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Structural Resonance Methods for Image Processing and Pattern Recognition

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Abstract

We have developed group-theoretical *methods of structural resonance* by analogy with "resonance methods" applied in the classical theory of oscillations and waves and in quantum physics. The "structural-resonance approach" is one of the advantageous interpretations of the *reconstructive computerized diagnostics* based on the previously developed group-theoretical statistical approach to solve ill-posed inverse problems. We have elaborated a unified method of reconstructing a “heterogenous” test-object with wide *semantic spectrum*, regarding this object as a “semantic mixture” of different mutually complementary *semantic contents*, matching each of them with its formalized *invariant structure*. To separate a cleared semantic content from the “mixture”, it is necessary to apply operators from its *group of automorphisms* to the informational image of the “mixture”. It results in randomization of all other semantic contents, so they can be easily suppressed statistically. We have defined measures of intensity for such resonance, carried out a detailed comparison of tomosynthesis and such structural-resonance “sense synthesis”, and compared “material reconstruction” and reconstruction of *structural-functional connections*. Finally yet importantly, we have discussed prospects of these methods for solving ill-posed problems in various fields of science and practice.

Keywords: Inverse problem, ill-posed problem, nonlinear backprojection, image processing, pattern recognition, spatial filtering, reconstructive computerized diagnostics, structural resonance

INTRODUCTION

Image processing and reconstructive algorithms

Nowadays, image processing (IP) is an integral component of almost any information technology (IT). IP application in science, technology and everyday life is practically unbounde [1], [2]. IP includes the sections important for practice such as pattern recognition (“image recognition”) [3], [4], [5], morphological analysis of images [6], visualization of objects in their evolution [7], analysis of spatial structures, [8], [9], [10], [11], remote sensing [12], [13], [14], processing of deformed images [15], [16] medical diagnostics, etc. There are no distinct borders between these sections. What is more, successful methods can easily overcome the border between the sections. It concerns, for example, the techniques of wavelet transformation [17]. One of the most significant factors in IP development is a strong need for methods of object reconstruction from incomplete data [18].

IP methods are most widely applied in nondestructive testing (NDT). Computing tomography (CT) turned to be crucial for both NDT and IP. For the first time, adequate methods of visualization of internal structure of the test-object have been developed for the NDT program purposes. The relationship between IP and CT was so close, that some experts interpreted CT as “a new perspective IP method”.

Specialists in NDT realized the essence of an *ill-posed inverse problem* in CT and importance of “regularization” for problem solution [19], [20], [21], [22]. However, they realized that regularization (in its classical meaning) of the CT problem can be employed only with a relatively great number of images that is atypical for NDT. Although the regularized problem is “intuitively close” to the initial one, it is still a different problem. We consider that much fine information concerning *structural-functional connections* in the test-object is irreversibly lost when solving the problem.

In NDT, conventional methods of CT are frequently not adequate for radiation testing. One of the examples is testing thin gaps between solid fuel and an engine body of the rocket. Another example is visualization of zones of crack formation in components of machines. These situations are practically important and widespread not only in NDT. A user (for example, a NDT specialist) needs *semantic information* about the situation with minimum semantic noise. A system of reconstruction and recognition is simply obliged to provide a user with information.

A typical test-object is the "mixture" of mutually contradictory "senses", more exactly "semantic contents" with different incompatible "logics". It is necessary to separate a pragmatically valuable "semantic content" from this "mixture" and to suppress other ("parasitic" ones) "semantic contents" in reconstruction. The "choice of purpose" is free, i.e. another "semantic content" can be referred to as a "pragmatically valuable" one. For example, visualization can be used for ferro-concrete reinforcement or granulation texture. There are different *aspects of reconstruction*.

Therefore, we claim that a reconstructive ill-posed inverse problem can have several stable solutions (if virtual "semantic contents" are really exist in the "semantic mixture"). The solution (which is not unique) is stable because an *invariant structure* is stable by nature. Nevertheless, classical Hadamard's condition of stability needs modification. It is necessary to formulate it anew from a group-theoretical statistical point of view.

During reconstruction, it is necessary to arrange precautions against dangerous "apriory information", which can contaminate the priceless initial data and, thus, ruin weak traces of "semantic contents" in the mixture. This cannot be applied to the *reconstructive computerized diagnostics* (RCD) elaborated by us since its procedures are based on testing the group-theoretical statistical hypotheses [23], [24], [25]. Reconstruction can be "material", i.e. expressed in the form of scalar field of some "material characteristic", namely "coefficient of linear attenuation of X-ray radiation" or "structural-functional". The results are represented as visualization of *structural-functional connections* in the test-object. This technique is much more informative and has much more possibilities than "material" one.

ESSENTIALS OF RECONSTRUCTIVE COMPUTERIZED DIAGNOSTICS

In [24], [25], we developed the ideology and unified methods of RCD based on the group-theoretical principles. The basis of RCD mathematical models is rapprochement between group theory and methods of mathematical statistics. As a result, we reconsidered the existing approaches to solving inverse ill-posed problems so that a number of the most difficult problems of this class can be solved. Initially, we created RCD for the needs of NDT and medical diagnostics (MD), but later the scope of the RCD applications spread far beyond these limits.

Group theory can be successfully applied in practical RCD due to achievements in abstract mathematics in the nineteenth and early twentieth centuries. The first non-Euclidean geometries [26], in particular "parabolic" geometry of Lobachevsky and "elliptical" geometry of Riemann, triggered other "new geometries" and, as a consequence, the need for their classification.

In 1872, F. Klein [27] in his "Das Erlanger Programm" created the most successful classification (a group-theoretical one). He posed the following problem: "There is a manifold and a transformation group defined in it; it is required to find the structures that belong to this manifold, whose properties do not change during transformations from a given group" [8]. One more problem (research area): "There is a manifold and a set of transformations defined in it. A theory of invariants corresponding to this group is being developed" [27].

These ideas are conceptually attractive when implementing any systematic classification and can be used as a basis for solution unification of a wide range of problems. We performed this to solve ill-posed reconstructive problems within a group-theoretical approach.

Group theory deals with a mathematical language adequate for describing entities, and as a result, it is a recognized means of representing different "semantic contents" with formalized "invariant structures", since "semantic content" is an integrity. That is why A. Poincaré started to use group theoretical methods to study such objects as "laws of nature" (chain of cause-effect relations) at the end of the 19th century. By the middle of the 20th century, group theory had become an irreplaceable tool not only for formalization, but also for conceptualization of various "contents" in theoretical physics. (In the problem-solving area of RCD, "semantic content" is an aspect of reconstruction).

In the 60', the leading scientists established the "symmetry" concept as philosophical category [28]. In the postwar period, many scientists used group-theoretical methodology extensively not only in physics, but also in other natural sciences and even in some humanitarian research [28].

Unfortunately, the situation was not favorable to the convergence of the group theoretical methods and information sciences. The matter is a perfect information science (as well as any of its particular aspects, for example, "theoretical diagnostics") which is inconceivable without the organic unity of its deterministic and probabilistic sides, whereas the group theory aims to describe stable ("stationary") systems. It is far from the concepts of "unpredictability" and "probability", which are found in the genesis of information. This does not mean that the systems described on the basis of group theory do not undergo changes. All of them are just at the level of *substratum* of the system. This horizon of events is indifferent to the group theory that describes an invariant *structure* of the system.

"Diagnostics" is nonsense if it is referred to a completely predictable object. In this case, it does not produce any information. Therefore, the creation of diagnostic systems entirely on the heritage of geometry is impossible. Its rapprochement with mathematical statistics on a group-theoretical basis is inevitable. "Symmetry" and "statistics" have a common genesis (remember the history of probability theory). In spite of the fact that the concepts "lived long time in different corners of mathematics", they were not completely isolated.

The development of contemporary information science requires means of formalization and computer processing of various “semantic contents”. The currently used "information theories" (for example, that elaborated by Shannon, which is a slight modification of Boltzmann's considerations about entropy) are completely indifferent to "meanings", “senses” or “ideas” [29], [30]. It is reasonable, because the main problem of such theories is transferring "impersonal" information through communication channels. Treating "subjectively colored" information would greatly hamper the development of technical means for communication in society.

However, computerization of "meaning processing" is a pressing problem. The contemporary development of "intelligent information technologies" (IIT) requires the problem solution urgently. It is also necessary to bring the IIT "internal language" closer to the natural human language. In this regard, the conceptualizing potential of the group theory [31] is unprecedented. Nevertheless, the procedure of representing "human meanings" with their formalized invariant structures always requires human participation.

Among shortcomings of the mathematical apparatus of the group theory, we note the lack of tools suitable for computer-aided solution of inverse problems (without which reconstructive diagnostics is impossible). On the contrary, in mathematical statistics [32], such "active elements" (various procedures for "statistical recognition", for testing statistical hypotheses, for generating statistical estimates, etc.) exist, which is an important prerequisite for developing group-theoretical statistical algorithms with different designation. Note, that such "computational muscles" are necessary to make reliable decisions using small statistical samples. In statistical physics, the situation is different, since "samplings" are huge, so it is "almost deterministic" discipline. Despite the above-mentioned effectiveness of classical mathematical statistics, it has essential (often even fatal) drawbacks. IP requires suppression of various noises. Suppressing "ordinary" stochastic noise is relatively easy. This problem is solvable with the use of arsenal of traditional mathematical statistics together with standard IP techniques, whereas the "struggle" against semantic noise ("parasitic signals") is extremely difficult. The arisen problem is the *formalized separation of semantic contents*.

The ideology of mathematical statistics itself is not sufficient to solve this problem. Group-theoretical ideology provides an opportunity to highlight additional aspects of this problem and thereby enables essential assistance to the statistics. We have accomplished this in the framework of RCD. Among new statistical tools, we have most developed methods for testing *group-theoretical statistical hypotheses* and methods for generating "nonclassical statistics" to enter these as nonlinear statistical estimates into statistical reconstructive procedures.

The most important difference between RCD and other "diagnostics" is that it does not work at the "substratum level", but at the level of entities. However, there exists also an important difference from other "structural methods". In order to obtain valuable diagnostic results, it is necessary to force structures to interact and evaluate the results of the interaction, i.e. this is not a static, but a dynamic level of the structural approach. When interacting, structures can change some of their invariants and a group of automorphisms. It causes a *decrease in the symmetry* of the structure. We have developed a variety of mathematical methods to estimate the reduction of symmetry. Interactions of structures can arise in real experiments. Of course, their modelling and calculation in RCD procedures are also possible and pertinent, especially in *group-theoretical pattern recognition*. In this case, the archetype (“pattern” in recognition procedure) is a group of automorphisms.

The systematic convergence of group-theoretical and statistical methods is primarily due to the need for adequate quantitative estimation of the results of "interaction of structures". (Traditionally, the representation of groups by linear operators was the main group-theoretical tool used for different applications, mainly in theoretical physics).

The present study is focused on the results obtained when developing RCD methods, which can be interpreted in terms of "structural resonance".

The concept of *resonance* has been traditionally used in general scientific issues. Nevertheless, resonance embraces different phenomena; therefore, its definitions by specialists, working in different fields of science and its applications, differ in "semantic nuances", and sometimes contradict each other. Similar variation can be observed in the *system approach* ("system analysis"). Despite the century of its existence, the fruitfulness of its methods and outstanding achievements, no generally accepted definition of the concept of a *system* exists. Dozens of contradictory definitions are available in literature. The only thing that allows professionals to understand each other is the integrity of the system, i.e. it is impossible to explain its properties based on the properties of its elements.

The general scientific definition of resonance also implies "anti-reductionism." This concept should omit what belongs to the system "substratum" and include what characterizes the system as an entity. It is also important that "resonance" is the main way for interaction between entities ("invariant structures"). However, to refine this concept, we will consider some RCD methods.

METHODS

Componentwise group-theoretical reconstructive methods

Among the componentwise group-theoretical reconstructive methods, the method of group-theoretical spatial filtering in the framework of "geometry in small" with various local transformation groups is the most frequently used by us in NDT practice.

In the beginning of the twentieth century, mathematicians and physicist-theoreticians have successfully developed the principles stated by Klein in the Erlangen program. The results published by E. Cartan during the period 1922–1925 were especially interesting [33]. There was a particular stimulus for suchlike research. For the past half-century, certain limitations in Klein's approach to the classification of manifolds have been clarified. (Some of the Riemannian spaces have only a trivial group of motions). To overcome this problem, E. Cartan developed the concept of a space in which group-theoretical transformations (according to Riemann) are authentic only locally, i.e. in infinitesimal area.

Thus, Cartan united the ideas of Riemann, Klein and Lie, who developed the theory of continuous groups. He modified G-space of Klein (i.e. the group of transformation G that operates on the set M) and entered such concept as *geometry of the group G*.

We reproduce the basic positions achieved in this synthesis in no detail. Suppose that operators from the local group of transformations G acts on the manifold M. In other words, it is the set of local mappings of this manifold to itself and to neighboring areas. In doing so, operators of mapping satisfy the axioms of the group theory. Then we shall speak that some geometric image α in M is equivalent to the geometric image β if the group G has an operator to transform α to β. The whole system of possible statements about these properties of geometric images (and quantities) that are invariant relative to all transformations of the group G is called the *geometry of the group G*. Naturally, we consider geometric images "in small" for a local group G.

These results are a valuable prerequisite for componentwise RCD within the framework of mathematical models of "geometry in small", more precisely the "geometry of the group G".

It should be noted that our RCD procedures are numerical algorithms. In this respect, they do not differ, for example, from numerical solutions of differential or integral equations. All infinitesimal operators act “in small”, but “in finite small”, and this is the basis for computing mathematics. We construct the componentwise methods “in finite small” that requires establishment of the appropriate parameters (for example, a step of digitization and sizes of sliding window, probably, multidimensional) to control computational process. This adaptation of parameters to the actual test-object is characteristic for statistical reconstructive procedures. Also, there can be parameters of statistical estimations, which are definitely reasonable only “in finite small”.

These common peculiarities are routine for group-theoretical statistical procedures. The choice of size of sliding window is important for group-theoretical componentwise methods. “Hopeless” degradation of the initial data for a test-object is not rare. As a rule, it is impossible to reconstruct fine topological properties of the object. Nevertheless, “algebraic properties” are more stable. The protective invariant structure that implicitly present in degraded data enable their reconstruction using statistical methods. However, estimation of the sliding window size is an inevitable stage in reconstructive procedure.

We have already described many times the procedure of transition from “infinitesimal” to “finite small” status in componentwise reconstructive algorithms (with replacement of the “key” Lie group by its finite subgroup, etc.).

Let us state an outline of the method. We consider a local group G in the manifold μ and intend to test a null hypothesis: “The geometry of the manifold μ at a point with coordinates $\xi_1, \xi_2, \ldots, \xi_A$ is the geometry of the group G“. This hypothesis is not a statistical one. It would be convenient to have a special tool for its testing – a functional “Φ ($\xi_1, \xi_2, \ldots, \xi$), so that Φ =0 if the hypothesis is valid and $\Phi > 0$ if not valid. It is not difficult. Φ ($\xi_1, \xi_2, \ldots, \xi$) would be used as non-negative measure of local dissymmetry. It is also possible, but it is more natural to apply for this aim the apparatus of mathematical statistics. In this case, it is necessary to define Φ in “finite small”. There are many specific statistical estimates for that. (In our further discourse, we ignore this fact and consider only Φ with all essential characteristics of suchlike measures. It is not dangerous, since in any application we can simply replace it by one of specific measures).

Φ is a "measure of difference" between hypothetical and real symmetries. We have designed it to evaluate the local dissymmetry determined during this study. It is also permissible to interpret Φ as a measure of the structural-resonance response of a local microimage to the influence of the structural factor with the symmetry of group G. In this case, this is a measure of "antiresonance".

If Φ=0 on the entire manifold (i.e., when the null hypothesis is valid "everywhere"), then the information image is degenerate (Malevich's "black square"). If some structural has violated the local symmetry, then the inequality $\Phi > 0$ must be satisfied. Simultaneous presence of both invariant and noninvariant (when the transformations from G apply to different local microimages) properties of the object in the "secondary image" Φ ($\xi_1, \xi_2, \ldots, \xi$) is required for it to be a meaningful message. In other words, a meaningful message is inconceivable without a background or "norm" with $\Phi = 0$ and "anomalies" with $\Phi > 0$.

The measure Φ immediately introduces aspects that are foreign to "pure geometry", to the theoretical model, since the manifold μ is "unpredictable". (It is not, for example, a "manifold with constant curvature" or a "manifold with constant variation of curvature"). A manifold μ with a hypothetical group *G* and measure Φ is neither a "Kleinian *G*-space" nor a "geometry of the Cartan group *G*." It would be so for the trivial condition $\Phi \equiv 0$ in its entire area of definition in μ.

This new mathematical object manifests essentially informational aspects, i.e. μ is the source and carrier of information, which is possible to reveal when testing hypotheses. We can say, that the "manifold μ with the group *G* and the measure Φ" comprises a "text" together with the key for its decoding. The decoded text is the "secondary image" Φ ($\xi_1, \xi_2, \ldots, \xi$). A manifold μ may include a number of "texts", which are "decoded" by different keys *G*. Essentially statistical aspects of μ inevitably follow informational aspects. In this case, we replace image Φ by the field of (specific and detailed) statistical estimates ("in finite small"). In this case, we replace Lie groups *G* by their finite groups to construct explicit statistical estimates.

The hypothesis for each element of the "secondary image" Φ ($\xi_1, \xi_2, \ldots, \xi_\Lambda$) is tested in the corresponding Euclidean tangent space ("in small"). Initially, there are no assumptions about the "connectivity" between these spaces; therefore, the result of testing is unpredictable. The real group (with which the hypothetical group *G* interacts) changes during the transition from one element of the image to another and, in general, each "Euclidean in small" space has its own real geometry of Cartan's type.

"Hard" implementation of testing of Klein's conditions, i.e. the requirement for strict adherence to the criterion Φ=0, is ineffective in practice. (In this case, the structures selected from μ based on the group *G* fall into the "set of measure null"). "Blurring" of the condition Φ=0 is appropriate and takes place with the use of statistical estimates. Of course, it is not typical for "pure geometry".

Semantic spectrum of the object of study

The group theory is a perfect tool for structural analysis and structural classification of objects; however, its possibilities are limited when the investigated test-object is noisy or when it represents a "semantic mixture" of various *semantic structures* described by the groups of automorphisms, and this is the most typical situation the researcher faces in any field of science and practice.

If an object contains many virtual "semantic contents", it is said to have a wide *semantic spectrum*. "Semantic content" is *unity of ratio and intuition*. Therefore, computer-aided processing of this object is not possible, although processing of its formalized "invariant structure" is possible. The formation of "semantic content" (as well as its understanding) requires *intellectual interpretation*. It is a function of human "synthetic" thinking [24], [25]. Almost synonymous terms such as "sense", "meaning", etc. can be used instead of "semantic content". A complete formalization of these objects is not possible.

To study the object, a researcher examines its *information image*, for example, a series of projections to form tomograms. (Note that projections are typical "semantic mixtures" with information from all layers of the object). A typical goal of the researcher is to isolate some special, (pragmatical) "semantic content" from the semantic spectrum; however, this is daily and routinely done by people for purely practical purposes.

A well-developed scientific theory with a high degree of generality cultivates some very narrow section of the semantic spectrum. Typically, it takes the form of the *theory of invariants* of a group of transformations characterizing this narrow "semantic content". For example, classical electrodynamics is the theory of invariants of the Lorentz-Poincare group, and classical mechanics is the theory of invariants of the "Galileo group", and so on.

Within a particular theory of invariants, the classes of "invariants" and "true" assertions do not coincide. It encompasses all "true" statements as a subset of the class of "invariant" assertions. For example, the set of all assertions that are invariant with respect to transformations from the Lorentz-Poincaré group is not necessarily classical electrodynamics. (There are other theories of invariants of this group). Those assertions that "are not invariant" are known "to be not true," but this does not mean that they are bound to be false. They are found in a zone of uncertainty. Within any theory, there are assertions that cannot be proved or disproved by the tools of this theory. (This goes back to the results by K. Gödel (1931)).

To identify the structural resonance, a certain structural factor with a group of transformations R should affect the "semantic mixture". In this case, the response to the impact of those invariant structures in the "mixture" for which R is a group of automorphisms will be different from that to the impact of the invariant structures for which R is not a group of automorphisms. These are the most general recommendations (group-theoretical ones) to be specified depending on the mechanism of representation of initial information about the "mixture" image.

Any diagnostic procedure has inherent errors of the first and second type. This procedure can involve error of the second type because the structural factor resonantly allocates a significantly broad class of objects for which *R* is a group of automorphisms, and no additional group-theoretical individualization of the target objects is performed. An error of the first type may occur in case the structural factor is chosen incorrectly, so a group of automorphisms for any structure in the "mixture" has only trivial (i.e. the group that contains only the identity operator) intersection with the group *R*.

The last decades have shown the interest of scientists in the establishment of structural invariants when studying classes of phenomena of a different nature. These invariants can be precise or inaccurate, and display "admixture" of the effect of alien factors. Mathematical methods and software tools to adequately work with inaccurate invariants are often required (to perform computer-based semantic filtering) on a regular and unified basis. The RCD methods are considered to be quite adequate for this case.

The examples of precise invariance can be "conservation laws" and "integrals of motion" in mechanics, invariance of such quantities as $\boldsymbol{EH}$ and $\boldsymbol{E^2}$ - $\boldsymbol{H^2}$ with respect to Lorentz transformations (where $\boldsymbol{E}$ and $\boldsymbol{H}$ are the electric and magnetic field intensities), etc. A rather widespread misconception is that inaccurate invariance is the prerogative of macrophysics and, above all, of the physics of continuous media, whereas fundamental microphysics deals exclusively with precise invariants. A great number of counterexamples can be provided. For example, strong interactions are characterized by "isotopic invariance" described by the transformations of the unitary symmetry group *SU* (2). It is disturbed by electromagnetic interaction (about 1%). The unitary symmetry of *SU* (3) is also approximate.

The "resonance of structures" was apparently first discussed by the chemists after the development of the “theory of resonance” by L. Pauling in 1928–1931. It supplements the classical postulates of the theory of the chemical structure and asserts that if for the classical theory admits several equally acceptable structural formulas a given chemical compound, not a separate structural formula, but imposition of these formulas, which Pauling called “resonance of structures”, corresponds to the actual state of the molecules of this compound (i.e. its chemical properties).

In modern science, the term "morphological resonance", which is close in meaning to “structural resonance”, especially after the studies of an English biologist Rupert Sheldrake and his "theory of morphogenetic fields."

The "resonance of structures" was appropriate for the image processing after the appearance of tomographic methods, especially methods of tomosynthesis [34], which deal with the system of superimposed projections.

The concepts "structure" and "resonance" cannot be explained if not invoke the concept "symmetry" [28]. The phenomenon of structural resonance arises from the interaction of the entity with the entity (structure with structure). The structures should be similar.

The proximity (or similarity) of the structures implies that their automorphism groups have a common non-trivial ("non-unit") subgroup. Overtones in the vibration of a string from the theory of oscillations can serve an analogy. During interaction of the overtones, the result will have a period of tone. (This is a special case of the Curie principle of dissymmetrization [24]). When the structures are similar, the parameters of the parametric group that describes them are “slightly different” (for example, similar layers in tomosynthesis). The structures are "close" if they are described by the same group, but differently noisy. For example, the projections for tomosynthesis transferred to the layer with a depth z may be similar. This proximity is also characteristic of a series of X-ray patterns of the same (fixed) object from the X-ray television microscope.

A classical theory of oscillations gives a great number of examples of particular cases of structural resonance. "Structural resonance" in the object typically occurs under the impact of some active structural factor. It is analogous to forced oscillations under the impact of some "driving force" in the oscillating theory.

According to the Curie principle of dissymmetrization [24], the symmetry group of the result of the effect of the factor on the structure is a set-theoretical intersection of symmetry groups of the factor and structure.

When the active factor is affected by the "semantic mixture", substructures with the same symmetry group as the factor itself (or higher) will be in the "privileged position". As to other substructures, they will hinder the resonance. Structural resonance manifests the internal conformity of the whole set of group-theoretical "aspects". It is possible to obtain them due to the factor impact on the substructure.

In order to estimate the degree of resonance "intensity”, it is necessary to form a complete set of results of group-theoretical transformations of the information image. After this, it is possible to calculate internal “consistency” of this set using the appropriate measure of similarity. Emphasize that structural resonance shows the degree of simultaneous mutual similarity of all "aspects".

Taking into account all these considerations, we can formulate the following requirements (*) to “measure of intensity” of structural resonance.

It should be 1) multiple, i.e. based on the system of “internal projections” (group-theoretical transformations of the "mixture" by operators from *R*); 2) "symmetric", i.e. invariant with respect to permutations of its arguments; 3) statistical in nature; 4) nonlinear and almost "multiplicative" (i.e. with a tendency to evaluate the result as a "logical product" of arguments).

Some deviations from "multiplicativity" are still necessary to eliminate the extremes of statistical estimates and make them work under noise conditions. These requirements are satisfied both by an estimate of the variance analysis [2] and by nonlinear averaging of projections in tomosynthesis [34]. (No doubt, it is possible to “invert” the measure of difference into the measure of similarity. The “measures of intensity" can be used both in scalar and in vector forms. Such possibility is especially valuable for componentwise methods.

Structural-resonance image processing

Consider an arbitrary heterogeneous object with a wide semantic spectrum, which is represented by its own information image, for example, a multi-dimensional image $I\ (\xi_1, \xi_2, \ldots, \xi_A)$ in the phase space S_A. Suppose that it consists of N "semantic components", while the n-th "semantic content" (n = 1, 2, ..., N) is described by its invariant structure with the automorphism group R_n.

The image processing is the formation of the image $S_n\ (\xi_1, \xi_2, \ldots, \xi_A)$ that describes the n-th "semantic content" on the basis of the original pattern $I\ (\xi_1, \xi_2, \ldots, \xi_A)$ in accordance with the requirements (*).

Assume that for any group of the pair of these groups with indices i and j the intersection

$R_i \cap R_j$ does not contain any group-theoretical operators except for the unity operator. (The interaction of this pair of strictly deterministic structures generates randomization. It can be used to create a generator of pseudorandom numbers.) Thus, it is not difficult to understand the dangerous proximity between deterministic and probabilistic states of the systems.

Create a structural factor on the group R_n to separate the n-th "semantic component" from the "semantic mixture". Transform $I\ (\xi_1, \xi_2, \ldots \xi_A)$ using the operators from the group R_n. In this case (assume the group is finite), each "semantic component" will have Q_n of the "inner aspects" (Q_n is the order of the group R_n). All these "aspects" for the n-th component will be identical, and their linear averaging will coincide with the contribution of the n-th component to $I\ (\xi_1, \xi_2, \ldots, \xi_A)$. As for any other component, the system of its "internal aspects" will be "randomized", in other words, operators from R_n transform them into pseudo-stochastic noise that can be easily suppressed using conventional methods of mathematical statistics. Thus, linear averaging over the aspects of the "alien" semantic component is highly randomized, not to mention nonlinear averaging. The group-theoretical statistical separation of "semantic structures", whose automorphism groups intersect only on the unity operator, can be almost ideal. In other situations (when $R_i \cap R_j$ comprises other operators together with the unity operator), more complicated measures of similarity or difference are constructed using both classical and non-classical [14], [15] statistics.

The suggested structural-resonance image processing is similar to tomography, but this is in some way "senso-graphy". The information image of the semantically heterogeneous "mixture object" can have a number of virtual, mutually complementary "semantic contents". They are uniformly defined by their formalized "semantic structures" and are a convenient means to control reconstruction, that is, to control structural and "semantic" resonance.

It is important that "semantic contents" in the semantic spectrum are mutually complementary, and therefore theoretical models that describe different "senses" are mutually complementary. They are logically incompatible. The invariants of one model cannot be "derived logically" from the invariants of the other model. Due to this, the "theory of everything" is unrealizable, as there is an additional reality beyond the boundaries of its description.

Thus, it is pertinent to remember the words of I. Prigogine (one of the founders of synergetic) and his collaborator I. Stengers: "The real lesson to be learned from the principle of complementarity, a lesson that can perhaps be transferred to other fields of knowledge, consists in emphasizing the wealth of reality which overflows any single language, any single logical structure. Each language can express only part of reality. For example, no single direction in the performing art and in the musical composition from Bach to Schoenberg does not exhaust all the music." (Non-exact citation, by memory).

The structural-resonance selection of "semantic content" (human or computer) is "focusing" of the content. In computerized technology of RCD, the "optics" for suchlike focusing is group-theoretical methods and models.

Note also the power of statistics focusing of "semantic content". There are no cutting of "alien" contents but reorientation of these to statistical support of basic content. It is one of the RCD advantages. In doing so, the choice of statistical measure of similarity (or diversity) is crucial, and it should be coordinated with the nature of the object under investigation.

The RCD structural methods are promising for the study of a wide range of quite different phenomena. The methods for constructing the primary information image also exhibit a high degree of flexibility. These primary images can be multidimensional. In simple situations, it is a two-dimensional image. Similar to tomosynthesis, reconstruction implies extraction of separate layers from the primary image, but the layers are "semantic". This approach eliminates significant boundaries between tomography and "conventional" image processing. The primary information image can be a series of images, video sequence, time slice in the "stream of events", a set of documents, musical work, medical history, etc.

RESULTS AND DISCUSSION

Examples of structural-resonance image processing

To illustrate the above, consider a simple example of tomosynthesis modeling in its usual coplanar geometry. A test-object (mathematically generated phantom) consisted of eight parallel flat layers equidistantly spaced one above other along the *z* coordinate, where *z* is the distance to the screen-recorder. The object was "irradiated" by eight "radiation sources" located in the corners and in the middle of the square sides in the plane *Z* = const, which was parallel to the recording plane. The second stage of computer simulation was reconstruction of different layers from the generated "shadowgraphs".

It is fruitful (and, even necessary) to consider tomosynthesis from the group-theoretical point of view. Let us pay attention to the following facts. Projections describe the same object. The procedure of measuring the projective data is similar for different projections in tomosynthesis. Projections must be "on equal terms" for any reconstruction procedure. This should be also evident for the results of reconstruction. ("The symmetry of the cause is conserved in the symmetry of consequent effect" [3]). Solution must be invariant with respect to any permutations of the projections; that is, the projections form a permutation group ("symmetrical group"). Any estimate of the solution must satisfy the same requirements for invariance.

Projections are typical "mixtures of semantic contents". Each projection includes information about the object layers. There are *N* functions of two arguments (*x* and *y*), which can be written in the form of functions of three arguments, since *z* = 0 on the registration plane.

$P_1(x, y, 0), P_2(x, y, 0), \dots, P_n(x, y, 0), \dots, P_N(x, y, 0).$

Each projection value corresponds to a beam-sum of the same value, a constant on the beam from the projection point on the screen (*x*, *y*, 0) to the radiation source with the corresponding number. Therefore, there is no difference in notation between projections and beam-sums. The estimation of “backprojection” is possible (in the general case, nonlinear [14], [15]) for the reconstructed tomographic image as a function of *N* beam-sums for all *N* beams passing through the point (*x, y, z*) of the object

$$T(x, y, z) = f(P_1, P_2, \dots, P_n, \dots, P_N) \qquad (1)$$

When reconstructing a separate layer of the object, all other layers are semantic noise (not stochastic noise), and the preliminary estimate (1) is designated to suppress this noise. There are two estimates of this type used for modeling

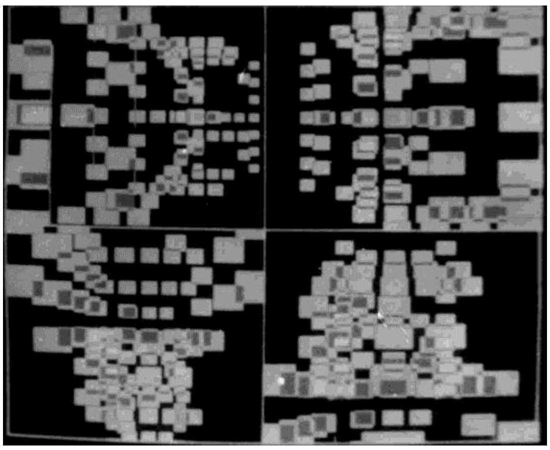
$$T(x, y, z) = (P_1 + P \dots + P_n \dots + P_N)/N \qquad (2)$$

i.e. usual linear averaging and estimation

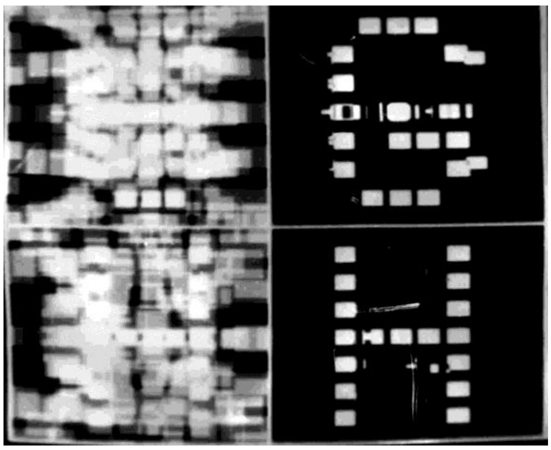
$$T(x, y, z) = min(P_1, P_2, \dots, P_n, \dots, P_N) \qquad (3)$$

or estimation using the "minimum projection method" [34].

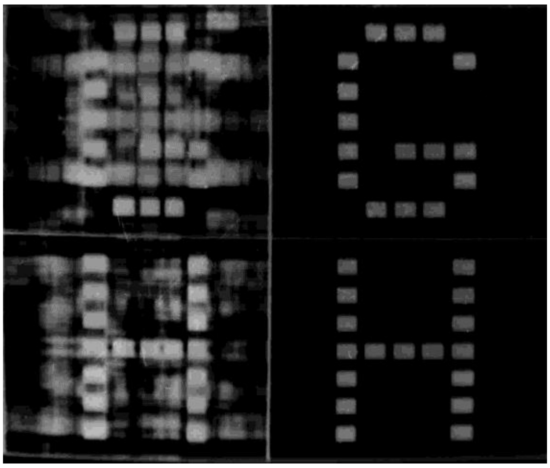
Figure 1 shows the results of numerical modeling for tomosynthesis. The information content of a single layer of eight layers is the image of a letter from the set A, B, C, ..., H. Figure 1a presents four "shadowgraphs" – the results of the test-object "irradiation". Figure 1b shows the results of reconstruction for two layers from four projections: to the left – using the classical method of inverse projections (2), to the right – using the "minimum projection method" (3). Figure 1c presents the results of reconstruction for the same layers from eight projections (without any noise). Figure 1d presents the results of reconstruction for the other two layers of the object from four projections.



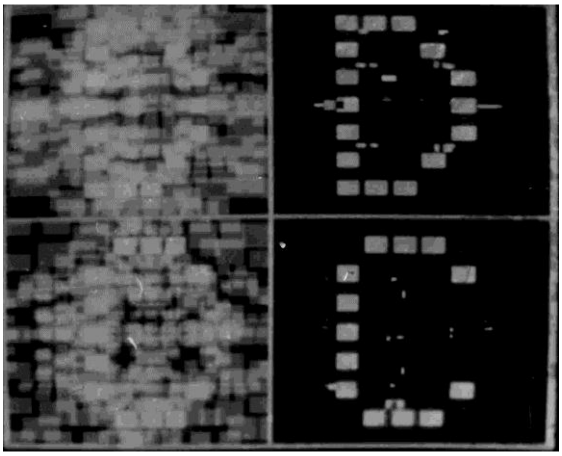
a)



b)



c)



d)

Figure 1: Comparison of backprojection estimations:

Classical estimation (b, c, d – on the left) and "minimum projection method" (b, c, d – on the right)

Separation of "semantic contents" (in this case "layers" or "sections") is difficult. The noise to be eliminated "adapts" to any reconstruction procedure (or estimate) that exhibits imperfections, that is, alien contents "seep" into the content under reconstruction. The results of modeling show that estimates (3) provide a better "breaking of adaptation" than that (2) when the solution (by definition) is non-negative. The method (3) gives an "upper estimate" for the solution. When formulas (1) are used for reconstruction, one can consider z = const as a parameter of the corresponding layer. "Layer-by-layer reconstruction" of the object in tomosynthesis is group-theoretical and parametric. The parameter z is a regulator of the reconstruction. However, "layer-by-layer" reconstruction is not obligatory for reconstruction. In fact, estimates of type (1) evaluate structural resonance response to the entire set of projections.

Estimates (1–3) are "measures of similarity." Estimates (2) and (3) satisfy the paragraphs 1–3 of the requirements (*). Nevertheless, estimates (2) do not completely satisfy paragraph 4, since they are not multiplicative and are more suitable for suppressing noises with normal distribution and zero mathematical expectation, but only in case the number of samples is great. This is a classical estimate of nonlinear backprojection used as a basis in tomosynthesis and in analog tomography. After the advent of computed tomography, it served a base estimate for the development of a huge number of reconstruction procedures with different types of filtering.

The estimate (3) is multiplicative (from the viewpoint of set theory). They work well on "sparse structures", which are often found in nondestructive testing, and convenient for reconstruction from small number of projections. The approximation (3) is a typical estimate of "nonlinear backprojection" [14]. The relationship between the methods of nonlinear backprojection (3) and the methods of "projection onto convex sets" (POCS) [35] we investigated in [36] and showed that estimates (3) are the POCS estimates. Then we developed their various modifications to improve noise immunity. We regularly use these nonlinear procedures (most commonly iterative ones) in nondestructive testing.

In "layer-by-layer variants" of tomosynthesis, it is necessary to carry out conditions of structural resonance in the z layer. Formulas (1–3) represent convenient means for this aim. Construct the "back projection operator» τ $(0, z)$ from the registration plane (z = 0) to the layer z = const. It is a special case of transfer operator τ (z_1, z_2). It carries a set of projections (with proper shifts and scale) from the layer z_1 to the layer z_2. The pure geometric transformation τ (z_1, z_2) need not calculation. Formulas (1–3) contain it "automatically". Operators τ (z_1, z_2) form a transitive group. The projections in the modified (to the layer z) set of projections are simply N variants of contaminated tomograms in the layer with depth z. The cause of the contamination is semantic noise (from N - 1 other layers). According to the requirements (*), these modified projections form a "symmetric" group of transformations in the layer z.

The synthesized tomogram is a response to "internal resonance" incide the set of modified projections. No doubt, the use of more advanced and flexible estimates instead of (2) or (3) will impove the quality of reconstruted tomograms. In particular, it is prospective to use for preprocessing of projections group-theoretical statistical estimates [23], [24], [25].

It is also suitable for 3D reconstruction, but without referring to visual representations of the "layers" of the reconstruction object. Both tomosynthesis (even in its nonlinear variants), and 2D computerized tomography are degenerate variants of tomography. (Degeneration in the system occurs when a certain quantity characteristic of the system has the same value in different states of the system). Degeneration in tomographic systems is primarily due to the limited viewing angle and lack of projections. For example, in 2D tomography all the rays lie in one plane, that is, their projections onto one of the coordinate axes are always zero. Therefore, a set of 2D tomograms is not always equivalent to a "real" 3D reconstruction. Nevertheless, the existing 3D systems, as a rule, are also "degenerate", mainly because of an acute lack of projections.

This circumstance is one of factors, which lead to “ill-posed” problems. Hadamard’s conditions for solution of well-posed problem (i.e 1) existence, 2) uniqueness, and 3) stability of solution) can be not valid. In the famework of RCD is ridiculous to demand uniqueness of solution. On the other hand, the enhanced attention to condition “2” is necessary by separated “directed reconstruction” of any “semantic content”. Many “pseudo-universal” algorithms appear due to implicit doubtful assumptions of developers. Besides, an attempts to “regularize” an ill-posed problem frequently leads to its infection by dangerous “apriory information”. As a result, implicit and hardly perceptible information about “semantic contents” can disappear from the initial data. In the RCD with its group-teoretical statistical approach, such situations are impossible.

When solving practical problems in the area of nondestructive testing (and in many other areas of computerized tomography and image processing), the testing conditions (often "nonstationary") force the use of simplified tomographic systems and compensation of their shortcomings "at the expense of mathematics." This leads to the need for the development of new reconstructive methods to solve “sharply” ill-posed diagnostic problems.

The "semantic mixture” in the object of study can be interpreted either as "purely material" or "structural-functional". The problems of tomosynthesis are solvable in the framework of the group-theoretical statistical approach. Nevertheless, representation of solution is in “material” form. Say, for visualization of “different reconstructed layers" specialists most commonly use such “material background” as “scalar field of the x-rays linear attenuation coefficient”.

However, the possibilities of solving the reconstruction problems at purely material level are limited. Among well-known lacks of this class of methods, we shall note the not overcome difficulties that lead to ill-posed problems, and sharply nonlinear (down to catastrophic) increase of computing expenses by solving of these problems depending on “information volume” (in bits) of object under reconstruction.

In the image processing with local group of transformations [23], we visualized only structurally functional relations in the test-object. The method for estimating the local element in the resulting image is structurally resonance. However, filtering conditions and automorphism groups for “semantic noise” were unknown. Therefore, it is pertinent to define group-theoretical “norm” for the background signal, and then to use a “measure of distinction” for statistical estimation (based on the analysis of variance) of “anomalies” with helpful semantic information. The basic theoretical models and the methods for recognition and visualization of crack formation zones in the components of nuclear power stations are similar [25].

RESULTS AND CONCLUSIONS

A great number of excessively difficult “ill-posed” reconstructive problems in nondestructive are need to be solved. In any case, modern tomography and IP cannot contribute to their solution. This situation is typical of technological development. Felix Klein compared mathematics with a huge Zeughaus (arsenal) that cannot not provide an appropriate weapon (i.e. mathematical method) for solving practical problems.

The successful solutions found for some of these problems within a group-theoretical statistical approach have resulted in the development of reconstructive computerized diagnostics and closely connected “methods of structural resonance for image processing and pattern recognition”. RCD is found in several streams of research. One of them involves group theory, modern geometry, mathematical statistics, methods to solve ill-posed problems and, finally yet importantly, methodology of structural approach to the system analysis. It does not mean that RCD is a conglomerate of diverse methods. This synthesis is organic, and it results in something qualitatively new. It is impossible to reduce the theoretical content to one of the predecessor from synthesized conceptions. Therefore, it can be referred to as a line of research with self-reliant conceptual basis.

RCD has been developing as a *technoscience*. In our opinion, it is relevant to define technoscience as the lines of “fundamental science” (or “basic”, “abstract” science) that aims at the development of new technologies. Two development stages can be distinguished: 1) venture basic research that can or even aims to maintain essentially new technologies, 2) occurrence of a “spectrum” of new studies in fundamental science stimulated directly from further development of new technologies. The indicator of successful development of technoscience (alongside with its direct practical output) is a new *multidisciplinary field*. Technoscience attempts to escape from impasses of a highly specialized science.

The contours of this multidisciplinary field for RCD are well known. It includes both a number of theoretical disciplines related to RCD and potential areas of application. Originally developed on reconstructive problems of nondestructive testing, RCD operated as “mathematical introscopy”. Nondestructive testing was, and remains a kind of “training range" for testing RCD methods.

The flexibility of the mathematical apparatus of the group theory enables implementation of different "aspects of reconstruction" in computer variants and designing of adequate algorithms for image reconstruction. A high quality of the reconstructed images depends on the statistical redundancy due to the group-theoretical "focusing of information" inherent in the statistical group-theoretical approach.

The sphere of RCD application is rapidly extending. New methods enabled to solve a great number of problems, especially in NDT. The solution of other problems depends on the level of the developed software. These include monitoring of fuel elements in nuclear reactors, development of multi-purpose systems for machine vision, including those for "android" robotic systems and for underwater stereoscopic systems, etc.

Now we use these methods to solve the problems of image processing in complex diagnostic systems in cardiology for early detection of the symptoms of sudden cardiac deaths.

One of the prospective programs relates to a "man-machine" system. It is the implementation of new methods to improve the dialogue between a human operator and technical components of the system. Thus, a group-theoretical statistical approach to reconstruction intended for NDT needs is no longer limited to this field. Apparently, this approach will be widely used in various multidisciplinary studies.

REFERENCES

[1] R. Gonzalez and R. Woods, Digital image processing (Transl. from English), Moscow, 2006, p. 1072.

[2] W. Pratt, Digital image processing, PIKS Inside, 3rd ed., New York: Wiley, 2001.

[3] R. Duda, P. Hart and D. Stork, Pattern Classification, 2nd ed., New York: Wiley, 2001.

[4] R. Duda and P. Hart, Pattern recognition of images and scene analysis. Transl. from English., Moscow: Mir, 1976, p. 511.

[5] A. Webb, Statistical pattern recognition, Chichester: Wiley, 2002.

[6] P. Soille, Morphological Image Analysys. Principles and Applications., 2nd ed., Berlin: Springer, 2000.

[7] S. E. Umbaugh, Computer Vision and Image Processing. A Practical Approach Using CVIP Tools., Upper Saddle River, New Jersey: Prentice Hall PTR, 1998, p. 504.

[8] H. Samet, The Design and Analysis of Spatial Data Structures, Reading, Massachusetts: Addison-Wesley, 1990.

[9] J. Ohser and F. Mucklich, Statistical Analysis of Microstructures in Material Science, Chicester: Wiley, 2000.

[10] C. Landron, O. Bouaziz, E. Maire and J. Adrien, "Characterization and modeling of void nucleation by interface decohesion in dual phase steels," *Scripta Materiala*, vol. 63, no. 10, pp. 973-976, 2010.

[11] S. A. Drury, Image Interpretation in Geology, London: Chapman & Hall, 1993, p. 517.

[12] I. R. Shott, Remote Sensing. The Image Chain Approach, 1 ed., New York: Oxford University Press, 1997.

[13] R. N. Stewart, Methods of Satellite Oceanography, Berkely: University of California Press, 1985.

[14] M. Y. Lyudaev, Application of adaptive filtering algorithm for processing aerial photographs (in Russian), vol. 7, Elektrosvyaz, 2009, p. 1513.

[15] G. Wolberg, Digital Image Warping // IEEE Computer Society, Los Alamos, 1990, Los Alamos: IEEE Computer Society press, 1990, p. 334.

[16] C. Broit, Optimal registration of deformed images. Ph.D. dissertation, Philadelphia, Pennsylvania: Univ. of Pennsylvania, 1981.

[17] K. Blatter, Wavelet analysis. Fundamentals of the theory (Transl. from Deutch), Moscow: Technosphere, 2004, p. 280.

[18] S. F. Gull and G. T. Daniell, "Image reconstruction from incomplete and noisy data," *Nature* , no. 272, pp. 686-690, 20 April 1978.

[19] A. N. Tikhonov, V. Y. Arsenin and A. A. Timonov, Mathematical problems of computed tomography (in Russian), Moscow: Nauka, 1987.

[20] F. Natterer, Mathematical aspects of computed tomography (in Russian), Moscow: Mir, 1990, p. 228.

[21] A. K. Luis and F. Natterer, "Mathematical problems of reconstructive computational tomography (in Russian)," *TIEER*, vol. 71, no. 3, pp. 125-147, 1983.

[22] A. N. Tikhonov, "Mathematical model (in Russian)," in *Mathematical encyclopedic dictionary*, Moscow, Great Russian Encyclopedia Publishing House, 1995, pp. 343-344.

[23] V. A. Baranov and U. Ewert, "Methods of Statistical Spatial Filtering of Images based on Local Group of

Transformations," *Russian Journal of Nondestructive Testing*, vol. 48, no. 2, pp. 123-128, 2012.

[24] V. A. Baranov and U. Ewert, "Symmetrical Aspects of the Causality Principle in Statistical Group-Theoretical Image-Reconstruction Methods," *Russian Journal of Nondestructive Testing*, vol. 48, no. 3, pp. 187-190, 2012.

[25] V. A. Baranov and U. Evert, "Group-theoretical statistical approach to the recognition and reconstruction of "semantic structures" in test-objects (in Russian)," *Control, Diagnostics*, no. 13, pp. 127-133, 2013.

[26] F. Klein, Lectures on the development of mathematics in the XIX century (in Russian), vol. 1, Moscow: Nauka, 1989, p. 456.

[27] F. Klein, "Das Erlanger Programm," *Ostwalds Klassiker der exakten Wissen-schaften*, no. 253, p. 84, 1974.

[28] A. V. Shubnikov and V. A. Koptsik, Symmetry in science and art (in Russian), Moscow: Nauka, 1972, p. 339.

[29] L. Brillouin, Science and Information Theory (in Russian)., Moscow: Fizmatgiz, 1960.

[30] R. L. Stratonovich, Theory of Information (in Russian)., Moscow: Sov. radio, 1975.

[31] B. L. Van der Waerden, Method of Group Theory in Quantum Mechanics (in Russian)., Moscow: LKI Publishing Hous, 2007, p. 200.

[32] J. A. Rice, Mathematical Statistics and Data analysis, Belmon, CA: Duxbury Press, 1995.

[33] B. A. Duburovin, S. P. Novikov and A. T. Fomenko, Modern Geometry. Methods and Applications (in Russian), Moscow: Nauka , 1986.

[34] V. A. Baranov, "Computerized Tomography," in *Variational Approach to Non-Linear Backprojection*, Novosibirsk , 1995.

[35] L. G. Gubin, B. T. Polyak and H. V. Raik, "The method of projections for finding common points of convex sets (in Russian)," *USSR Computational Mathematics and Mathematical Physics*, vol. 7, p. 1, 1967.

[36] V. A. Baranov, «International Symposium on Computerized Tomography for Industrial Applications,» в *Convex projections reconstruction on the basis of non-linear backprojection techniques*, Berlin, 1994.