

# Comparative Analysis of Simulation Results and Test of the Dynamics of the Wheelset

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## Abstract

The result of the above functions is a matrix in which the first column contains the nodal values of the variable and the rest - the values of the unknown functions of the system, calculated at these points. The order of the columns with the found values of the desired functions is determined by the sequence in which they are listed in the vector. The advantage of this approach is that this system nonlinear characteristics can be easily taken into account.

The calculation of dependence of lateral movement  $y$  and  $\psi$  rotation angle from the time when the wheelset wobbling, received by using  $\text{rkfixed}(y, 0, 10, 10000)$  function, producing calculations according to the method of Ruge-Kutta of 4th order, received by using  $\text{Rkadapt}(y, 0, 10, 100)$  function, producing calculations according to the method of Ruge-Kutta of 4th order, received by using  $\text{Bulstoer}(y, 0, 10, 100)$  function, implementing the method of Bulirsch-Stoer,

The number of steps it took to find a solution with the help of embedded  $\text{rkfixed}$  ( $n_1 = 10000$ ) function, proved the greatest, than the function  $\text{Rkadapt}$  ( $n_2 = 100$ ), implementing adaptive control of equations solving process according to the method of Ruge-Kutta of 4th order and  $\text{Bulstoer}$  ( $n_3 = 100$ ) function, generating calculations according to the method of Bulirsch-Stoer.

To characterize the accuracy of the calculations let's find the absolute error of lateral displacement by  $\text{Rkadapt}$  method and by  $\text{Bulstoer}$  method.

Thus, the absolute error of lateral movement calculations calculated by  $\text{Rkadapt}$  method slightly smaller than by  $\text{Bulstoer}$  method, whereas the absolute error of the rotation angle calculations is the reverse situation. Proceeding from the above errors calculation results, it can be judged about the fact that the solutions begin to stabilize after six seconds.

**Keywords:** modelling, comparative analysis, wheelset dynamics solutions, nonlinear characteristics.

## INTRODUCTION

The study of the carriage dynamics is a complex task. Even when driving on a straight path, when movement is performed at a low speed, there are problems associated with wobble oscillations [1]. At high speeds, it may be considerable fluctuations in the vertical and / or twisting movement.

In solving the problems of rolling stock dynamics and its interaction with the railway track, one always seek to possible simple solutions, allowing correct and accurately reflect the studied dynamic processes. For this purpose, modern branches of science and technology are used.

For example, the mathematical program package MathCAD provides a set of built-in functions for the numerical solution of differential equations. Two of these functions:  $\text{rkfixed}$ ,  $\text{Rkadapt}$  producing calculations according to the method of Ruge-Kutta of 4th order and  $\text{Bulstoer}$  function realizing method Bulirsch-Stoer.

Functions  $\text{rkfixed}(y, t_0, t_1, n, F)$ ,  $\text{Rkadapt}(y, t_0, t_1, n, F)$  and  $\text{Bulstoer}(y, t_0, t_1, n, F)$  include the

following parameters:  $y$  - the vector of initial conditions with the dimension corresponding to the order  $k$  of the differential equation or the number of first-order differential equations;  $t_0$  - the starting point for the variable;  $t_1$  - the final point of the calculation;  $n$  - the number of fixed points, where an approximate solution is sought;  $F$  - vector-function containing the right-hand parts of equations.

The result of the above functions is a matrix in which the first column contains the nodal values of the variable and the rest - the values of the unknown functions of the system, calculated at these points. The order of the columns with the found values of the desired functions is determined by the sequence in which they are listed in the vector. The advantage of this approach is that this system nonlinear characteristics can be easily taken into account.

The difference between `rkfixed` and `Rkadapt` functions is as follows. The first of them is looking for an approximate solution with a constant step, the second - provides adaptive control of the solution process: with a small step in a rapid change in the functions and larger - with slow. Bulstoer function is recommended in case the system solutions are sufficiently smooth and continuously changing. Under this condition Bulstoer function allows to get more accurate decisions than `rkfixed`, spending less time.

## MATHEMATICAL MODELLING

Let's consider the mathematical solutions of problems arising in the study of the dynamics of the train carriage and the train on the example of the wheelset. The motion equations of this system are nonlinear. Fluctuations of skidding and wobbles are as follows [1-3]:

$$\begin{cases} m\ddot{y} + \frac{2f_{11}}{v}(\dot{y} - v\psi) + \frac{2f_{12}}{v}\dot{\psi} - \frac{2f_{11}}{r_0}\Delta_2(y) + W_A\Delta_L(y) + k_y y + c_y \dot{y} = F_y(t), \\ I_\omega \ddot{\psi} + \frac{2a^2}{v}f_{33}\dot{\psi} + \frac{2af_{33}}{r_0}\left(\frac{r_L - r_R}{2}\right) + \frac{2f_{22}}{v}\dot{\psi} - \frac{2f_{12}}{v}(\dot{y} - v\psi) - \\ - \frac{2f_{22}}{r_0}\Delta_1(y) - a\psi W_A\delta_0 + k_\psi \psi + c_\psi \dot{\psi} = F_\psi(t), \end{cases}$$

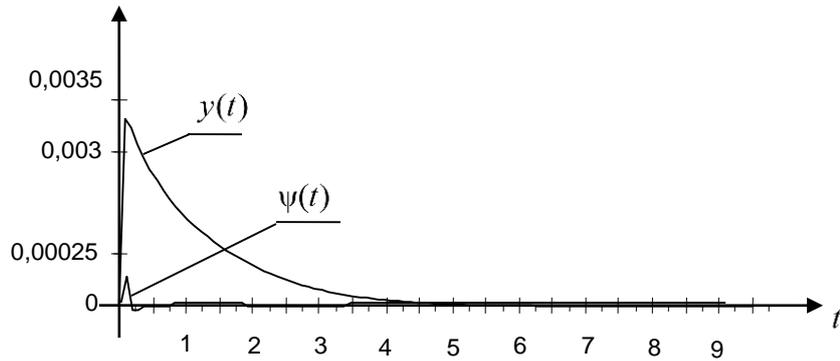
Where  $m = 536 \text{ kg} \cdot \text{s}^2 / \text{m}$  - the mass of the "axis - wheelset" system;  $I_\omega = 1865 \text{ N} \cdot \text{ms}^2$  - the moment of inertia of the wheelset relative to the axis  $z$ ;  $a = 762 \text{ mm}$  -

the distance of contact point of wheel with rail to the  $z$ -axis;  $r_0 = 508 \text{ mm}$  - the radius of the wheel;  $f_{11} = 1,6 \cdot 10^7 \text{ N}$  - the coefficient of longitudinal creep of the rail;  $f_{12} = 0,52 \cdot 10^5 \text{ N} \cdot \text{m}$  - the coefficient of combination of transverse and rotary creeps;  $f_{22} = 189 \text{ N} \cdot \text{m}^2$  - the coefficient of rotary creep;  $f_{33} = 1,7 \cdot 10^7 \text{ N}$  - the coefficient of transverse creep;  $W_A = 250000 \text{ N}$  - the axle load;  $k_y = 875984 \text{ N/m}$  - the coefficient of transverse stiffness;  $c_y = 17520 \text{ N} \cdot \text{s/m}$  - the coefficient of transverse damping coefficient;  $k_\psi = 21,14 \cdot 10^6 \text{ N} \cdot \text{m/rad}$  - the stiffness coefficient of wobble fluctuations;  $c_\psi = 3527 \text{ N} \cdot \text{ms/rad}$  - the damping coefficient of wobble fluctuations;  $v = 20 \text{ m/s}$  - the velocity of the wheelset axis;  $\delta_0 = 0,05^\circ$  - the conic angle;  $F_y(t)$  - the transverse force;  $F_\psi(t)$  - the perturbing moment, causing wobble.

The equations were solved numerically using the built-in functions MathCAD [4-7]: `rkfixed`, `Rkadapt` and `Bulstoer`, calculations were carried out with the same precision. Then, graphics of parameters  $y$  and  $\psi$  were printed.

1. The calculation of dependence of lateral movement  $y$  and  $\psi$  rotation angle from the time when the wheelset wobbling, received by using `rkfixed(y, 0, 10, 10000)` function, producing calculations according to the method of Ruge-Kutta of 4th order, where  $y = (0, 0, 1, 1)^T$  - the vector of initial conditions;  $t_0 = 0$  - the starting point for the variable;  $t_1 = 10$  - the final point of the calculation;  $n = 10000$  - the number of fixed points, where an approximate solution is sought;  $F(t, y)$  - the vector-function containing the right-hand parts of the equations.

The calculation results of dependence of lateral movement  $y$  and  $\psi$  rotation angle from the time when the wheelset wobbling, are shown in Fig. 1.



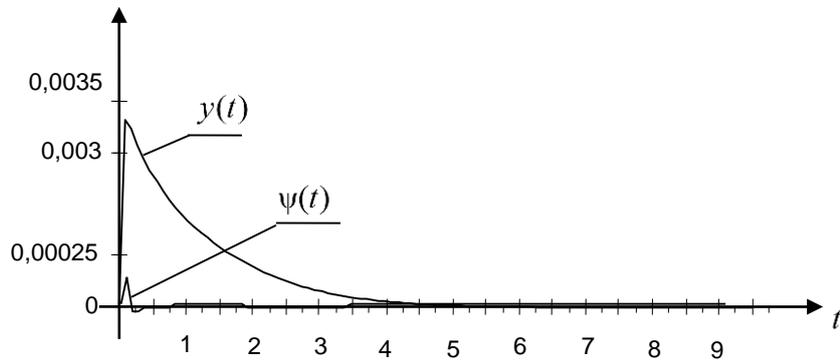
**Figure 1:** Dependences of lateral movement  $y$  and  $\Psi$  rotation angle from the time when the wheelset wobbling, obtained by the function rkfixed

2. The calculation of dependence of lateral movement  $y$  and  $\Psi$  rotation angle from the time when the wheelset wobbling, received by using `Rkadapt(y, 0, 10, 100)` function, producing calculations according to the method of Ruge-Kutta of 4th order, performing adaptive control of the solution process : with a small step in a rapid change in the functions and larger - with slow. Function parameters defined above.

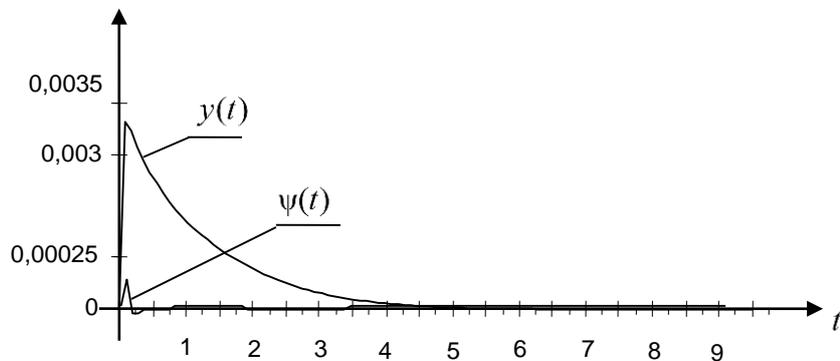
The calculation results of dependence of lateral movement  $y$  and  $\Psi$  rotation angle from the time when the wheelset wobbling, are shown in Figure 2.

3. The calculation of dependence of lateral movement  $y$  and  $\Psi$  rotation angle from the time when the wheelset wobbling, received by using `Bulstoer(y, 0, 10, 100)` function, implementing the method of Bulirsch-Stoer. Function parameters defined above.

The calculation results are shown in Figure 3.



**Figure 2:** Dependences of lateral movement  $y$  and  $\Psi$  rotation angle from the time when the wheelset wobbling, obtained by the function Rkadapt



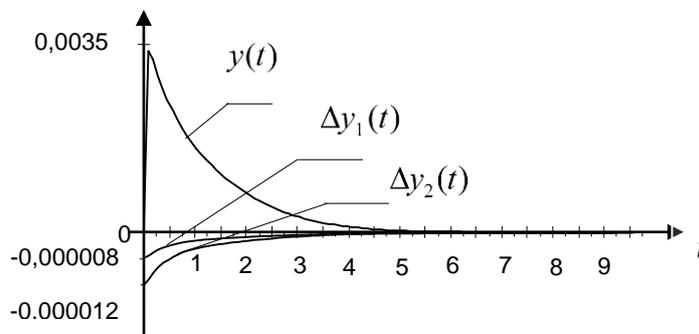
**Figure 3:** Dependences of lateral movement  $y$  and  $\Psi$  rotation angle from the time when the wheelset wobbling, obtained by the function Bulstoer

The number of steps it took to find a solution with the help of embedded rkfixed ( $n_1 = 10000$ ) function, proved the greatest, than the function Rkadapt ( $n_2 = 100$ ), implementing adaptive control of equations solving process according to the method of Ruge-Kutta of 4th order and Bulstoer ( $n_3 = 100$ ) function, generating calculations according to the method of Bulirsch-Stoer.

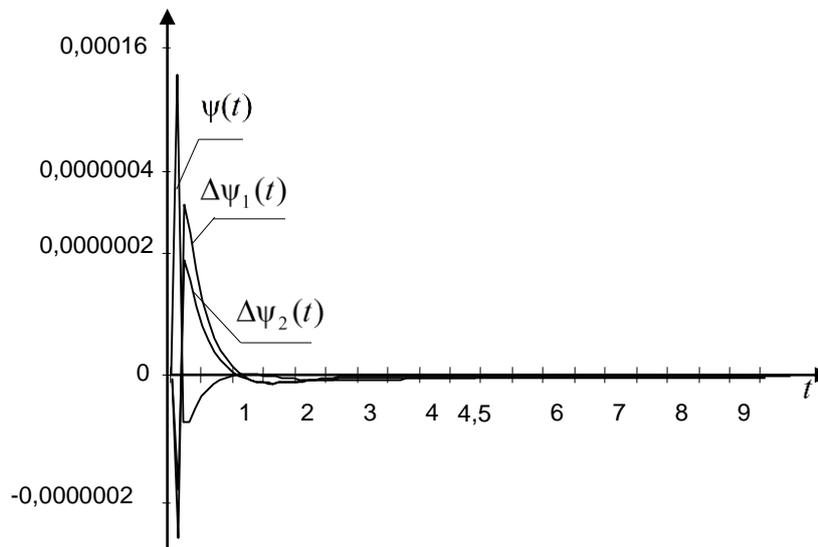
To characterize the accuracy of the calculations let's find the absolute error of lateral displacement  $\Delta y_1(t) = y(t) - y(t)_1$  - by Rkadapt method and  $\Delta y_2(t) = y(t) - y(t)_2$  - by Bulstoer method. The calculation results are shown in Figure 4.

The absolute error of the rotation angle calculations  $\Delta \psi_1(t) = \psi(t) - \psi_1(t)$  - by Rkadapt method,  $\Delta \psi_2(t) = \psi(t) - \psi_2(t)$  - by Bulstoer method. The calculation results are shown in Figure 5.

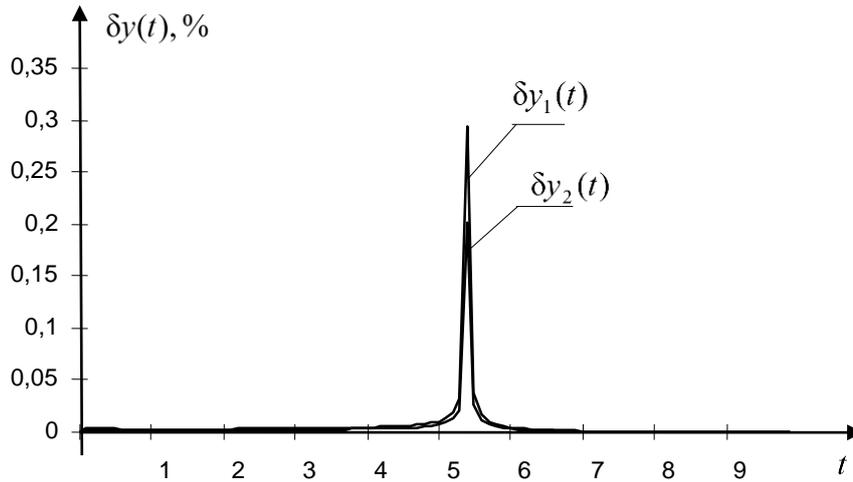
The relative error in the calculations of lateral displacement  $\delta y_1(t) = \left| \frac{\delta y(t) - \delta y_1(t)}{\delta y(t)} \right| \cdot 100\%$  - by Rkadapt method,  $\delta y_2(t) = \left| \frac{\delta y(t) - \delta y_2(t)}{\delta y(t)} \right| \cdot 100\%$  - by Bulstoer method. The calculation results are shown in Figure 6.



**Figure 4:** The absolute error of lateral movement calculations.

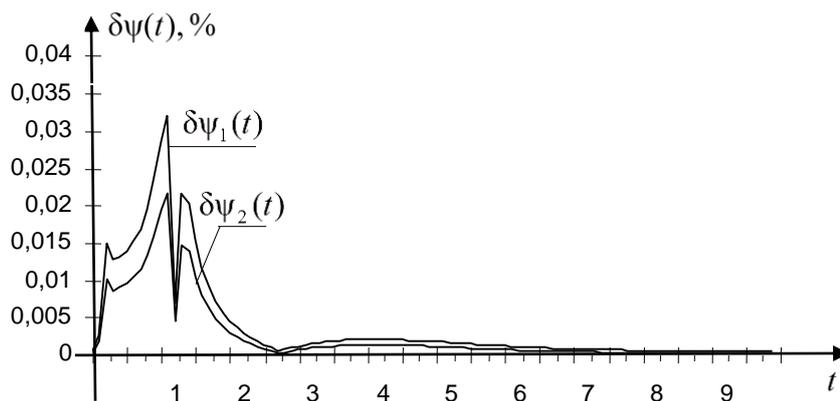


**Figure 5:** The absolute error of the rotation angle calculations.



**Figure 6:** The relative error in the calculations of lateral displacement.

The relative error of the rotation angle calculations - method Rkadapt, - method Bulstoer. The calculation results are shown in Figure 7.



**Figure 7:** The relative error of the rotation angle calculations

## CONCLUSION

Thus, the absolute error of lateral movement calculations calculated by Rkadapt method slightly smaller than by Bulstoer method, whereas the absolute error of the rotation angle calculations is the reverse situation. Proceeding from the above errors calculation results, it can be judged about the fact that the solutions begin to stabilize after six seconds. The number of integration steps of motion equations of the wheelset by Rkfixed about 100 times more than the methods of Rkadapt and Bulstoer with the same accuracy of the calculations, therefore it is not recommended to use Rkfixed method for solving such problems.

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