Development and Calculation-experimental Analysis of Pressure Pulsations and Dynamic Forces Occurrence Models in the Expansion Joints of Pipelines with Fluid

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Abstract

This article is a continuation of papers [1-3], concerning expansion-joints vibration deformation in pipelines with a fluid. Physical models and analytical dependences which describe the occurrence of pressure pulsations and dynamic forces determining vibration transmission through expansion joints in a wide frequency range up to hundreds of hertz have been developed and experimentally verified. It has been shown that the pressure pulsation and dynamic forces arising in the expansion joint are connected with a quadratic dependence on the oscillation frequency and may grow because of the increasing frequency by three-four orders of magnitude over a wide frequency range as compared to low frequencies. Vibration transfer through an expansion joint is characterized by its transfer vibration rigidity which also increases as square of the disturbance frequency which may serve as a diagnostic criterion of such rigidity model in an expansion joint during its experimental research. The suggestions on the direction of further research to reduce pressure pulsations, dynamic forces and vibration transmission through expansion joints over a wide frequency range have been made.

Keywords: pipeline, expansion joint, vibration, vibration rigidity, dynamic force, vibration frequency, working fluid, pressure pulsation.

DESIGN FEATURES OF NON-SPACING (PRESSURE-BALANCED) PIPELINE EXPANSION JOINTS.

The necessity to study the physical models of vibration transmission through expansion joints of pipelines with liquid have been shown in papers [1-3]. Vibration transmission through pipelines in some cases is described in [1-7]. Vibration insulation inserts (expansion joints) in the pipelines are used for its decrease. Vibration isolation properties of expansion joints are characterized by the magnitude of the transfer vibration rigidity $C(f)$: the smaller it is, the better vibration isolation is. It is defined as the ratio of the dynamic (vibration) force $Q(f)$ at the output of the expansion joint to the vibration amplitude $A(f)$ at its input at a frequency $f$ [4]:

$$C(f) = \frac{Q(f)}{A(f)} \quad (1)$$

In the expression (1), all quantities are complex. Modulus of stiffness is useful for comparison. With the vibration frequency growth for almost all types of expansion joints there is a significant increase of $C(f)$ to the static rigidity [1-4].

Possible ways of vibration and dynamic forces transmission through expansion joints in pipelines with fluid are:

a) transmission through the expansion joint structure and its resilient elements;

b) transmission through pressure pulsations $P_H$ (Figure 1) already existing at the inlet of the expansion joint (e.g., by an operating pump) affecting the expansion joint and pipeline walls behind him;

c) the occurrence of dynamic forces from the internal static pressure $P_0$ of the expansion joint (Figure 1a) due to changes in areas of the resilient elements (RE), which the pressure effects at the vibration deformation of the RE (see Figure 1c);

g) the occurrence of pressure pulsations $P_k$ (Figure 1a) in the expansion joint itself caused by its vibration deformation as a part of the pipeline. Below we consider the physical models for the case g).
VIBRATION DEFORMATION MODELS OF THE EXPANSION JOINTS WITH WATER

The deformation schemes analysis of various types of pressure-balanced expansion joints in Figure 1 and in the paper [1] shows that their deformation is accompanied by a flow of working fluid between the local internal volumes without changing the total volume of the expansion joint. The more movable liquid mass and the higher its acceleration is, the greater the pressure pulsations generating this process are. Effecting the inner surfaces of the expansion joint, the pressure pulsations create dynamic forces transmitted to the pipeline and the foundation.

Internal configuration of the pressure-balanced expansion joints structure is rather complicated. Presented the calculation and experimental analysis of the fluctuation occurrence in these expansion joints is aimed at identifying the visual, simply calculated and structurally controlled physical models of the pulsation occurrence and obtaining the calculation dependencies to assess the significance of each of these models in each case. It is necessary while developing measures to reduce pressure pulsations and dynamic forces transmitted by expansion joints.

In the published research papers there are no data to develop such models and their calculation schemes, to determine the reasons for significant growth of expansion joints vibration rigidity of pipelines with water with growing frequency. Only the dependence of experimental transient vibration rigidity of expansion joints on the frequency is stated in the paper [4] as well as the methods for determination of experimental transient vibration rigidity of expansion joints. In accordance with the paper [4] a complex matrix describing the transfer of forces and pulsations through expansion joints has the
dimensions 13×13. With the elements of such a matrix having been experimentally defined, it is unclear which of them and how should be influenced to reduce the transmission of vibrations from the expansion joint at a given frequency. It is necessary to develop physically visual models describing the appearance of the working fluid pressure pulsations and dynamic forces transmitted by the expansion joints during their vibration deformation. The analysis of the vibration deformation of the expansion joints structures allowed suggest the following models of the pressure pulsations occurrence in their cavities during vibration displacement of nozzles for calculation and experimental studies.

Model 1, Figure 2. Pressure pulsation is determined only by expansion joint (or pipeline) vibrations as a rigid body moving with the fluid.

Model 2, Figure 3. Pressure pulsation is defined by local changes of volume in separate expansion joint sections or cavities (bellows gofers, rubber-cord casing cavities) during its vibration deformation. Model 2 with a "long piston" vibrating with the acceleration is depicted in Figures 3 a, b. It enables one to distinguish two excitation models. Model 2 is a flow of fluid along the piston (Figure 3).

Model 3 "piston under the fixed cap" – fluid displacement from the gap between the piston and the plug (Figure 4).

Figure 2: Model 1 of combined expansion joint (pipeline) and the fluid oscillations.

Figure 3: Model 2 occurrence of pressure pulsations in the expansion joint with a "long piston". a) overflow scheme, b) calculation scheme

Figure 4: Model 3 occurrence of pressure pulsations in the expansion joint with the "piston under the fixed cap". a) overflow scheme, b) calculation scheme
CALCULATION OF PRESSURE PULSATIONS AND DYNAMIC FORCES IN EXPANSION JOINTS

For models 1,2,3 at Figures 2-4, the distribution of pressure pulsations transmitted to the foundation (fixed part of the expansion joint), dynamic (vibration forces), transitional vibration rigidity of the expansion joint (1) depending on the design parameters, vibration frequency and amplitude under the following assumptions are calculated.

In operating conditions the forces acting from the pump on the inlet (movable) nozzle of the expansion joint (Figure 1) are unknown. The kinematic excitation of the input nozzle of the expansion joint is considered, the output nozzle is assumed to be stationary. Maximum permissible value of vibration speed of the power equipment at the general level in the frequency range from 10 to 1000 Hz in operation is limited to 11 mm/s for safety reasons (normal operation up to 4 mm/s). This corresponds to the vibration amplitude A = 0.2 mm at a frequency of 10 Hz. Considering that the vibration amplitude decreases with increasing frequency and the characteristic expansion joints gaps are in the magnitude of more than 1 mm, the maximum vibration amplitude is considerably less than the characteristic gaps and characteristic linear dimensions of the expansion joint, which allows solving the problem in a linear setting. Consequently, the viscosity can be neglected, because at a rate up to 11 mm/s Reynolds number and friction forces will be small.

The walls of the expansion joint (pipeline) are absolutely rigid. Resilient elements (goffers of rubber-cord shells, bellows) provide predetermined displacements according to the given schemes, but not extensible: with the static pressure p0 changing inside the expansion joint there is no change in the volume of the cavities formed by them. Static and vibration rigidity of the resilient elements in the direction of displacement can also be neglected (assuming it to be zero), assuming that the transmitted forces are defined only by pulsations (structural rigidity is absent).

Model 1 in Figure 4. The excitation of pressure pulsations in the working medium plugged at both ends of the pipe at its harmonic vibrations along the X axis according to the law

\[ A = A_0 \sin(\omega t). \]

Considering the pipe section shown in Figure 4 to be a quadripole, neglecting attenuation in the pipe, but we can write in matrix form [8,9]:

\[
\begin{pmatrix}
P_1 \\
Q_1
\end{pmatrix} = \begin{pmatrix}
\cos(kH) & i\frac{\rho c}{S} \sin(kH) \\
i\frac{S}{\rho c} \sin(kH) & \cos(kH)
\end{pmatrix} \times \begin{pmatrix}
P_2 \\
Q_2
\end{pmatrix}.
\]

where \( k = 2\pi f/c \) the wave number, \( c \) – speed of sound in water, \( f \) – frequency of fluctuations, \( P_1 \) and \( P_2 \) are pressure pulsations, \( Q_1 \) and \( Q_2 \) outflow (volume velocity) at the upper and lower covers, \( S \) – area of the pipe, \( \omega = 2\pi f \) – circular frequency, \( H \) is the height of the pipe, \( \rho \) - fluid density.

Substituting in expression (2) for the outflow

\[ V_1 = V_2 = i \omega S A \]

we get

\[ P_1 = \rho c \omega A \left( \frac{\cos(kH) - 1}{\sin(kH)} \right) = -P_2 \]

Having written, by analogy with (2) the equation for \( P(X) \) and \( Q(X) \) we get, considering conditions (3) and (4) the distribution of pressure pulsations along the pipe

\[ P(X) = \rho c \omega A \left( \frac{\cos(kX) - \cos(kH)}{\sin(kH)} - \sin(kX) \right) \]

The characteristic dimensions of the expansion joint and the cross-section of the pipeline do not exceed 1 meter. The speed of sound in water is 1200-1500 m/s. Wavelength \( \lambda = \frac{c}{f}, \) where \( c \) is the speed of sound, \( f \) - frequency. For wavelengths of more than 2 meters (less than half the wavelength at the length of the expansion joint), the upper frequency is 600 Hz. For the frequency range up to 200 Hz, which is sufficient for practical purposes, wave phenomena can be neglected in the expansion joint medium with an error of less than 7% (see Figure 5). Then for small values of \( kH \) (for expansion joint \( H \) sizes to be less than 1 meter and the frequency \( f \) to be less than 200 Hz) approximate expressions for the module of ratio \( P_1/a, \) force \( Q_\alpha \) and vibration transition rigidity \( C(f) \) of the fluid column with height \( H \) will be simple and graphic:

\[ P_1/a = \rho H/2 \]
\[ Q_\alpha/a = 2\rho IS = \rho HS \]
\[ C(f) = Q_\alpha/A = \rho H \omega^2 S \]

\[ f, \text{ Hz} \]

Figure 5: The error while calculating pulsations with the simplified formula (6) for the pipe 1 meter long with water
Let us calculate the pressure pulsations in the working medium of a pipe with a piston immersed in the working medium, to which a harmonic force \( F = P_0 \sin(\omega t) \) is applied. The calculation scheme for the pipe is shown in Figure 3. Accept the water mass to be \( M_v \) in the gap between the piston and the pipe wall.

\[
\begin{align*}
M_v &= \rho_v (S - S_p) \left( H_p + 2 \cdot 0.44 \sqrt{S - S_p} \right),
\end{align*}
\]

where \( S \) and \( S_p \) – the area of the pipe and the piston respectively; \( \rho_v \) - density of water; 2-0.44 are the coefficients of the attached mass (adopted by analogy with coefficients for the Helmholtz resonator [10]). \( M_p \) is piston mass with the added mass of water

\[
M_p = \left( \rho_p H_p + \rho_v \cdot 2 \cdot 0.44 \sqrt{S - S_p} \right) S_p.
\]

The equations of the masses movement can be written as a

\[
-\nu M_p X_p + (P_0 - P_0') S_p - i \omega \eta_p X_p + F = 0. \quad (9)
\]

\[
-\nu M_v X_v + (P_0 - P_0') S_v - i \omega \eta_v X_v = 0. \quad (10)
\]

The equations of quadripoles of the upper and lower portions of the pipe:

\[
\begin{align*}
\begin{pmatrix} P0' \\ Q0' \end{pmatrix} &= \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \times \begin{pmatrix} P1' \\ Q1' \end{pmatrix}, \\
\begin{pmatrix} P0 \\ Q0 \end{pmatrix} &= \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \times \begin{pmatrix} P1 \\ Q1 \end{pmatrix}.
\end{align*}
\]

Equation of continuity at the inlet of the upper and lower sections of the pipe:

\[
\begin{align*}
-Q0' + i \omega X_p S_p + i \omega X_v S_v &= 0, \quad (13) \\
-Q0 - i \omega X_p S_p - i \omega X_v S_v &= 0 \quad (14)
\end{align*}
\]

and setting acoustic loads at the boundary of the upper and lower sections of the pipe we get:

\[
\begin{align*}
Z_h &= \frac{P1'}{Q1'}, \quad (15) \\
Z_t &= \frac{P1}{Q1}. \quad (16)
\end{align*}
\]

Substituting (15) and (16) to (11) and (12) we find the acoustic load at the inlet of the upper and lower portions of the tubes (above and below the piston respectively)

\[
\begin{align*}
P0' &= Z_{vx} \times \begin{pmatrix} A_2 Z_{zu} + B_2 \\ C_2 Z_{zu} + D_2 \end{pmatrix}, \quad (17) \\
P0 &= Z_{vx} \times \begin{pmatrix} A_1 Z_{zu} + B_1 \\ C_1 Z_{zu} + D_1 \end{pmatrix} \quad (18)
\end{align*}
\]

From the continuity equations (13) and (14) we get:

\[
\begin{align*}
Q0' &= i \omega X_p S_p + i \omega X_v S_v = 0, \quad (19) \\
Q0 &= -i \omega X_p S_p - i \omega X_v S_v = 0. \quad (20)
\end{align*}
\]

Substituting (19) and (20) in (17) and (18) we get the expression for \( P0' \) and \( P0 \)

\[
\begin{align*}
P0' &= i \omega X_p S_p Z_{vx}' + i \omega X_v S_v Z_{vx}', \quad (21) \\
P0 &= -i \omega X_p S_p Z_{vx} - i \omega X_v S_v Z_{vx}. \quad (22)
\end{align*}
\]

Subtracting (22) from (21) we obtain

\[
\begin{align*}
P0 - P0' &= -i \omega X_v S_v (Z_{vx} + Z_{vx}') - i \omega X_v S_v (Z_{vx} + Z_{vx}'). \quad (23)
\end{align*}
\]

Substituting (23) into the equations of motion (9) and (10) and calculating the resulting system of equations, we get the amplitude of piston and water oscillations \( X_p \) and \( X_v \) which occur in the gap between the piston and the pipe wall. Using the calculated amplitudes \( X_p \) and \( X_v \), one can calculate pressures \( P0' \), \( P0 \) from formulas (21) and (22) and the volume velocities \( Q0' \), \( Q0 \) from formulas (19) and (20). Pressure \( P1 \) and a volume velocity \( Q1 \) at the boundary of the lower section can be calculated from the inverse matrix

\[
\begin{align*}
P1 &= \frac{P0}{Q1} \\
\begin{pmatrix} P1 \\ Q1 \end{pmatrix} &= \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}^{-1} \times \begin{pmatrix} P0 \\ Q0 \end{pmatrix}.
\end{align*}
\]

The pressure \( P_x \) and a volume velocity \( Q_x \) at a random point of the lower portion of the pipe are calculated with a matrix equation, taking into account that, according to (16) \( Q1 = \frac{P1}{Z_h} \)

\[
\begin{align*}
P_x &= A_x B_x \times \begin{pmatrix} P1 \\ Q1 \end{pmatrix}, \\
Q_x &= C_x D_x \times \begin{pmatrix} P1 \\ Q1 \end{pmatrix}.
\end{align*}
\]

Dynamic influence force on the foundation is calculated as the product of pulsation \( P1 \) to the area of the pipe \( S \). For small values of \( kL \) analogously to the formulas (6-8) approximate expressions for the dependence modules \( P1/a \) and vibration transition rigidity \( C(f) \) for the piston height \( H \) will be

\[
\begin{align*}
P1/a &= \rho H S_p / 2(S - S_p), \quad (24) \\
C(f) &= Q_1/A = \omega^2 \rho H S_p / (S - S_p). \quad (25)
\end{align*}
\]

Model 3. "piston under the fixed cap". The excitation of pressure pulsations in the working fluid in the small gap by the vibrating mechanical element in the form of a piston, by extruding working fluid from the gap \( h \). We shall calculate the pressure pulsations in the fluid in the small gap \( h \) under the piston immersed in the working fluid, to which a harmonic force is applied \( F = F_0 \sin(\omega t) \). Calculation scheme is shown in Figure 4. The equation of continuity of fluid

\[
-x_n S_n - x_n S_n = 0; \quad (26)
\]

mass movement equation of fluid \( M_n \) in the gap between the piston and the pipe

\[
-M_n a_n + P1 S_n = 0. \quad (27)
\]

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The equation of the fluid concentric mass movement under the piston in the gap $h$ may be written as an equation for the hydrodynamics of an ideal fluid

$$\rho a_r = -\text{grad}(P),$$

(28)

where

$$\text{grad}(P) = \frac{dP}{dr} r,$$

(29)

continuity equation of fluid with velocity $V_a$ under the piston with velocity $V_p$ at a radius $r$

$$2\pi r h V_a = -\pi r^2 V_p.$$

(30)

From equations (26) and (27), we get

$$P_l = -\frac{S_e}{S_a} M_a a_n.$$

(31)

From equations (28) and (29), we get

$$\rho a_r dr = -dP.$$

(32)

From equation (30), we get acceleration $a_n$

$$a_n = -\frac{r}{2h} a_n$$

(33)

Substituting (33) to (32) we get

$$\rho \frac{r}{2h} a_n dr = dP.$$

(34)

Having integrated the resulting equation and determined the constant of integration from the conditions that, in accordance with (31) when $r = R_a$ pressure

$$P = P_l = -\frac{S_e}{S_a} M_a a_n,$$

we get the pressure distribution in the gap below the piston

$$P = -\left(\frac{S_e}{S_a^2} M_a + \rho \frac{R_a^2 - r^2}{4h}\right) a_n.$$

(35)

The pressure force $Q$ effecting the foundation is equal to

$$Q = -P_l S_a - \int_0^{\frac{L}{2}} P 2\pi r dr =$$

$$= \left[\frac{S_e}{S_a}\left(1 + \frac{S_e}{S_a}\right) M_a + \rho \frac{\pi R_a^4}{8h}\right] a_n.$$

(36)

The first term in (36) is determined by the inertia of the fluid in the gap between the piston and the wall of the pipe, the second the displacement of fluid from the gap $h$ under the piston. The contribution of the second term in the force amplitude with small gaps $h$ can be large enough and according to hyperbola it is increased with decreasing gap. The vibration rigidity $C(f)$ of the fluid, given that $a_n = \omega^2 X_a$ is equal to

$$C_d = \left|\frac{Q}{X_a}\right| = \frac{S_e}{S_a} \left(1 + \frac{S_e}{S_a}\right) M_a + \rho \frac{\pi R_a^4}{8h} \omega^2$$

(37)

**CALCULATION RESULTS AND COMPARISON WITH THE EXPERIMENT**

Experimental verification of the formulas (2-8) for model 1 in the frequency range of 0-200 Hz and (9-25) for model 2 in the frequency range 0-1600 Hz was conducted in the water-filled pipe with length $H$ of 1 meter. Model 1 is the pipe suspended elastically with a frequency less than 1 Hz, model 2 is fixedly mounted on the force sensor produced by the firm PCB. The pipe vibration was excited by wide-band white noise with an vibrator. We measured the dependence of pressure pulsations in the water on an acceleration of $a = \omega^2$ of nozzles or a piston. Pressure pulsations were measured by hydrophones placed at several points along the pipe, the acceleration pulsations were measured by accelerometers. Signal processing and calculation of vibration rigidity, as the ratio of force to the displacement of the piston, were conducted with a multichannel analyzer «Pulse» produced by the firm "Brüel & Kjær."

Figure 6 for model 1 shows a comparison of the calculated dependences on the frequency ratio $P_x/a$ at different distances from the pipe bottom. Calculation was made for the water density $\rho = 998 \text{ kg/m}^3$ and the experimentally measured speed of sound in the pipe water $c = 1250 \text{ m/sec}$. A good correspondence of calculated and measured data indicates their reliability.

![Figure 6: Comparison of calculated and experimental values $P(x)/a$ at a frequency of 60 Hz at different $L_x$](image_url)

Figure 6 shows for model 1 a comparison of the calculated and experimental values $P(x)/a$ at a frequency of 60 Hz at different $L_x$.
experimental spectra of pressure pulsations. Good correspondence between the calculated and measured data indicates both the accuracy of calculation formulas and measurement results.

Figure 7: Comparison of calculated and measured pressure pulsations spectra at a distance X = 0.875 mm from the bottom of the pipe upon pipe excitation with the immersed piston.

For model 3 experiment and calculations were conducted in the frequency range of 0-1600 Hz in the pipe diameter Drp = 0.08 m, filled with water to a height H=0.069m. The piston diameter Dn = 0.05 meters, height h_n = 0.005 meters. The vibration rigidity was measured in the experiment. Figure 8 shows a comparison of the calculated and experimental spectra of vibration rigidity. The good correspondence indicates the accuracy of the calculation formulas and the experimental results.

Figure 8: Comparison of calculated and measured dynamic rigidity with the piston 50 mm in a 80 mm pipe, h = 1 mm, H_b = 68 mm.

If all three models work simultaneously, using the formulas (8), (25) and (37) we get an expression for the transition vibration stiffness

$$C(f) = \frac{Q}{A} = \omega^2 \rho k_{geom},$$  \hspace{1cm} (38)

where k_{geom} – the factor determined only by the geometry of the expansion joint. Expression (38) shows that the transfer vibration rigidity C(f) the pressure pulsation caused in a compensator vibration due to its deformation, directly proportional to the fluid density and the square of the frequency of deformation. The latter may be a diagnostic sign of the prevalence of just such a model of transition vibration rigidity.

CONCLUSIONS AND RECOMMENDATIONS.

Physical models and analytic relationships describing the occurrence of pressure pulsations and dynamic forces in pipeline expansion joints have been developed that determine the transmission of vibration through them in a wide frequency range up to hundreds of hertz. The validity of formulas obtained has been confirmed by the good correspondence between calculation and experiments carried out.

It is shown that transition vibration rigidity of an expansion joint, caused by pressure pulsations, is connected with the expansion joint and its elements vibration frequency by the quadratic dependence. With increasing frequency, rigidity can grow three - four orders over a wide frequency range in comparison with low frequencies, which may be a diagnostic criterion of such expansion joint rigidity model in its experimental study.

The further trend in the research of vibration transmission through the fluid expansion joints should be research of the influence of resilient elements structure compliance and their volumetric compliance, as well as methods of reducing the expansion joint vibration rigidity using passive and active
methods of pressure pulsation and dynamic forces reduction.

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