersecting at right angles at the airplane's center of gravity.} \]

\[ T_t \text{ total air temperature: the temperature of an airflow measured as the airflow is brought to rest without removal or addition of heat; also designated as TAT or } T_t. \]

\[ OAT \text{ outside air temperature: the temperature of the static outside temperature without the effects of airspeed; also designated as OAT. } \]

There are several options as to where on an aircraft to place an air data boom with each configuration having its own trade-offs. The two most conventional placement areas are extending from the nose cone and attached to the wing. These two configurations are the easiest to implement effectively which were considered for this research. Paramount to the success of an air data boom system is its ability to experience free-stream conditions which are not subject to disturbances from the aircraft. The closer the boom is to these ideal conditions the more accurate data it will be able to collect.

The nose cone mounted configuration was the other prominent choice for boom placement. Its largest con is that the nose cone and fuselage of the aircraft can present intense pressure gradients and vortices upstream which may affect the accuracy of the boom.

CALIBRATION FOR ANGLE OF ATTACK AND ANGLE OF SIDESLIP PARAMETERS

Angle of attack and sideslip were measured with vanes fitted on the instrumentation boom attached to the left-hand side of the helicopter and extended beyond the nose of the helicopter. The vanes were scaled with a coarse scale range of ±180 degrees and a fine scale range of ±40 degrees for angle of attack and ±45 degrees for angle of sideslip. These were combined to form the parameters AOS_VANE and AOA_VANE [5]. Downwash from the rotor affected the vanes below approximately 40 knots causing incorrect readings at low speed. Below 40 knots true airspeed, there is an error trap in the processing that disables the rate correction and sets the true angle equal to the calibrated parameter. The air data boom used in this experiment showing its alpha and beta head is shown in figure 3.
Alignment of the boom with respect to the fuselage of the helicopter was checked on the ground. For alpha, the local incidence of the alpha/beta vane was measured with an inclinometer and compared to helicopter pitch attitude measured on the floor behind the pilot seat. The delta was used in the alpha calibration to align the raw alpha measurements to the aircraft pitch attitude. For beta, the aircraft was leveled in roll (0.0 degree roll attitude) and a laser alignment tool was used to locate the vertical center of the helicopter fuselage projected out in front of the helicopter. A plum bob was used to project the angle of the boom on the hangar floor and measurements were made to calculate a boom angle with respect to the vertical plane. This angle was used in the beta calibration to correct the raw beta measurement to the aircraft vertical plane.

Flight analysis of the angle of attack measurement showed that the vane angle needed a downwash correction to accurately represent the true wind angle relative to the aircraft body axis. No downwash correction was necessary for the angle of sideslip measurement. Calibration of the angle of attack vane was performed by analyzing maneuvers that captured a range of alpha which can be considered a quasi-steady state environment. The measured pitch attitude and a derived flight path angle are used to calculate the angle of attack from the following equations:

\[
\text{Angle of Attack (alpha): } \alpha = \theta - \gamma \\
\text{Where: } \alpha = \text{angle of attack (deg)} \\
\theta = \text{pitch attitude (deg)} \\
\gamma = \text{flight path angle (deg)}
\]

\[
\text{Flight Path Angle (gamma): } \gamma = \sin^{-1}\left[\frac{\text{h} \text{dot} \mu (T_A/T_{\text{std}})/V_{\text{true}}}{}\right]
\]

Where: \(\gamma\) = flight path angle (deg)
\(\text{hdot}\) = rate of climb (ft/min)

\(T_A = \text{ambient temperature (K)}\)
\(T_{\text{std}} = \text{standard day ambient temperature (K)}\)
\(V_{\text{true}} = \text{true airspeed (ft/sec)}\)

The calibration was evaluated over the normal aircraft flight envelope. This relationship can be summarized as follows:

\[
\alpha = f(\alpha_{\text{true}})
\]

Where: \(\alpha_{\text{true}} = \text{filtered vane angle of attack (deg)}\)

The angle of attack correction analysis model was extended to \(\pm 180\) degrees. This correction is shown in Table I.

<table>
<thead>
<tr>
<th>Filtered Angle (deg)</th>
<th>Alpha Vane (deg)</th>
<th>Corrected Angle (deg)</th>
<th>Angle</th>
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</thead>
<tbody>
<tr>
<td>-180.0</td>
<td>-180.00</td>
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<td></td>
</tr>
<tr>
<td>-90.0</td>
<td>-81.14</td>
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<td>+90.0</td>
<td>+76.63</td>
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<tr>
<td>+180.0</td>
<td>+180.00</td>
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The sideslip correction analysis was extracted from analysis of both steady-heading sideslip and dynamic directional oscillation maneuvers. The sideslip angle correction analysis model was extended to \(\pm 180\) degrees. This correction is shown in Table II.

<table>
<thead>
<tr>
<th>Filtered Angle (deg)</th>
<th>Alpha Vane (deg)</th>
<th>Corrected Angle (deg)</th>
<th>Angle</th>
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</thead>
<tbody>
<tr>
<td>-180.0</td>
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**FREQUENCY-RESPONSE METHOD USING CIFER®**

The frequency-response identification method is particularly well suited to support the development and validation of flight-vehicle dynamic systems. The direct comparison between flight-test frequency responses and those from simulation models provides an excellent means of model validation and update for the system components (e.g., actuators, sensors, airframe, flight-control software) as well as the end-to-end behavior. Feedback stability and noise amplification properties are determined from the broken-loop frequency response and characterized by metrics such as crossover frequency and associated gain-and-phase margins. Command tracking performance is determined from the closed-loop frequency
response and characterized by metrics such as bandwidth, time delay, and equivalent-system eigenvalues. The system-identification approach presented here allows the direct and rapid (including real-time) identification of these frequency responses and metrics without the need to first identify a parametric (state-space) model structure, as it is required when applying time-domain methods [6]. Careful tracking of the broken-loop and end-to-end closed-loop frequency response behavior, from the preliminary design studies through detailed design and simulation and into the flight test, provides an important "paper trail" for documenting system performance and solving problems that might appear in the later phases of development.

The availability of comprehensive and reliable computational tools has substantially enhanced the acceptability of frequency-domain techniques in the flight-control and flight-test communities. Benefits derived from applying these techniques include the reduction of flight-test time required for control-system optimization and handling-qualities evaluation, especially for complex control-law architectures, as well as improvements in the final system performance. Frequency-domain methods offer a transparent understanding of component and end-to-end response characteristics that can be critical in solving system integration problems encountered in flight test.

The Army/NASA Rotorcraft Division (Ames Research Center) jointly developed the Comprehensive Identification from Frequency Responses (CIFER®) integrated facility for system identification [7] based on the frequency response approach shown in figure 4. This tool is composed of six core analysis programs built around a sophisticated database, along with a set of user utilities to provide a highly interactive, graphics-oriented environment for dynamics studies. The foundation of the CIFER® approach is the high-quality extraction of a complete MIMO set of nonparametric input-to-output frequency responses. These responses fully characterize the coupled characteristics of the system without a priori assumptions. Advanced chirp z-transform (CZT), multi-input conditioning, and composite window techniques, developed and exercised with over 20 years' worth of flight project applications, provide significant improvements in frequency-response quality relative to standard fast Fourier transforms (FFTs). Sophisticated nonlinear search algorithms are used to extract parametric models of varying complexity from this MIMO frequency response database that are used in simulation, handling-qualities, and flight control studies.

The key features of the CIFER® tool are as follows: 1) identification algorithms that have been extensively applied and proven on many flight projects; 2) implementation of frequency-response identification in a step-by-step sequence of core programs; 3) checks of user inputs against key guidelines; 4) chirp z-transform and composite window optimization for high-quality frequency-response identification; 5) multi-input frequency-response solution; 6) highly flexible and interactive definition of identification model structures; 7) fully automated weighting-function selection based on frequency-response accuracy; 8) reliable parameter accuracy metrics; 9) integrated procedure for identification and model-structure determination; 10) time-domain verification of models, including identification of offsets and biases; and 11) a suite of specialized utilities that support many of the applications just mentioned, which is uniquely suited to the difficult problems associated with flight-test data analysis.

![Figure 4: Flowchart of Frequency-domain method for System Identification](image)
Frequency Sweep Data

As an illustration of the first two blocks in figure 4 (Frequency Sweep Inputs and Aircraft), the basic input used for identification of the AS365N2 in cruise is a frequency sweep. Figure 5 shows flight-test data recorded at an indicated airspeed of 100 knots. The first plot shows the pilot lateral input and the second the roll-rate response. The aircraft is trimmed at the specified conditions, and the control is excited using a sinusoidal function with varying frequency. Although a greater amplitude of control input is usually desirable for a higher frequency excitation due to the tendency of lower response magnitude, this study employs constant amplitude inputs for convenience. The input amplitude is maintained ±5% of the trimmed control position.

SISO and MISO Frequency-Response Calculations

The key step in the identification procedure is the extraction of accurate frequency-responses for each input/output pair. Time histories for multiple frequency sweeps on a particular control are concatenated. Single-input/single-output (SISO) frequency responses for each input/output pair are determined using the Chirp-Z transform (an advanced Fast Fourier Transform) and overlapped/windowed spectral averaging [8]. When multiple control inputs are present in the excitation, as is the case of BO-105 helicopter data and most other open-loop helicopter tests, the contaminating effects of partially correlated inputs must be removed. This is accomplished by inverting the spectral matrices of all inputs x to a single output y at each frequency point f_k. The required "conditioned transfer-function matrix" T(f_k) is obtained as

\[
T(f_k) = \frac{\hat{G}_{xy}(f_k)}{\hat{G}_{xx}(f_k)}
\]

(4)

Where,

\[\hat{G}_{xx} - \text{input auto-spectrum estimate}\]

\[\hat{G}_{xy} - \text{cross-spectrum estimate}\]

Figure 6 and 7 present the CIFER analysis results in form of Bode plots that represent the response of the aircraft in frequency domain [9]. Determination of the \(\omega = \omega_{180}\) and \(\omega = 2\omega_{180}\) points often becomes difficult because of phase curve roughness caused by low coherence data and the effects of dynamics above the bandwidth. In such cases, as Tischler et al. [10] state, the phase data can be plotted on a linear frequency scale and approximated using a least-squares fit. The Q-plot of FRESPID, MISOSA and COMPOSITE results are shown in figure 8a, 8b, 9a, 9b and 10a, 10b respectively. The case plotting is shown in appendix A and B. The determined bandwidth and phase delay are shown in Table III and Table IV for hovering and forward flight conditions respectively.

![Figure 5: Frequency sweep input and Roll Attitude Response](image-url)
Figure 6: Frequency domain responses and least squares fit (Hover)
Figure 7: Frequency domain responses and least squares fit (100 knots)
Figure 8a. FRESPID: Frequency Response Identification for lateral sweep

Figure 8b. FRESPID: Frequency Response Identification for pedal sweep

Figure 9a. MISOSA: Multi-Input Conditioning for lateral sweep

Figure 9b. MISOSA: Multi-Input Conditioning for pedal sweep
In the frequency domain analysis, the linearity of the input-output dynamics can be represented by the coherence function [11]. As the simulation does not account for noises in signal, the coherence in this case purely reflects the fraction of the output spectrum that is linearly related with the input spectrum. The coherence becomes unity for a perfectly linear system. In order to show validity of the data in compliance with the bandwidth criterion, it is suggested that the coherence should be at least 0.6 [12]. The coherence function estimate is given by

\[ \gamma_{xy}(f) = \frac{\hat{G}_{xy}(f)^2}{\hat{G}_{xx}(f) \hat{G}_{yy}(f)} \] (5)

**Transfer-Function Modeling**

Direct transfer-function fitting of individual input/output frequency responses leads to single-axis, transfer-function models. This provides a direct and minimal-dimensional realization of the input-to-output dynamical behavior of a system, useful for many applications, such as handling-qualities analyses [13] and classical design of flight control systems [14]. A key application of transfer-function modeling is in the determination of an appropriate model structure for state-space model formulation. Systematic evaluation of the matching quality of candidate transfer-function models over desired frequency ranges provides valuable information on the order of the system, level of coupling, and initial guesses for many state-space parameters. A detailed application of this approach to model structure determination is presented in [15].

<table>
<thead>
<tr>
<th>Table III: Bandwidth and phase delay (Hover)</th>
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<tr>
<td>Pitch</td>
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<tr>
<td>(\omega_{BW_{phase}})</td>
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<tr>
<td>(\omega_{BW_{gain}})</td>
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<td>(\tau_p)</td>
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<tr>
<th>Table IV: Bandwidth and phase delay (100kts)</th>
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<tr>
<td>Pitch</td>
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<tr>
<td>-------</td>
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<tr>
<td>(\omega_{BW_{phase}})</td>
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<tr>
<td>(\omega_{BW_{gain}})</td>
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<td>(\tau_p)</td>
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Figure 10a. COMPOSITE: Multi-Window Averaging for lateral sweep

Figure 10b. COMPOSITE: Multi-Window Averaging for pedal sweep

Figure 11: Helicopter as an input-output system
State-Space Model

State-space models are often the desired end product of system identification. Such models are needed for example in: 1) control system design and optimization; 2) simulation model development, troubleshooting, and improvement; and 3) comparison of wind tunnel and flight characteristics. The ultimate product of a more intensive system identification effort can be a parametric model composed of the complete differential equations of motion that characterize the MIMO behavior of a fixed-wing or rotary-wing aircraft [16].

The linear equations of motion for small perturbations about a trim flight condition are represented in state-space form as

\[ x = Ax + Bu(t - \tau) \]  

(6)

Where, the control vector \( u \) is composed of the control-surface deflections (inputs) of figure 11, and the vector of aircraft states \( x \) is composed of the response quantities (speeds, angular rates, and attitude angles). The time-delay vector \( \tau \) allows a separate time-delay value for each control axis as a lumped representation of the higher-order dynamics (e.g., actuators, linkages, etc.) that are not explicitly included in the state-space model. Typically, the set of available flight-test measurements \( y \) is composed of a subset of the states; \( y \) can also include combinations of the states, such as the angles of attack and sideslip as measured by a nose boom sensor. The measurement vector can also include additional quantities, such as the accelerometers shown in fig. 11, which respond directly to control inputs. The general form of the measurement vector can be written as

\[ y = Cx + Du(t - \tau) \]  

(7)

System identification determines the values of the matrices \( A, B, C, D \) and the vector \( \tau \) that define the state-space model.

Time Domain Verification

The last step in the procedure of figure 4 is model verification. For this step, the identified state-space model is driven with flight data which is not used in the identification process, in order to check the model’s predictive capability. A key concern is that the model, which is identified based on one input form, can predict the response characteristics to other input forms. A multiple-input/multiple-output time-domain program integrates the state-space model Eq. (6) and (7), and determines the unknown state equation biases and zero shifts in the data. This is done by minimizing the weighted least-squares error between the model and vehicle responses.

CONCLUSION

The flight test plan defines the tests necessary to collect the ground and flight data on an AS365N2 helicopter, using as a guideline the test point matrix provided by Sejong University. This data is not intended to fully qualify a flight training device to any level as defined by the FAA. The data recorded was processed using a flight test analysis software package. The recorded (raw) flight test data was then converted to engineering units with corrections to air data, acceleration, rates, etc., applied as necessary as determined by the data processor’s analysis of specific pre-flight and post-flight measurements. A frequency-response method for rotorcraft system identification by using an integrated software package called CIFER® has been presented.

ACKNOWLEDGMENTS

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REFERENCES


NOMENCLATURE

$\alpha$ angle of attack (deg)
$v_{\text{vane}}$ filtered vane angle of attack (deg)
$\theta$ pitch attitude (deg)
$\gamma$ flight path angle (deg)
$\hat{u}$ input auto-spectrum estimate
$\hat{u}^{\text{cross}}$ cross-spectrum estimate
$h_{\text{dot}}$ rate of climb (ft/min)
$T_a$ ambient temperature (K)
$T_{\text{std}}$ standard day ambient temperature (K)
$T(f_k)$ conditioned transfer-function matrix
$V_{\text{true}}$ true airspeed (ft/sec)

Appendix A: Case Plotting for FRESPID, MISOSA and COMPOSITE using aileron as input

Appendix B: Case Plotting for FRESPID, MISOSA and COMPOSITE using rudder as input