A Compliant Mechanism for Aerostatic Thrust Bearings Controlled by Piezo-actuators

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Abstract: Because of their remarkable performance, the use of compliant mechanisms has also been extended to the field of aerostatic bearings. This paper presents the design, modelling and experimental validation of a compliant mechanism used for a piezoelectric tool actuator to actively control an aerostatic thrust bearing. The proposed design procedure consists in a topological analysis of the mechanism, so as to define the kinematic functionality of the mechanism, a quasi-static analysis to tune its functional stiffness, and a modal analysis of the mechanism to identify its mode shapes. Experimental validation tests were performed to verify both the features of the compliant mechanism and the functionality of the system.

Keywords: Design, compliant mechanism, flexure hinges, gas bearings, air bearings, active control.

NOMENCLATURE

\( A \) \([m^2]\) Cross-sectional area
\( A_r \) [-] Elongation at failure
\( C_{U,Fj} \) \([m/N]\) Generic compliant coefficient
\( C_{Hi} \) Compliance matrix of the \( i^{th} \) flexure hinge
\( C_{Beam,i} \) Compliance matrix of the \( i^{th} \) beam
\( E \) \([Pa]\) Young’s modulus
\( F_j \) \([N]\) Generic force
\( F_{pzt} \) \([N]\) Piezo-actuator force
\( F_0 \) \([N]\) Piezo-actuator blocking force
\( F_{0,pzt} \) \([N]\) PZT blocking force
\( h \) \([m]\) Generic height
\( H \) \([m]\) Controlled (or working) height
\( HB \) \([Pa]\) Brinell hardness
\( I \) \([m^3]\) Cross-sectional moment of inertia
\( K_{in/out} \) \([N/m]\) Mechanism functional stiffness
\( K_{pzt} \) \([N/m]\) Piezo-actuator axial stiffness
\( K_{i,a} \) [-] Axial stress concentration factor
\( K_{i,b} \) [-] Bending stress concentration factor
\( K_{6x6} \) Local stiffness matrix
\( K_{15x15} \) Global stiffness matrix
\( \Delta K \% \) [%] Functional stiffness percentage difference
\( k_l \) \([N/m]\) Load stiffness
\( l, L \) \([m]\) Generic length
\( L \) \([m]\) Maximum bearing pad length
\( \Delta L \) \([m]\) PZT stroke variation
\( \Delta L_0 \) \([m]\) PZT stroke at nominal free condition
\( \Delta L_{max} \) \([m]\) PZT maximum stroke
\( M_{vz} \) \([Nm]\) Reaction Torque
\( M_z \) \([Nm]\) Generic torque
\( M_{6x6} \) Local mass matrix
\( M_{15x15} \) Global mass matrix
\( P_s \) \([Pa]\) Supply pressure
\( R_{p,0.2} \) \([Pa]\) Theoretical elastic limit of the material
\( r \) \([m]\) Flexure hinge radius
\( R_{Hi} \) Rotational matrix
\( t \) \([m]\) Height of the flexure hinge minimum section

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\[
[T_{F, H_i}] = \text{Force transposition matrix}
\]
\[
[T_{F, H_i}] = \text{is the product of } [R_{H_i}] \text{ and } [T_{F, H_i}]
\]
\[
U_i \quad [m] \quad \text{Scalar Displacement component}
\]
\[
\{U_P\} \quad [m] \quad \text{Displacement vector of the } i^{th} \text{ beam at the point } P
\]
\[
\{U_{P, H_i}\} \quad [m] \quad \text{Displacement vector contribution related to } i^{th} \text{ flexure hinge at the point } P
\]
\[
V_x \quad \text{Vertical reaction force}
\]
\[
V_{pzt} \quad [V] \quad \text{Input voltage applied to the piezo-actuator}
\]
\[
w \quad [m] \quad \text{Flexure hinge thickness}
\]
\[
Z \quad [m] \quad \text{Air pad vertical dimension}
\]
\[
v \quad [-] \quad \text{Poisson’s coefficient}
\]
\[
\sigma \quad [Pa] \quad \text{Normal stress}
\]
\[
\sigma_{adm.} \quad [Pa] \quad \text{Admissible normal stress}
\]
\[
\sigma_r \quad [Pa] \quad \text{Ultimate stress}
\]
\[
\epsilon \quad [-] \quad \text{Strain}
\]
\[
\varphi \quad [rad] \quad \text{Beam orientation}
\]
\[
\rho \quad [kg/m^3] \quad \text{Material density}
\]
\[
\theta \quad [rad] \quad \text{Infinitesimal angular rotation}
\]
\[
\omega \quad [Hz] \quad \text{Frequency}
\]

INTRODUCTION

Mechanisms are mechanical devices consisting of rigid members connected through joints used to transform the energy provided by one or more input devices to output motions [1], [2]. However, this kind of system possesses substantial limitations regarding friction and wear, which significantly compromise their use for small scale application where these phenomena are considerable. Paros [3] was the first who proposed the use of elastic members, known as flexure hinges or simply flexures, to extend the use of mechanisms to small scale applications. This insight made it possible to mitigate limitations such as friction, backlash, wear, weight savings and losses which affect the traditional joints, thus extending the use of mechanisms to small scale applications. Nowadays, this methodology has been further studied and enhanced leading to the current use of compliant mechanisms which, unlike the conventional ones, are usually monolithic structures exploiting the input force from actuators to achieve frictionless output motions [4]. Compliant mechanisms have been and frequently are employed in a wide range of high precision applications, e.g., electrostatic suspensions [5], precision CNC turning centres [6], micro-robots [7], micro-grippers [8] and surgical tools [9]. The current state of the art shows that coupling compliant mechanisms and piezoelectric actuators represents a consolidated method, especially at the micro- and nano-scales, for achieving very accurate positioning servosystems [10]. Because of this remarkable performance, these servo-mechanisms have also been used to enhance both the static and dynamic features of aerostatic bearings [11]. They can be adopted both in linear [12], [13] and rotative bearings [14], [15] to actively modify the pressure of the air gaps, thus achieving significant performance enhancements. Shimokohbe et al. [16], [17], [18] proposed an active air journal bearing for ultra-precision applications characterised by infinite radial stiffness and high damping capability. Al-Bender et al. [19], [20], [21] designed an active bearing surface whose deflection is controlled by three piezoelectric actuators which permit producing variations in the air gap pressure. Matsumoto et al. [22–23] proposed a similar type of active geometrical compensation [11], which is known as support compensation [21], where the strokes of the piezoelectric actuators are exploited to modify the thickness of the bearing in order to compensate for variations in the height of the air gap.

This paper presents the design, modelling, and experimental validation of a similar mechanism, which is driven by a multilayer stacked piezo-actuator and was proposed in [24–26]. Unlike the other cited compliant structures, the ease of integration with the controlled bearing, as well as the structure’s simplicity, are distinctive feature of this solution. The design of this device is described starting from its specifications by explaining and pointing out all the crucial aspects of the proposed procedure.

SPECIFICATIONS AND GOALS

Figures 1(a) and 1(b) show a sketch and a picture of the considered active bearing with the designed compliant mechanism. As can be seen, this bearing consists of a conventional aerostatic thrust bearing 1 (designed by MAGER®) which has been integrated with a multilayer stack piezo-actuator 2 (PI®P-888.31 PICMA Multilayer Piezo-actuator) with the aid of the designed compliant mechanism 3 and two connection elements 4. The configuration of this actively controlled bearing has been conceived for the use of a compensation method which goes under the name of the support compensation method [22–23]. This mechanism has the task of suitably preloading the actuator and constraining its stroke along the vertical direction.
As discussed above, the ease of integration and simplicity of the structure are the distinctive features of this kind of servo-mechanism. Figure 2 shows a scheme useful to grasp the operating principle of the actively compensated aerostatic thrust bearing (ATB). $h$ and $Z$ are the air gap height and ATB vertical dimension, respectively, while $H$ is the ATB controlled height ($H = h + Z$). The external load is designated by $F$. The main goal that the system has to achieve is to preserve the initial value of the controlled height $H$ even in the presence of external disturbances, e.g., load variations. For example, when the force $F$ increases (decreases) by $dF$, with a constant supply pressure $P_s$, $H$ also decreases (increases) by $dH$ as a result of structural deformations and the reduction of the air gap. The initial value of the controlled height $H$ is restored through a variation of the PZT stroke depending on the driving voltage, which is supplied at the actuator terminals. This driving voltage is opportunely imposed by a PI controller depending on the present positional error $dH$, which is computed by comparing the actual value of the ATB controlled height $H$ to the optimal one $^1$. The first requirement taken into account at the beginning of the design was the size of the mechanism, which must not exceed the dimensions of the bearing surface (60x30 mm$^2$) to guarantee a correct integration. The other specifications are imposed on the bearing’s capacity of compensation and the choice of a suitable safety factor allowing for only elastic deflections of the mechanism when the actuator modifies its stroke. The compliant mechanism was designed to have a capacity of compensation of about $\pm 5 \mu m$ along the vertical direction, i.e., the mechanism must provide a range of vertical displacements of $\pm 5 \mu m$ as output motion. The mechanism material is the 16NiCr11 steel ($E = 210$ GPa, $\nu = 0.3, A_r = 15\%$, HB = 339–409, $\sigma_r = 1380$ MPa, $R_p,0.2 = 882$ MPa).

Concerning the load and boundary conditions, the mechanism was designed to work in a clamped–clamped configuration with vertical forces applied in the middle of its length. The adopted design procedure comprised the following steps:

1. a topological (or kinematic analysis);
2. a quasi-static analysis;
3. a modal analysis and
4. an experimental verification.

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$^1$ The value at which the conventional bearing exhibits its highest performance
The topological (or kinematic analysis) was necessary to establish the kinematic functionality of the mechanism, thereby providing the desired output motion as a consequence of the application of a defined input force. The quasi-static analysis was used to determine the functional stiffness of the compliant mechanism and refines the size of the flexure hinges in dependence on its stress. The functional stiffness of the mechanism, as described further, was fundamental to obtain the proper integration with the piezo-actuator, thereby fulfilling the desired specifications. Finally, the modal analysis of the mechanism made it possible to identify the main mode shapes of the system, whose knowledge is essential in the presence of input forces with high frequencies. The results from these analyses were verified with the help of experimental tests.

THE COMPLIANT MECHANISM DESIGN

Topological Synthesis (Kinematic Analysis) and Preliminary Calculations

A topological synthesis (or kinematic analysis) is the first step in designing compliant mechanisms. This kind of study is necessary to define the kinematic functionality of the mechanism, depending on the desired output motion and the prescribed input force. Figure 3 shows a sketch of the preliminary rigid body mechanism. As specified in Section 2, the aim of the kinematic analysis was to define a mechanism configuration which allows only pure vertical translations of the beam \( AB \) as output motions. In this case, it is straightforward to select the correct geometrical (\( l_1 = l_3 \)) and kinematic conditions (\( \theta_1^0 = \theta_2^0 = \theta_3^0 = 0 \)) which make it possible to fulfill the desired specifications.

Quasi-Static Analysis

As said before, the quasi-static analysis is necessary to determine the presence of critical stresses inside the mechanism and assess its functional stiffness \( K_{in/out} \). This latter is defined as the ratio between the input force \( F_{in} \) and the output displacement \( U_{out} \). The quasi-static analyses were performed both through an analytical model and an FE model. The compliance based matrix method (CBMM) [27] and the software ANSYS®14 APDL were employed to implement the analytical and FE models of the proposed compliant mechanism. Figure 4(b) shows the geometry of the compliant mechanism by indicating the dimensions, applied forces, constraints, and displacements which were employed for this kind of analysis. As the mechanism presents a transverse symmetry axis for the load, constraints, and geometry, the quasi-static analysis was simplified by considering only one-half of the system and imposing the corresponding symmetry constraints.

Analytical Model Using the CBMM

The CBMM [27] is used to exploit the superposition principle and the theory of elasticity to assess the motion and the functional stiffness of the compliant structures. This method considers compliant mechanisms as chains consisting of (flexible) flexures interposed between rigid or compliant links and computes the global displacement of a point of interest as the sum of the displacement contributions related to each flexure hinge deflection. It is possible to distinguish three different kinds of reference frames. The first one (\( OXY \)) is unique, and is a fixed and absolute reference frame, unlike this, the other two are relative to the flexures and are their centres of rotation (\( H_i \)). The first (\( H_i \)) has its axes parallel to the absolute reference frame, whereas the second one is rotated by the angle \( \phi_i \). A load \( P_j \) applied to the \( i^{th} \) beam at the point \( j^{th} \) deforms all the flexure hinges and the beams situated between the \( j^{th} \) point and the fixed end of the compliance chain. A global displacement at the \( k^{th} \) point of interest \( P_{k} \), belonging to the \( i^{th} \) beam, is computed as the sum of each flexure and beam deflection contribution.

\[
\begin{align*}
\{U_k\} &= \{U_{1,Hi}\} + \{U_{2,Hi}\} + \ldots + \{U_{P,Hi}\} + \ldots + \{U_{P,Hi}\} \\
\end{align*}
\]

(1)

The partial displacement contribution of a flexure hinge \( U_{P,Hi} \) at a generic point of interest \( P_{k} \) is computed considering this flexure as fixed at its first end and all other members (both flexure and beams) as an unique fictitious beam. The relation between the load and displacement at a generic hinge is expressed by the compliance matrix \( [C_{Hi}] \) [28]:

\[
\begin{bmatrix}
U_{x,Hi} \\
U_{y,Hi} \\
\theta_{z,Hi}
\end{bmatrix} =
\begin{bmatrix}
C_{Ux,Fx} & 0 & 0 \\
0 & C_{Uy,Fy} & -C_{Uz,Mz} \\
0 & -C_{\theta z,Fx} & C_{\theta z,Mz}
\end{bmatrix}
\begin{bmatrix}
F_{x,Hi} \\
F_{y,Hi} \\
M_{z,Hi}
\end{bmatrix}
\]

(2)

where the coefficients \( C_{\theta z,Fx} \) and \( C_{\theta z,Fy} \) are equal, on the basis of the reciprocity principles [29], and the minus sign
in front of them indicates that $F^j_{y,H_i}$ and $M^j_{z,H_i}$ generate the opposite end deflection $U^j_{y,H_i}$. The expressions for the $[C^i_{H_i}]$ are given in [30]. It is straightforward that in the presence of more load points, the partial displacements related to the $i^{th}$ hinge are computed by exploiting the linear superposition principle.

$$\{U^i_{P,H_i}\} = [C^i_{H_i}] \sum_{j=1}^{l} [\hat{T}^f_{Fj,H_i}] \{F^j_{Fj}\}$$

Therefore, the previous expression can be rearranged as follows:

$$\{U^i_{P,H_i}\} = [C^i_{H_i}] \sum_{j=1}^{l} [\hat{T}^f_{Fj,H_i}] \{F^j_{Fj}\}$$

To obtain the displacement of the point of interest $U^i_{P}$, Equation 5 has to be modified further, by expressing $U^i_{P}$ as a function of the global displacement of the considered point of interest $U^i_{P}$, pre-multiplying by the matrices $[R_{H_i}]$ and $[T^u_{Fj,H_i}]$.

$$[T^u_{F,Hi}] = \begin{bmatrix} 1 & 0 & -\Delta y_{H_i,F} \\ 0 & 1 & \Delta x_{H_i,F} \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, Equation 5 becomes

$$\sum_{i=1}^{n} [R^i_{H_i}] [T^u_{P,H_i}] \{U^i_{P}\} = [C^i_{H_i}] \sum_{j=1}^{l} [R_{H_i}] [T^f_{Fj,H_i}] \{F^i_{Fj}\}$$

from which the displacement sought can be obtained as

$$\{U^i_{P}\} = \sum_{i=1}^{n} \sum_{j=1}^{l} [T^u_{P,H_i}]^{-1} [R^i_{H_i}]^{-1} [C^i_{H_i}] [R_{H_i}] [T^f_{Fj,H_i}] \cdot \{F^i_{Fj}\}$$
Application of the CBMM  In this case, this method was applied by considering the longitudinal symmetry (both on geometry and loads) of the compliant mechanism (see Figure 4(b)) by using the corresponding boundary conditions 3. The main goal of this step was to compute the input–output relation between the external force from the piezo-actuator at the point $F$ and the related vertical displacement $UyF$. The global displacement $UyF$ was evaluated as the sum of $UyF,H1$ and $UyF,H2$, which are the displacement contributions related to the flexures $H1$ and $H2$ by using the procedure described above. The final expression for $\{U_F\}$ is

$$\{U_F\} = \left[\begin{array}{c} C_{FH1} \\ C_{beam1} \\ C_{FH2} \end{array} \right] \cdot \left[\begin{array}{c} F \\ V \end{array} \right]$$

To solve Equation 9, firstly, the unknown reactions $V_x$ and $M_{V_z}$ were computed and, secondly, the global displacement $\{UyF\}$ was obtained by considering $V_x$ and $M_{V_z}$ as external forces. When $F_{PZT}$ is equal to 1 N, the solution of 9 leads to

$$\{V\}^T = \left[\begin{array}{c} 0 \\ 0 \end{array} \right] \cdot 0.013 \cdot Nm$$

In fact, in order to optimise the automatic meshing process, a manual pre-mesh was made. The external load from the piezo-actuator was modelled as a concentrated force applied at the middle of the central beam, as illustrated in Figure 4(b). The screw connections for integrating the mechanism with the bearing were modelled by imposing null displacements at the lower surfaces of the beams and at the end of the mechanism (see Figure 4(b)). Figure 5 shows the deformed shape and the field of stress of the mechanism when the external load is 1 N. Moreover, a further investigation was made about the use of a mesh refinement around the flexure centre of rotations (see Figure 6). Figure 6 shows the values of the functional stiffness computed as functions of the number of elements used in the FE model. These values were computed both by employing uniform (red line) and refined meshes (blue line). As expected, the model with the refined mesh turned out to be less stiff than the other one due to the higher compliance of its flexures (which had a higher number of DOFs). As shown in Figure 6, the stiffness discrepancy between the two models is considerable for a small number of elements, e.g., $\Delta K_{rel}=8.92\%$ with 3000 elements, and it reduces as the number of elements increases. The selected threshold was selected to be around 20,000 elements ($\Delta K_{rel}=1.28\%$) since increasing further the number of elements was almost ineffective ($\Delta K_{rel}=0.45\%$ with 40,000 elements). The final estimated functional stiffness was 4.86 N/$\mu$m.

FE Model

The FE model of the compliant mechanism was implemented by using ANSYS® 14 APDL, where the compliant mechanism was modelled as a 2D structure composed of quadrangular elements (PLANE82). This kind of element is an 8-node element having two degrees of freedom at each node: translations in the nodal $x$ and $y$ directions [31]. This choice was made since this type of element makes it possible to take into account the specimen’s thickness when the plane stress option is activated. In this case, this property was crucial for obtaining an accurate model since, due to the dimensions of the mechanism, both the plane stress and plane strain approximations can fail. The compliant mechanism was discretised through a semi-automatic mapped mesh with quadratic elements and mesh refinements were performed close to the centre of rotation of the flexure hinges to increase the accuracy of the model. As discussed above, the quasi-static analysis of the mechanism is useful in order to define the mechanical coupling with piezoelectric actuators by appropriately tuning the functional stiffness of the mechanism. The relation between the PZT supply voltage $V_{PZT}$ and its stroke was experimentally evaluated (see Figure 7). The PZT’s free characteristic behaviour relation was investigated through a test where the actuator was placed in a vertical frictionless prismatic guide (see Figure 7(a)) and the input voltage was cycled from 0 to 120 V. The PZT’s maximum stroke and hysteresis were evaluated both in free and encased conditions (see Figure 7(b)). As can be seen, in encased conditions the PZT’s stroke is reduced compared to the free condition because of the pre-load by the compliant mechanism. Figure 8 shows the generic piezo-actuator’s characteristic curves. These curves show the trend of the actuator’s stroke as a function of the respective generated pulling force. As is evident, an increase of the generated force is

3 Symmetry and asymmetry boundary conditions [29]
always coupled with a reduction in the displacement, up to
the maximum force (blocking force $F_0$), when the actuator
stroke drops to zero. Different kinds of external loads can
be applied to piezo-actuators and they result in different
graphical representations. For this case, it is necessary to
consider the simultaneous presence of constant (masses)
and variable (springs) loads. When static loads are applied
to the actuator (see Figure 8(a)), in the presence of a con-
stant supply voltage, the piezo-actuators reduce their stroke
to $\Delta L_{\text{mass}}$ depending on the stiffness (see Equation 12).

$$\Delta L_{\text{mass}} = \frac{mg}{k_{\text{pzt}}}$$  \hspace{1cm} (12)

where $m$ is the value of the mass applied to the actuator
and $g$ is the acceleration due to gravity. Otherwise, when
springs load the actuator, the consequent stroke reduction is
higher when the spring is stiffer. In the case of a load-spring,
it is necessary to distinguish two different possibilities on
the basis of the stiffness of the adopted spring. When large
strokes are requested, soft springs should be adopted and
the resulting maximum stroke can be computed as follows:

$$\Delta L \simeq \Delta L_0 \left( \frac{k_{\text{pzt}}}{k_{\text{pzt}} + k_t} \right)$$  \hspace{1cm} (13)

Figure 8(b) shows the working curve of the actuator–
spring system, which has the slope $F_{\text{eff}}/\Delta L$, after the
application of the compliance constraints, and corresponds
to the stiffness $k_t$. When large forces have to be generated,
the load stiffness $k_t$ must be greater than that of the
actuator $k_{\text{pzt}}$ [32]. Considering the piezo-actuator’s
static features ($k_{\text{pzt}} = 267 \text{ N}/\mu\text{m}$, $F_0 = 3500 \text{ N}$ and
$\Delta L_{\text{max}} = 13 \pm 20% \mu\text{m}$) and that the functional stiffness
of the designed compliant mechanism ($k_t$) is about $5 \text{ N}/\mu\text{m},

the mechanism–actuator coupling leading to a stroke reduc-
dation is about $0.3 \mu\text{m}$ fulfilling the initial specifications (see Section ).

**Modal Analysis**

A modal analysis of the compliant mechanism was carried
out in order to identify the system’s natural frequencies and
the related mode shapes, since their knowledge is crucial in
the presence of input forces provided at high frequencies.
The modal analysis was performed both using the FE model
described above and a lumped parameter model.

**Lumped Parameter Model**

Figure 9 shows the scheme of one-half of the lumped
parameter model of the compliant mechanism. As can be
seen, it comprises two flexure hinges ($H_k$ for $k = 1, 2$).
FIGURE 7. Piezoelectric actuator characterization.

(a) PZT static characterization test setup (T: stroke transducer).  
(b) PZT stroke versus driving voltage.

FIGURE 8. Load case with spring and constant loads: working graph with working curve.

(a) Load case with constant load: working graph with working curve.  
(b) Load case with spring preload: working graph with working curve.

FIGURE 9. Scheme of an half of the 15 DOFs model.
and two short beams \((B_j \text{ for } j = 1, 2.)\) which were both considered as beams with three DOFs for each node (the translations \(U_X\) and \(U_Y\) along the \(X\) and \(Y\) directions and the rotation \(\theta_Z\) around the \(Z\) axis). The model of the mechanism was simplified by considering its symmetry with regard to the geometry and boundary conditions. The model has 15 DOFs. It is clamped on the right end and is constrained by a double pendulum on its left end. This model comprises four beams (two flexure hinges and two beams) characterised by classic mass and stiffness matrices [33]: The flexure hinges were modelled as beams with a constant rectangular cross-section whose height is equal to the minimum cross section of the flexure hinge \(t_k = h_k\). The global mass \([M_g]_{15x15}\) and stiffness \([K_g]_{15x15}\) matrices of the system were assembled by using a classical mapping procedure [34]. After mapping, the final dynamic governing equation of the mechanism, written in a compact way, is

\[
[M_g]_{15x15}\{\ddot{x}\} + [K_g]_{15x15}\{x\} = \{0\} \quad (14)
\]

which can be easily solved by finding the eigenvalues and the eigenvectors. The first computed natural frequency of the mechanism was about 2701 Hz.

**FE Model**

The main results obtained from the modal analysis of the FE model are presented in Table 1 along with the first 5 natural frequencies of the mechanism, their ratios of effective masses to the total mass, and their related mode shape types.

The effective modal mass provides a method for judging the relevance and the direction of a vibrational mode when a system is excited. Modes with relatively high effective masses along a certain direction can be readily excited and provide significant contributions to the system’s motion. On the contrary, modes with low effective masses do not provide important contributions to the system’s motion because they are only slightly affected by external excitations. Figure 10 shows the deformed shapes of the first 5 mode shapes of the mechanism.

### Table 1.

Table showing info about the first five mode shapes of the compliant mechanism; the Modal mass X, Y and Z are the ratio of the effective mass to the total mass of the mechanism.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [Hz]</th>
<th>Modal mass X</th>
<th>Modal mass Y</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2302</td>
<td>0.2611E-05</td>
<td>0.5156</td>
<td>Flex.</td>
</tr>
<tr>
<td>2</td>
<td>5620</td>
<td>0.2668E-03</td>
<td>0.1574E-06</td>
<td>Flex.</td>
</tr>
<tr>
<td>3</td>
<td>21797</td>
<td>0.7632</td>
<td>0.3118E-02</td>
<td>Axial.</td>
</tr>
<tr>
<td>4</td>
<td>30910</td>
<td>0.2366E-01</td>
<td>0.1104</td>
<td>Flex.</td>
</tr>
<tr>
<td>5</td>
<td>34932</td>
<td>0.3454E-01</td>
<td>0.3463E-02</td>
<td>Flex.</td>
</tr>
</tbody>
</table>

| sum  | 0.8217         | 0.6326       |              |      |

**Experimental Verification and Comparison**

The numerical and analytical models described before were verified through different types of experimental tests. The first step was to verify the mechanism’s geometrical dimensions with the aid of an electron microscope. The experimental static characterization and the modal testing of the compliant mechanism were the second and third validation tests.

**Verification of the Mechanism’s Geometry**

This verification was useful to assess the difference between the nominal and real dimensions of the designed mechanism. These values were successively used to adjust
Table 2. Final geometrical parameters of the mechanism (see Figure 4(b)).

<table>
<thead>
<tr>
<th>Final Geometrical Parameters [mm]</th>
<th>h</th>
<th>r</th>
<th>i</th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>ø</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9.78</td>
<td>1.525</td>
<td>0.955</td>
<td>8.2</td>
<td>7.8</td>
<td>13.47</td>
<td>7.82</td>
<td>8.41</td>
<td>16</td>
</tr>
</tbody>
</table>

the input parameters to the numerical (FE) and analytical (CBMM) models of the designed compliant mechanism. In this way, it was possible to precisely evaluate the accuracy of the models by comparing their results to the experimental ones. In particular, the geometrical dimensions measured through the verification were the height \( h_j \) and the length \( x_j \) of the \( j \)th beam and (see Figure 4(a)) the height \( t_k \) and the radius \( r_k \) of the \( k \)th flexure hinge. Table 2 shows the final values which were used to evaluate the accuracy of the adopted models.

**Static Test**

A static compression test was carried out in order to identify the functional (or output [30]) stiffness of the mechanism, defined as the ratio of the input force applied to the mechanism to the output displacement at the point of interest. Figure 11(a) shows a photograph of the compression test set-up. In this test, the compliant mechanism was integrated with the aerostatic bearing without the presence of the piezo-actuator. The external force applied by the testing machine MTS® Q/Test 10 was measured by an embedded load cell\(^4\), whereas the mechanism’s deformation at the point of interest \( P \) was measured through an LVDT Dial Gauge: MAHR® 1318 (error limits: 0.2 \( \mu \)m or 0.3\% of the indicated probe value\(^5\)). The choice to use this kind of sensor was made due to the limited available space for positioning the sensor and the static nature of the test. The results from the compression test are plotted and compared to those obtained from the FE and CBM models in Figure 11(b) and Table 3 respectively.

**Modal Testing**

Figure 12(a) shows the experimental set-up of the modal testing carried out in order to verify the results from the numerical and analytical modal analyses by investigating the compliant mechanism’s natural frequencies. As can be seen, the mechanism was integrated with the aerostatic bearing through its screw connections without the presence of the piezo-actuator. Regarding the boundary conditions of this test, the bearing structure was fixed to a stationary part through a mechanical anchoring for reproducing the boundary conditions of the numerical and the analytical models. The system was excited through an indirect hammer excitation\(^6\) and the response was evaluated using an accelerometer which was fixed in the middle of the central crossbeam of the mechanism. Figure 12(b) shows the results of this modal testing. The obtained natural frequency was equal to 1962 Hz and was appropriately corrected by considering the additional and non-negligible mass of the accelerometer \( m_{acc.} = 5 \times 10^{-3} \text{kg} \) which is not negligible compared to that of the investigated mechanism \( m_{mech.} = 19 \times 10^{-3} \text{kg} \). This a posteriori correction was made by multiplying the experimental natural frequency \( \omega_{test} \) by the square root of the ratio between the mass considered in this test \( m_{test} = m_{mech.} + m_{acc.} = 24 \times 10^{-3} \text{kg} \) to the effective mass of the mechanism \( m_{mech.} = 19 \times 10^{-3} \text{kg} \), as shown in Equation 15

\[
\frac{\omega_{mech.}}{\omega_{test}} = \sqrt{\frac{k_{mech.}}{m_{mech.}}} \cdot \sqrt{\frac{m_{test}}{k_{test}}} = 1.124 \quad (15)
\]

Finally, the values of the corrected experimental natural frequency turned out to be equal to 2205 Hz. Table 4 compares the natural frequencies obtained from the experimental, numerical, and lumped modal analyses.

\(^4\) The certificate of calibration demonstrates that the load cell has a repeatability of 0.408 % for tensile loads and 0.469 % for compression loads, an accuracy of 0.116 % for tensile loads and -0.197 % for compression loads and a resolution of 0.204 % for tensile loads and 0.204 % for compression loads.

\(^5\) the larger of the two values in question is valid

\(^6\) To avoid the overload condition of the accelerometer (Bruel and Kjaer\(^\text{®} \) type 4508: reference sensitivity at 159.2 Hz, 20 ms\(^{-2}\) RMS) the input excitation was applied upon the test bench.

Table 3. Comparison of the experimental, analytical and numerical stiffness of the designed compliant mechanism.

<table>
<thead>
<tr>
<th></th>
<th>Stiffness [N/( \mu \text{m} )]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>4.65</td>
<td>–</td>
</tr>
<tr>
<td>Numerical (FEM)</td>
<td>4.86</td>
<td>4.32</td>
</tr>
<tr>
<td>Analytical (CBMM)</td>
<td>5.24</td>
<td>11.25</td>
</tr>
</tbody>
</table>

Table 4. Comparison of the experimental, numerical natural frequencies of the designed compliant mechanism.

<table>
<thead>
<tr>
<th></th>
<th>Natural Frequency [Hz]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>1962</td>
<td>–</td>
</tr>
<tr>
<td>Experimental corrected</td>
<td>2205</td>
<td>–</td>
</tr>
<tr>
<td>Numerical (FEM)</td>
<td>2302</td>
<td>4.21</td>
</tr>
<tr>
<td>Lumped (15 DOFs)</td>
<td>2701</td>
<td>18.36</td>
</tr>
</tbody>
</table>
CONCLUSIONS

Compliant mechanisms, thanks to their distinctive features, e.g., low friction, low wear, no backlash, and weight savings, are currently employed in a wide range of small scale high precision applications. Because of their remarkable performance, the use of compliant mechanisms has also been extended the field of aerostatic bearings [11]. This paper presents the design, modelling, and experimental validation of a compliant mechanism used for a piezoelectric tool actuator aimed to actively control an aerostatic thrust bearing through a support compensation methodology [11], [24]. The proposed design procedure consists in:

- a topological (or kinematic) analysis of the mechanism, for defining the mechanism’s kinematic functionality;
- a quasi-static analysis to tune the functional stiffness of the mechanism, thereby allowing a correct mechanical coupling with the piezo-actuator, and
- a modal analysis of the mechanism to identify the first natural frequency of the mechanism.

This description is enriched by useful theoretical explanations of the adopted numerical and lumped-parameter models and practical advice on the design issues encountered. The results of the experimental tests confirm the system’s functionality and the accuracy of the presented models of the compliant mechanism.

REFERENCES


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