Performance of Image Similarity Measures under Burst Noise with Incomplete Reference

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Abstract

A comprehensive study on the performance of image similarity techniques for face recognition is presented in this work. Adverse conditions on the reference image are considered in this work for the practical importance of face recognition under non-ideal conditions of noise and/or incomplete image information. This study presents results from experiments on the effect of burst noise has on images and their structural similarity when transmitted through communication channels. Also addressed in this work is the effect incomplete images have on structural similarity including the effect of intensive burst noise on the missing parts of the image. The AT&T face image database was used in this work which consisted of images with dimensions 92x112 pixels and 256 grey levels per pixel. To quantify the error and evaluate system performance the Structural Similarity Index Measure (SSIM), Feature Similarity Index Measure (FSIM) and the Sjhcorr2 algorithms are considered. Peak signal to noise ratio (PSNR), window length in pixel and maximum burst length in pixel were also used in this test.

INTRODUCTION

Image discrimination has become an interesting subject over the past decade due to its implementation in many fields such as safety, identity authentication and video monitoring. Different ways for image discrimination, especially for Face recognition, have been proposed [1, 2] and many algorithms for recognition have recently been proposed based on similarity measures between two images [3].

Similarity techniques can be classified into three categories: statistical techniques, information theory techniques and hybrid techniques [4]. In statistical approaches the similarity can be defined as the variance between statistical features of the two images being compared. An easy way to measure the similarity between two images is to calculate the Mean-Squared Error (MSE) but this simple method performs poorly in the application of facial recognition [5, 6]. Two objective measures that address some of the issues associated with MSE are the Structure Similarity Index Measure which was proposed by Wang and Bovik [3, 7] and the Feature Similarity Index (FSIM) which was proposed in [8]. In information theory approaches the similarity can be defined as the variance between information-theoretic characteristics in the two images [9]. The Sjhcorr2 method is a hybrid measure based on both information-theory based features as well as statistical features used for assessing the similarity among images [10].

Several factors affect the security and accuracy of data transmitted through communications systems over physical channels, one major issue which will be specifically examine in this work is burst noise which affects the reliability and rate in which the data can be transmitted [12].

This paper is organized in the following manner: Section 2 introduces similarity techniques and discusses the difference between the different methods, Section 3 looks at burst noise and the effects this can have in a communication channel, Section 4 will examine the statistical models used in the experiments presented in this work, Section 5 shows the experimental results and discussion on the outcomes obtained, Section 6: will conclude the paper and propose new directions for this work to head.

WELL-KNOWN SIMILARITY TECHNIQUES

There are two main approaches for measuring image similarity: statistical methods including the Structural Similarity Measure and the Feature Similarity Measure, and information – theory based methods including the Symmetric joint histogram method and Hybrid methods including Sjhcorr2 measure.

A Structural Similarity Measure

The measure proposed by Wang and Bovik. (2004) which was called SSIM, used a distance function to measure the similarity of two images based on statistical features, Equation 1 shows this measure:

\[
Ssim(a, b) = \frac{(2\mu_a+\alpha)(2\mu_b+\alpha)}{(\mu_a^2+\alpha^2+\alpha)(\mu_b^2+\alpha^2+\alpha)} \frac{(2\sigma_{ab}+\beta)}{(\sigma_a^2+\beta^2+\beta)(\sigma_b^2+\beta^2+\beta)}
\]

(1)
where $\mu_a$ and $\mu_b$ represents the means and $\sigma_a^2$ and $\sigma_b^2$ represents the variance of $a$ and $b$; $\sigma_{ab}$ is the covariance of $a$ and $b$, and $c_0$ and $c_2$ are constants inserted to avoid division by zero, and are defined as $c_0 = (T_1 L)^2$ and $c_2 = (T_2 L)^2$. $T_1$ and $T_2$ are small constants and $L$ is the maximum pixel value [3, 7].

### A Feature Similarity Index Measure

A Feature Similarity Index Measure (FSIM) proposed for complete RIQA depended on the fact that HVS realizes an image fundamentally according to its low-level characteristics. The phase congruency (PC) is used as the primary characteristic and the gradient magnitude (GM) is utilized as the secondary characteristic. The phase congruency and the gradient magnitude play roles in distinguishing the IQ [8]. The computation of the FSIM consists of two stages, the first stage is to compute the local similarity, $S_L$, defined as:

$$ S_L(x) = [S_P(X)]^p + [S_GM(X)]^p $$  \hspace{1cm} (2)

where $S_P$ represents the PC similarity and $S_GM$ represents the GM similarity. The second stage is to compute the FSIM between $F_1(x)$ and $F_2(x)$:

$$ \text{FSIM} = \frac{\sum_{x,y} S_L(x,y) \cdot \text{PC}_{\text{max}}(x,y)}{\sum_{x,y} \text{PC}_{\text{max}}(x,y)} $$  \hspace{1cm} (3)

where $\text{PC}_{\text{max}}(x)$ is the maximum between $\text{PC}_1(x)$ and $\text{PC}_2(x)$ [11].

### The Symmetric Joint Histogram - 2D Correlation (Sjhcorr2) Measure

Sjhcorr2 is a hybrid measure based on information-theory based features and statistical features used for assessing the similarity among greyscale images, proposed in [10]. The Sjhcorr2 measure again consists of two parts, the first part being the information theory part:

$$ R(x,y) = \frac{\sum_i \sum_j (H_{ij} - \bar{H})(T_{ij} - \bar{T})^2}{2\hat{\sigma}^2} $$  \hspace{1cm} (4)

where $R(x,y) \geq 0$, $H_{ij}$ is the symmetric joint histogram of image $x$ and image $y$, while we have $T_{ij} = H(x,y)$ as the self-symmetric joint histogram of the first image $x$. The above value can be normalized as follows:

$$ S(x,y) = \frac{R(x,y)}{R_{\text{max}}(x,y)} $$  \hspace{1cm} (5)

The final version of the first part can be stated as follows:

$$ r(x,y) = 1 - S(x,y) $$  \hspace{1cm} (6)

The second part of the measure is represented by 2-D correlation between the reference image $x$ and the noisy image $y$ and is given as follows:

$$ p(x,y) = \text{corr2}(x,y) = \frac{\sum_i \sum_j (x(i,j) - \bar{x})(y(i,j) - \bar{y})}{\sqrt{\sum_i \sum_j (x(i,j) - \bar{x})^2 \sum_i \sum_j (y(i,j) - \bar{y})^2}} $$  \hspace{1cm} (7)

where $\bar{x}$ and $\bar{y}$ are the mean values of $x$ and $y$ respectively. The effect of information-theoretic features could be incorporated with the effect of correlative features as:

$$ \text{SjhCorr2}(x,y) = K_1 q(x,y) + K_2 p(x,y) $$  \hspace{1cm} (8)

where $K_1 + K_2 = 1$, $0 \leq \text{SjhCorr2}(x,y) \leq 1$.

### BURST NOISE

Burst noise is a type of internal electronic noise (undesirable electrical energy) that is produced inside a communication system or at a receiver. It occurs because of imperfections in semiconductors material and heavy ion implants. Burst noise consists of sudden, step-like transitions between two or more current levels, to as high as several hundred microvolts at random and unpredictable times and is known to last for several milliseconds. Low burst noise is achieved by using clean device processing, and therefore is beyond the control of the designer [12]. The effect burst noise has on images is represented by a strings of pixel errors, each with random length and random gaps between bursts (distribution of burst length and waiting times is Uniform, Poisson or Rayleigh). Each string of pixel errors can be represented as a pixel value of 0's.

### THE STATISTICAL MODELS OF BURST NOISE

In signal processing systems, the integrity and quality of systems can be realized by understanding the statistical characteristics of the noise processes associated with the system. Considering the case of burst noise, there are several different statistical models shown below that can be used to model this type of noise [13]:

#### Uniformly-Distributed Model

The probability density function (pdf) of the uniform distribution is [14]:

$$ p_U(x) = \begin{cases} 1 / (B - A) & A \leq x \leq B \\ 0 & \text{elsewhere} \end{cases} $$  \hspace{1cm} (9)

A random variable $R$ on a symmetric interval $[-A,A]$ can be generated using a random variable $U$ (uniformly distributed) as follows:

$$ R = -A + 2A \cdot U $$  \hspace{1cm} (10)

#### Poisson -Distributed Model

The probability of $M$ (Poisson random variable) events in an interval is given by the equation [15]:

$$ P(M) = e^{-\gamma} \frac{\gamma^M}{M!} $$  \hspace{1cm} (11)

where $\gamma$ is the average number of events per interval, $M$ takes the values of 0, 1, 2..., while $e$ is Euler’s number, and...
\( M! = M \times (M - 1) \times (M - 2) \times \ldots \times 2 \times 1 \) is the factorial of \( M \).

**Rayleigh-Distributed Model**

If \( x \geq 0, A = b^2, b \) is the Rayleigh parameter (real positive parameter) and \( p \) is power of Rayleigh noise \( p = \mathbb{E}(X^2) \) (determined power of noise in dB) then the probability density function (pdf) of the Rayleigh distribution is [16]:

\[
p(x) = \frac{x}{A} e^{-\frac{x^2}{2A}}
\]  
(12)

\[
b = \sqrt{\frac{p}{2}}
\]  
(13)

**RESULTS AND DISCUSSION**

The following types of noise have been considered in the simulations and testing performed: Gaussian noise, impulsive noise and multiplicative noise. To test the performance of the system SSIM, Sjhcorr2 and FSIM has been implemented in MATLAB. Better results could be obtained using local analysis [18, 19] or hybrid analysis [20].

Performance of Similarity vs. Missing Part Window Length:

The performance of the similarity measures are tested using different window lengths, this window length is defined as the missing area of image, for example a window length of 10 pixels will result in a missing area of 10*10 pixels. Whenever the window length was increased, note that Sjhcorr2 measure, follow it FSIM withstand more than SSIM. Fig 1 shows the test images and the performance of the similarity measures vs. window length.

Performance Similarity vs. Maximum Burst Length and PSNR

Maximum Burst Length (MBL) represents the largest possible length of the error string, in the experiments presented in this work the length of the error bursts were less than or equal to this maximum burst length value with the length selected at random. There are three cases for waiting time when the burst length is uniform (or Poisson or Rayleigh). It can be observed that a Sjhcorr2 measure, followed by FSIM, gives a similarity despite maximum burst length being large or a low PSNRdB (amount of similarity differs for each case of burst length).

![Figure 1](image_url)

**Figure 1:** Similarity versus missing window length for two poses of the same person. The information-theoretic measure gives higher similarity.
Waiting Time (WT) is Uniform:
Performance of the similarity measures are tested according to maximum burst length and PSNRdB results are shown in Figure 2.

Waiting Time is Poisson:
Performance of the similarity measures are tested according to maximum burst length and PSNRdB results are shown in Figure 2.

Waiting Time is Rayleigh:
Performance of the similarity measures are tested according to maximum burst length and PSNRdB results are shown in Figure 2.

Figure 2: Performance of similarity measures when the test image is affected by burst error (both burst length and waiting time are uniform). Right: Performance vs. max burst length. Left: Performance versus PSNR (dB).

Figure 3: Performance of similarity measures when the test image is affected by burst error (both burst length and waiting time are Poisson). Right: Performance vs. max burst length. Left: Performance versus PSNR (dB).

Figure 4: Performance of similarity measures when the test image is affected by burst error (both burst length and waiting time are Rayleigh). Right: Performance vs. max burst length. Left: Performance versus PSNR (dB).
Performance of Similarity under Gaussian, Impulse and Multiplicative Noise for Incomplete Images:

Another test was performed for incomplete images under Gaussian noise which is one of the most common noise types encountered in signal processing and communication systems, multiplicative (speckle) noise which causes severe damage to signals and systems due to its dangerous effects on all content of a signal, and impulsive (salt & pepper) noise which is common in image processing. Fig 5 shows the test images and the performance of SSIM, FSIM and Sjhcorr2 under these noise conditions.

Similarity vs. Persons (Face Recognition)

The incomplete face image (with a window length of 20) was compared with the face images in the database (The same snapshot that has been compared is complete present in the database). Note that the facial image was distinguished by the three measures (a). But when the length of the window increased to 70 the Sjhcorr2 scale could not distinguish the image and gave the largest similarity with person 10 (b). The incomplete face image (Gaussian noise with PSNRdB = 10.1487) was compared with the database, note that the facial image was distinguished by the three measures (c) but when Impulsive noise with PSNRdB = 5.7728 the Sjhcorr2 scale and FSIM scale could not distinguish the image (FSIM gave the largest similarity with person 37, sjhcorr2 gave the largest similarity with person 4 (d)).

![Performance of similarity measures for an incomplete (window length=60) reference image with noise (Gaussian, impulse and multiplicative). Observe that Sjhcorr2 measure can detect similarity even at low PSNR.](image)
Figure 6: Similarity (under noise and incomplete information) vs. persons
The incomplete face image with maximum burst length of 7 was compared with the database (The tick that has been compared does not exist in the database), note that the facial image was distinguished by the three measures (where (a) had a burst length that was uniform and a waiting time which was also uniform, (b) had a burst length that was uniform and a waiting time that was Poisson distributed and (c) had a burst length that was uniform and a waiting time that was Rayleigh distributed).
Table 1 shows the rate of incomplete images being recognized using the similarity techniques and the error rate according to window length under noise and under the effect of burst noise. These images were taken from the AT&T Face Database containing 40 test images and 40 training images.

The proportion of images correctly and incorrectly recognized were calculated as ratios using the following equations:

\[
\text{ratio of recognition} = \frac{\text{number of correct recognition attempts}}{\text{total number of attempts}} \times 100 \% \tag{14}
\]

\[
\text{ratio of error} = \frac{\text{number of failures in recognition}}{\text{total number}} \times 100 \% \tag{15}
\]
<table>
<thead>
<tr>
<th>Measure</th>
<th>Window length</th>
<th>PSNR</th>
<th>MBL</th>
<th>SSIM Ratio of success</th>
<th>SSIM Ratio of failure</th>
<th>FSIM Ratio of success</th>
<th>FSIM Ratio of failure</th>
<th>Sjhcorr2 Ratio of success</th>
<th>Sjhcorr2 Ratio of failure</th>
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<tr>
<td>Measure</td>
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<td>PSNR</td>
<td>MBL</td>
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<td>15%</td>
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<tr>
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<td>2.5%</td>
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CONCLUSIONS

Through experiments conducted on face images from the AT&T database it can be concluded that:

A. Similarity between two images:
   1. The FSIM measure and sjhcorr2 measure can withstand the most despite increases in the length of the window and Maximum Burst Length.
   2. Although the two images differ, similarity measures (FSIM and Sjhcorr2) give a high similarity.
   3. When the noise is high, FSIM gives a similarity of approximately 20%, followed by Sjhcorr2.
   4. In the burst noise, study a narrow spectrum of PSNRdB (not up to a very low PSNR).

B. Recognition of incomplete face image:
   1. The best measure is structure similarity, the rate of recognition was found to be 90% if the length of the window is large and 97.5% if the length of the window is small.
   2. In the case of very high noise, the rate of recognition for all measurements is less than 50% which increases with less noise up to 90% if the window length is large and 97.5% if the window length is small.
   3. In the case of burst noise, the scale FSIM failed in all cases of burst length and waiting time and for all values of MBL. A scale of SSIM and Sjhcorr2 gave recognition rates of above 50% for all values of maximum burst length.

4. Similarity measures withstand more when the distribution of the length of the burst is uniformly distributed and the distribution of the waiting time was Poisson.

REFERENCES


